Multi-objective methods for welding flux performance optimization

Več namenske metode za optimizacijo uspešnosti varilnega praška

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Abstract: The traditional welding flux development is by lengthy and costly trial and error experiments and the optimum welding flux formulation is not guaranteed. This paper presents discussions on promising multi-objective decision making (MODM) methods that can mitigate the limitations of the traditional approach to welding flux design. The methods are weighted-sum scalarization (WSS), desirability indices, goal programming and compromise programming. The steps a welding flux designer (WFD) may follow to determine the best compromise welding flux, welding flux design situations where each may be useful and trade-off explorations were mentioned. No attempt was made to determine the relative merits of the approaches because the usefulness of each depends on the welding flux design situation. The descriptions only serve as a guide for the WFD to decide which method best suits his needs.

Izvleček: Klasični razvoj varilnih praškov poteka z dolgotrajnimi in dragimi preizkušanji in odpravami napak. Pri takšnem načinu ni zagotovljena optimalna sestava varilnega praška. V članku so predstavljene več namenske metode odločanja (MODM), ki odpravijo nekatere omejitve tradicionalnega pristopa raziskav varilnega praška. Uporabljene metode so skalarizacija uteženih vsot (WSS), indeksi zaželjenosti, ciljno programiranje in kompromisno programiranje. Navedeni so koraki za zagotovitev naboljšega varilnega praška, ki naj bi jih sledil načrtovalec varilnega praška (WFD). Prav tako so omenjeni različni preiskani kompromisi za nekatere razmere pri načrtovanju varilnih praškov. Članek ni poskušal odgovoriti na vprašanje relativne vrednoti pristopov, ker je uporabnost vsakega odvisna od razmer za katere razvijamo varilni prašek. Opisane metode naj bi služile samo kot vodilo WFD, za izbiro najustreznejše metode za trenutne potrebe.

Key words: welding flux, weighted-sum scalarization, desirability indices, goal programming, compromise programming

Ključne besede: varilni prašek, skalarizacija uteženih vsot, indeksi zaželjenosti, ciljno programiranje, kompromisno programiranje

INTRODUCTION

Welding flux design like many real world problems involves multiple objectives which are more often than not conflicting. In the welding flux formulation problem, the welding flux designer (WFD) aims at developing a flux that will deposit weld-metal with the required quality characteristics and at the same time fulfil the operational and environmental requirements. Some of the frequently encountered objectives of WFDs depending on the type of metal, are to get a flux that will deposit weld-metal with maximum acicular ferrite content, maximum charpy impact toughness, maximum tensile strength, minimum diffusible hydrogen content, as well as minimum spatter, minimum fume, minimum toxic content of fume during welding, etc … The conflict of these objectives arises because improvement in one objective can only be made to the detriment of one or more of the other objectives. Because of the conflicting nature of the objectives, it is not feasible to get a flux formulation which optimizes all of them simultaneously. Therefore compromises and balances are often provided and designed into the flux.

The traditional method of achieving the compromises and balances is through tedious trial and error experiments. According to ADEYEYE & OYAWALE[1] the limitations of the traditional trial and error method are mainly due to the paucity of computational models that can be used for the prediction and optimization of flux properties. The traditional approach is time consuming and costly. Consequently, the lead-time for a new welding flux is usually long. The optimality of the flux so designed is difficult to ascertain because of the ever present trial and error and absence of quantitative means of testing optimality. The inability to identify and quantify the direct and interaction effects of the input variables such as levels of flux ingredients is another drawback of the traditional trial and error approach. With the obvious need to overcome these drawbacks, Kanjilal et al,[2-6] introduced a new approach which has great potential to revolutionize welding flux design technology. They used a design of experiment (DoE) method known as mixture design to layout the experiment. Data from the experiment were used to develop regression models that relate the input/predictor variables to the output/response variables. With their approach welding flux design can now be based on quantitative footing, direct and interaction effects of variables that determine the properties of welding flux can be identified and quantified and more insight gained. Some of the phenomena that hitherto could not be explained can now be explained in terms of synergetic or antagonistic interaction effects of input variables.

In a recent paper, Adeyeye & Oyawale[7] proposed a methodology in which the mixture design method used by Kanjilal et al.[2–6] was integrated with mathematical programming optimization technique. This new methodology extended the work of Kanjilal and co-investigators. The methodology was able to identify the optimum welding flux formulation and also assist the WFD to know either it was feasible to achieve desired flux performance level within the input space or not with minimal experimental efforts. However, their study was limited to a situation where the WFD is interested in a single objective. The multiple objectives welding flux design situation is encountered more frequently than the single objective case. The WFD therefore needs well tested and validated optimization tools that can handle multiple objectives and also assist him to explore various trade-off options. There are many optimization methods in multi-objective decision-making (MODM) area which could be used for this purpose. MODM is not new in other areas of arc welding technology but it appears relatively unknown to WFDs and as a result MODM applications in welding flux formulation is yet to be explored.[8–10] In this article, some of the MODM optimization methods are discussed and various welding flux design situations where they could be useful are presented.

Welding flux design problem

The design of welding flux that meets operational requirements, weld-metal quality requirements, environmental, manufacturability and storage requirements is far from trivial. Operational characteristics such as arc stability, deposition rate, slag control, etc … determine the productivity and cost of the welding process. Welding flux design therefore seeks to maximize the contribution of the welding flux to the society while minimizing its cost to the manufacturer, user and the environment. Each lifecycle stage of the flux is taken into consideration during the design stage. Health and safety of the welder and other workers at the welding environment are also important.

The flux is therefore expected to produce minimum fume, no or minimum noxious odours and minimum amount of toxic materials in the fume. Some of the commonly encountered requirements are presented in figure 1. Most of the requirements are bundles of other requirements and can be broken down to secondary and tertiary requirements. For instance, weld-metal quality depends on mechanical property, microstructure, bead morphology etc … all of which are also determined by other requirements (Figure 1). The requirements presented in Figure 1 are not exhaustive; depending on the situation more requirements may be added. The requirements the WFD selects for a particular flux depend on the welding method, the particular metal to be welded and the service requirement of the weldment.

These requirements are incompatible because it is not possible to improve one quality characteristic without decreasing the achievement or satisfaction of one or more of the other quality characteristics. The problem of flux design therefore, is that of determining the flux ingredients levels that will achieve the best compromise between the various requirements. Studies have shown that the types and levels of flux ingredients and process parameters are key factors that determine these requirements.[11–17] Application of optimization models to welding process parameter optimization has received much attention while so far application of such models to welding flux formulation is scanty in the literature.[8–10, 18, 19] As mentioned earlier, the traditional approach of achieving compromises and balances is by lengthy trial and error experiments. The flux so designed can not be guaranteed to be the best compromise flux because it is not possible to explore all possible combinations of flux levels because of cost and time limitations.

An integration of Kanjilal and co-workers method with the MODM approach will mitigate the problems of the WFD. As the WFD can not face testing all possible combinations of flux levels and measure the quality of resulting flux, a model capturing the relationship between each quality characteristics and flux levels should be assumed over the domain of interest through regression equations. The proven method a WFD may use to capture such relationship is the mixture experiment approach. The details of the mixture experiment approach abound in the literature.[20-27] Various model forms that may be used to fit regression models of the responses are presented in the paper of ADEYEYE & OYAWALE.[1] A key assumption is that each of the responses defining the quality of the flux is related to the same set of varying factors. The objective is to find factor setting that will achieve the best compromise flux formulation.

The specific steps a WFD may follow are presented as follows: (i) Determination of the welding process for which the flux will be used and its specific requirements. For instance, extrudability, strong and tough coating are not requirements for SAW where as they are very important requirements for SMAW. (ii) Determination of the type of metal the flux will be used for and its specific characteristics and requirements. (iii) Determination of the service requirement of the weldment. This will assist the WFD to establish the mechanical properties, weld-metal chemistry and metallurgical features which the welding flux should achieve when used to weld. (iv) Preparation of a list of requirements with the preferences of the WFD. Typical preferences of WFD may be: a welding flux that will maximize penetration, minimize diffusible hydrogen content and achieve a target of say 300ppm oxygen content in the weld-metal. (v) Laying out the experiment using the mixture experiment design procedure.[27–30] (vi) Performing the experiment as prescribed by the algorithm in step v above. (vii) Using the data from the experiment to develop response models that capture the relationship between each of the requirements and flux component levels over the domain of interest. [1] (viii) Using the appropriate MODM method that suits the particular welding flux design situation to find the factor setting that achieves the best compromises and balances.

Steps i to vii above have been addressed in the literature.[1–7, 18, 19] Step (viii) is our focus in this paper. Some of the common well tested and validated MODM methods the WFD can couple with the mixture experiment to achieve the best compromise welding flux formulation are discussed in the following section.

Description of various MODM approaches applicable to welding flux design

This section presents brief discussions of the most widely used MODM methods that can be integrated with the mixture experiment methodology to mitigate the problems associated with the traditional welding flux design approach. The methods are scalarization techniques, goal programming and compromise programming. These MODM methods were selected for discussion because they are suitable for welding flux design problems and also sufficiently flexible for incorporating the flux formulators preferences concerning the relative importance of each objective or quality characteristic.

- Flux requirement
  - Weld - metal requirement
    - Composition
      - *Chromium*
      - *Manganese*
      - *Carbon*
      - *Difussible – hydrogen*
    - Microstructure
      - Ferrite number
      - Acicular ferrite
      - Bainite e.t.c...
      - Inclusion volume fraction
      - Inclusion size
      - Inclusion number density
    - Mechanical property
      - Yield strength
      - Tensile strength
      - Charpy impact strength
      - Hardness
      - Elongation
    - Weld bead
      - Penetration
      - Reinforcement
      - Bead width
      - Dilution
  - Environmental requirement
    - Fume
      - Generation rate
      - Particle number
      - Mass distridution
    - Noxious odour
    - Toxic materials
      - Cr^{6+}
      - Mn
      - Ni, *etc..*
  - Manufacturability requirement
    - Extrudability
    - Good bonding
    - Uniform mixing
  - Storage requirement
    - Durability of coating
    - Moisture pick - up
    - Shelf - life
  - Operational requirement
    - Arc striking
    - Arc stability
    - Penetration control
    - Spatter
    - Slag control
    - Slag detachabilty
    - Deposition rate
    - Electrode overheating

Figure 1. Typical welding flux requirements

Notations

a : Achievement or satisfaction function

D : Overall, global or composite desirability index

I : Set of responses, quality characteristics or objectives

J : Set of terms in the response functions/equations

p : A topological metric i.e. a real number belonging to the closed interval $[0, \infty]$

Q : Set of priority levels

x : *n*-dimensional decision/predictor variables

C_s : Set of feasible constraints

d_i : Desirability of response i, for each $i \in I$:

DL_i : Absolute distance between the value of response i and its ideal value, for each $i \in I$

DL_{ni} : Normalized distance between the value of response i and its ideal value, for each $i \in I$

DL_p : Composite/overall distance function for all normalized distances for metric p

$f_i(x)$: Regression equation/function for response i, for each $i \in I$:

$f_{qi}(x)$: Regression equation/function for response i, at priority level q, for each $i \in I$ and $q \in Q$

$f_i^*(x)$: Best, ideal or anchor value for response i, for each $i \in I$

$f_i^{**}(x)$: Worst, anti-ideal or nadir value for response i, for each $i \in I$

L_i : Lower limit for the value of response i, for each $i \in I$:

$L_{n\infty}$: Largest distance for $p = \infty$

n_i : Negative deviation/underachievement for response i, for each $i \in I$:

n_{qi} : Negative deviation/underachievement for response i, at priority level q for each $i \in I$ and $q \in Q$

p_i : Positive deviation/overachievement of response i, for each $i \in I$:

p_{qi} : Positive deviation/overachievement for response i, at priority level q, for each $i \in I$ and $q \in Q$

s_i : Is an exponent chosen to reflect how rapidly the deviation from the target value of response i towards its lower limit becomes undesirable, for each $i \in I$

t_i : Is an exponent chosen to reflect how rapidly the deviation from the target value of response i becomes undesirable, for each $i \in I$

T_i : Target value/aspiration level for response i, for each $i \in I$:

U_i : Upper limit for response i, for each $i \in I$:

u_i : Weight assigned to the negative deviation of response i, for each $i \in I$:

u_{qi} : Weight assigned to the negative deviation of response i, for priority level q for each $i \in I$ and $q \in Q$

v_i : Weight assigned to the positive deviation of response i, for each $i \in I$:

v_{qi} : Weight assigned to the positive deviation of response i, for priority level q, for each $i \in I$ and $q \in Q$

w_i : Weight assigned to response/objective i, for each $i \in I$:

Z_q : Achievement/satisfaction function for priority level q, for each $q \in Q$

Z_q^* : Optimal value of the satisfaction/achievement function for priority level q, for each $q \in Q$

β_{ij} : Coefficient of term j in response function i, for each $j \in J$ and $i \in I$

γ_i : Is an exponent chosen to reflect how rapidly the deviation from the target value of response i towards its upper limit becomes undesirable, for each $i \in I$

Scalarization Techniques
We shall discuss two types of scalarization techniques, namely;

- Linear Aggregation/ Weighted Sum Scalarization (WSS)
- Nonlinear Aggregation (Desirability indices)

Linear aggregation/weighted sum scalarization (WSS)
The WSS approach consists of adding all the response equations together using a weighting coefficient, w_i for each response. The weighting coefficient denotes the relative importance of the responses. Since a minimizing objective can be converted to a maximizing objective by multiplying it by -1, the multi-objective optimization problem can be transformed into a single/combinational problem of the form below without any loss of generality.

$$\max imize, WSS = \sum_{i \in I} w_i f_i(x)$$

Subject to

$$x \in C_s \tag{1}$$

Where $w_i > 0, \forall i$ and $\sum_{i \in I} w_i = 1$

Consider a case where the WFD wants to decide the flux ingredient levels that will give the best compromise flux formulation given the following objectives;

a. Maximize acicular ferrite (AF) content, $f_1(x)$

b. Maximize charpy impact toughness, $f_2(x)$

c. Minimize diffusible hydrogen content, $f_3(x)$

Once the response equations have been established according to steps i to vii in section 2 above, the WSS may be used to achieve the desired flux component levels as follows;

Step 1: Convert the minimizing objective, $f_3(x)$ to maximizing objective by multiplying it by –1 (i.e. minimize $f_3(x) = \text{maximize} - f_3(x)$).

Step 2: Normalize the objectives. This is necessary because the objective/response functions have different units. For instance the unit of AF in the microstructure is in fractions (%), diffusible hydrogen is in mL per 100 g of weld-metal, while that of charpy impact toughness is in joules. In such cases the WFD must first convert all objectives into the same dimensions or dimensionless before combining them into one. Also the values of different functions or the coefficients of the terms in the functions may have different order of magnitude. Consider the hypothetical response/objective functions

$$f_1(x) = 0.5x_1 + 0.2x_2 + 0.8x_1x_2$$
$$f_2(x) = 16.5x_1 + 25.0x_2 + 20.8x_1x_2$$

Using the WSS approach without normalization may lead to a situation where $f_2(x)$ dominates $f_1(x)$. Therefore, if we want w_i to closely reflect the relative importance of the functions, all functions must be expressed in units of approximately the same numerical value. The objective functions may be converted to their normal forms as follows; [31, 32]

Normal form of:

$$f_i(x) = \left(w_i \Big/ \sqrt{\sum_{j \in J} \beta_{ij}^2} \right) f_i(x); \qquad (2)$$

for each $i \in I$ and $j \in J$

Step 3: Aggregate the objective functions into a single function as follows and add the structural constraints;

$$Maximize, WSS = \sum_{i \in I} \left(w_i \Big/ \sqrt{\sum_{j \in J} \beta_{ij}^2} \right) f_i(x)$$

Subject to; (3)

$x \in C_s$

Note that each minimizing objective must be converted to maximizing objective before combining them into one.

Step 4: Solve the resulting model using the appropriate software or algorithm.

The WSS method is suitable for flux design situations in which the WFD is interested in determining flux ingredient levels that maximizes desirable quality characteristics while at the same it minimizes undesirable characteristics. Trade-off options may be explored by the WFD by using various weight structures.

Nonlinear aggregation (desirability indices)

Instead of linear aggregation the WFD can use nonlinear aggregation methods such as computing the product of the objective functions values which is a modelling approach based on the theory of utility functions. A utility function assigns to each combination of values that may occur in the response space a scalar value- the so called utility. We discuss the commonest of the nonlinear aggregation method and how it can be applied in welding flux design. The desirability function (DF) approach is very common among the nonlinear aggregation methods. It was first proposed by Harrington[33] and further modified by Derringer and Suich[34] and Kim and Lin[35]. In the DF approach, the quality of a compromise/balance between the responses can be measured by the desirability concept. The adequacy of each of the responses, $f_i(x)$ are first quantified by a value d_i between 0 and 1. A desirability of zero (i.e. $d_i = 0$) represents a property level that is expected to render the welding flux unacceptable for use. A desirability of 1(i.e. $d_i = 1$) represents a property level at which the specifications of the WFD is perfectly satisfied. The procedure a WFD may follow to determine the factor setting that give the best compromise flux formulation are presented below:

Step 1: Transform each response $f_i(x)$ to the same scale using a desirability function denoted by d_i, such that $d_i \in [0,1]$. If $d_i = 0$, the welding flux is not at all acceptable according to the specifications of i[th] response and if $d_i = 1$, the welding flux satisfies the specifications completely. There are many forms of desirability function which the WFD may use depending on the goal of optimization. Generally, the goal of optimization is to maximize desirable responses, minimize undesirable responses and hit the target level of some. Derringer and Suich[34] desirability functions are the most widely used and are presented below.

(i) The Larger-the-best (LTB) Case: In the LTB case the WFD is interested in maximizing the response. For instance, studies have shown that the larger the amount of AF in the microstructure the better for low alloy C-Mn steels, deep penetration is also desirable, etc … For such cases the individual desirability function is given by;

$$d_i = \begin{cases} 0, & f_i(x) < L_i \\ \left(\dfrac{f_i(x) - L_i}{T_i - L_i} \right)^{t_i}, & L_i \le f_i(x) \le T_i\,, \\ 1, & f_i(x) > T_i \end{cases}$$

for each $i \in I$ (4)

With T_i in this case denoting large enough value for the i[th] response. That is a property level at which a small increase will not further improve the flux. It may be fixed based on previous experience, preliminary experiment, literature, etc ...

(ii) The Smaller-the-best (STB) Case: For responses such as diffusible hydrogen, fume generation, toxic content of fume, spatter, etc…the smaller their amount the better. WFDs usually aim at welding fluxes that minimizes such responses. The desirability function for such responses is given by;

$$d_i = \begin{cases} 1, & f_i(x) < T_i \\ \left(\dfrac{f_i(x) - U_i}{T_i - U_i}\right)^{t_i}, & T_i \le f_i(x) \le U_i \\ 0, & f_i(x) > U_i \end{cases}$$

for each $i \in I$ (5)

With T_i in this case representing small enough value for the response at which a small decrease will not further improve the welding flux. t_i is suitably chosen to reflect rapidly the deviation from the target becomes undesirable.

(iii)Nominal-the-best (NTB): In the case of NTB, the specifications consist of a target value T_i and the deviations around it are minimized. d_i takes the value of 1 if the quality characteristic attains the target value and decreases if it deviates from the target. If T_i lies on the midpoint i.e. $\frac{U_i + L_i}{2}$ of the speci fication interval, the specification is called a two-sided symmetric specification, otherwise a two-sided asymmetric specification. The desirability function is expressed as;

$$d_i = \begin{cases} \left(\dfrac{f_i(x) - L_i}{T_i - L_i}\right)^{s_i}, & L_i \le f_i(x) \le T_i \\ \left(\dfrac{f_i(x) - U_i}{T_i - U_i}\right)^{r_i}, & T_i \le f_i(x) \le U_i \\ 0, & otherwise \end{cases}$$

for each $i \in I$ (6)

s_i and γ_i are suitably chosen to reflect how rapidly a deviation from the target becomes undesirable.

Step 2: Construct the overall (i.e. global/composite) desirability index D. This can be done by aggregating the individual d_i in a single value, D, still in the $[0,1]$interval representing the overall desirability of the welding flux. The most widely used composite desirability is the weighted geometric mean given by;

$$D = \left[\prod_{i \in I} d_i^{w_i}\right]^{1/\sum_{i \in I} w_i} \tag{7}$$

Where, w_i is a weighting coefficient indicating the relative importance of the i^{th} response and $\sum_{i \in I} w_i = 1$

Step 3: Find the flux ingredient levels that maximizes the overall desirability D, in the domain of interest, that is;

$$\max imize, D = \left[\prod_{i \in I} d_i^{w_i} \right]^{1/\sum_{i \in I} w_i} \quad (8)$$

Subject to;

$x \in C_s$

Step 4: Use the flux ingredient levels of step 3 to formulate the welding flux. If the WFD wants to explore the available trade-off options, then various values of w_i, s_i and γ_i are used and the WFD selects the solution that best suits his needs.

Goal Programming (GP)

The GP approach is suitable for welding flux design situation where the WFD has some specific numeric values (target values) established for the quality characteristics/responses and wants a welding flux formulation that minimizes the weighted some of the deviations of the quality characteristics from their respective target values. There are two cases of GP, namely; (i) Non preemptive Goal Programming (NGP) (ii) Pre-emptive Goal Programming (PGP).

(i) Nonpre-emptive goal programming (NGP):

In NGP, the quality characteristics/responses are presumed to be of roughly comparable importance. Since it is not possible to achieve all the goals because of their conflicting nature, there will be deviations from their target values for all or some of the responses. These deviations are unwanted and therefore, they should be minimized. The unwanted deviations are assigned weights according to their relative importance to the WFD and minimized as an Archimedian sum. The specific steps the WFD may follow are as follows:

Step 1: Establish the desired target levels ($T_i, L_i \,\&\, U_i$) for each of the responses/quality characteristics, (e.g. acicular ferrite ≥50 %, oxygen content is 240 µL/L and diffusible hydrogen content ≤ 8 mL per 100 g).

Step 2: Assign weights to each response and their respective negative (n_i) and positive (p_i) deviations

Step 3: Construct the goal constraints of the problem. The goal constraint is usually given by;

$f_i(x) + n_i - p_i = T_i, L_i, or U_i$

for each $i \in I$ (9)

Step 4: Construct the achievement function of each response as illustrated in the table below.

Table 1. Construction of achievement function

Objective	Description	Achievement Function
$f_i(x) \geq L_i$	Under-achievement or negative (n_i) deviation (i.e. values below L_i) is unwanted and must be minimised.	Minimize n_i
$f_i(x) \leq U_i$	Over-achievement or positive deviation (p_i) (i.e. values above U_i) is unwanted and must be minimised.	Minimize p_i
$f_i(x) = T_i$	Both negative (n_i) and positive (p_i) deviations are unwanted and must be minimised	Minimize($n_i + p_i$)

Step 5: Construct the overall achievement function and add the goal constraints to the structural constraints of the problem. The complete NGP model of the problem may be stated as;

$$\min imize, a = \sum_{i \in I} (u_i n_i + v_i p_i)$$

Subject to;

$$f_i(x) + n_i - p_i = T_i, L_i, or U_i$$

$$\text{for each } \; i \in I \qquad (10)$$

$$x \in C_s$$

$$n_i \times p_i = 0 \;\text{ for each }\; i \in I$$

(It is not possible to have both p_i and n_i together for any response i). The weights u_i, v_i take the value zero if the minimization of the corresponding deviational variable is not important to the WFD.

Step 6: Solve the model in step 5 to find the flux ingredient levels that minimizes the weighted sum of the deviations.

Step 7: Use the values obtained to develop the needed welding flux. Trade-off exploration may be achieved by using different weight structures.

(ii) Pre-emptive goal programming (PGP)

The PGP method is suitable for welding flux formulation situation in which some quality characteristics/responses are of overwhelming importance when compared to others. There is a hierarchy of priority levels for the responses, so that the responses of primary importance receive first priority attention, those of secondary importance receive second priority attention and so forth.

The achievement function is minimized in a lexicographic sense. A lexicographic minimization may be defined as a sequential minimization of each priority while maintaining the minimal value Z_q^* reached by all higher priority level minimization. The steps the WFD may follow are the same as that of NGP except that in step 5, a hierarchy of priority levels are established and the solution is in sequential order.

Step 5: Establish the priorities in hierarchical order and construct the achievement or satisfaction function, Z_q for each priority level as below;

$$Z_q = \sum_{i \in I} (u_{qi} n_i + v_{qi} p_i) \quad (11)$$

for each $q \in Q$ and $Q \leq I$

The weights u_{qi} and v_{qi} take the value zero if the minimization of the corresponding deviational variable is not important to the WFD at that priority level.

Step 6: Minimize the achievement/ satisfaction function in lexicographic order i.e.

$$lex \min \left[z_1, z_2, \ldots\ldots z_Q \right]$$

Subject to;

$$x \in C_s \text{ (Structural constraints)} \quad (12)$$

$$f_{qi}(x) + n_{qi} - p_{qi} = T_{qi}, L_{qi}, or U_{qi}$$

(Goals on the q^{th} priority level) for each $i \in I$ and $q \in Q$

$z_j = z_j^*$ for j=1 to q-1(Solutions of higher level priorities).

Where, $z_1 >>> z_2 \ldots >>> z_Q$ and z_j^* is the optimal level that was achieved for the achievement function z_j of any priority level $j < q$.

When we deal with goals on the same priority level, the approach is just like the one described for NGP. The solution methodology ensures that the optimal solution of a higher priority goal is not sacrificed in order to achieve a lower priority goal. For each priority level, z_q is minimized while requiring that all higher priority satisfaction or achievement levels are maintained as hard constraints.

Step 7: use the values obtained from the solution of the last priority level to develop the needed welding flux. Trade-off options or sensitivity analysis are done by using different weight structures within priority levels and different priority structures for the responses.

Compromise Programming (CP)

Compromise Programming (CP) was first proposed by ZELENY[36, 37] and subsequently used by many researchers.[38, 39] CP identifies the best compromise solution as the one that has the shortest distance to an ideal point where the multiple objectives/ responses simultaneously reach their optimum values. The ideal point is not practically achievable but may be used as a base point. The operative structure of CP may be summarised in the following way;

Step 1: For each response function, determine the ideal (best or anchor) value $f_i^*(x)$ and the anti-ideal (worst or nadir) value $f_i^{**}(x)$ within the solution space for each $i \in I$.

Step 2: Define the distance or degree of closeness DL_i between the i^{th} response and its ideal value. The distance is defined by $DL_i = f_i^*(x) - f_i(x)$ when the i^{th} response is maximized or as $DL_i = f_i(x) - f_i^*(x)$ when the i^{th} response is minimized. When the units used to measure the responses are different (e.g. acicular ferrite (%), toughness (joules), yield strength (kN/mm^2), diffusible hydrogen (mL per 100 g)…) normalised distances rather than the absolute deviations must be used (Romero et al, 1987). Thus the normalised degree of closeness is given by;

$$DL_{ni} = \frac{f_i^*(x) - f_i(x)}{f_i^*(x) - f_i^{**}(x)}, \text{ for each } i \in I \tag{13}$$

Step 3: Construct the composite function of the normalized distances. The corresponding composite distance functions are introduced through a family of p-metrics. The basic structure of the composite function is given by;

$$DL_p = \left[\sum_{i \in I} \left(w_i^p \left(\frac{f_i^*(x) - f_i(x)}{f_i^*(x) - f_i^{**}(x)} \right)^p \right) \right]^{1/p} \tag{14}$$

P= a topological metric i.e. a real number belonging to the closed interval $[0, \infty]$

Step 4: Seek the solution that minimizes DL_p. The problem may be stated as;

$$Minimize, DL_p = \left[\sum_{i \in I} \left(w_i^p \left(\frac{f_i^*(x) - f_i(x)}{f_i^*(x) - f_i^{**}(x)} \right)^p \right) \right]^{1/p} \tag{15}$$

Subject to,

$x \in C_s$

L_1* *metric (p = 1): The equation (15) above is the general model. If the WFD considers all distances from the ideal point to be of equal importance, then $p = 1$ and the best compromise flux formulation is obtained by solving;

$$\min imize, DL_1 = \sum_{i \in I} w_i \left(\frac{f_i^*(x) - f_i(x)}{f_i^*(x) - f_i^{**}(x)} \right) \quad (16)$$

Subject to,

$x \in C_s$

DL_∞ metric (p=∞): If only the largest deviation counts to the WFD, then the problem becomes a min-max problem and $p = \infty$. The WFD determines the best compromise flux formulation by solving;

$$\min imize, D_\infty = L_{n\infty}$$

Subject to;

$$w_1 \left(\frac{f_i^*(x) - f_i(x)}{f_i^*(x) - f_i^{**}(x)} \right) \le L_{n\infty} \quad (17)$$

$$w_I \left(\frac{f_I^*(x) - f_I(x)}{f_I^*(x) - f_I^{**}(x)} \right) \le L_{n\infty}$$

$x \in C_s$

The other best compromise solutions fall between the solutions corresponding to L_1 and L_∞. For instance if the WFD weighs each deviation in proportion to its magnitude, then $p = 2$ and equation (15) is solved to obtained the needed flux ingredient levels. The parameter p represents the concern of the WFD over the maximum deviation. The larger the value of p, the greater the concern becomes. As $p \to \infty$, the alternative with the largest deviation completely dominates the distance measure. Sensitivity analysis or trade-off exploration may be done by the WFD by using different values of w_i and p.

CONCLUSION

MODM methods that a WFD can integrate with mixture experiments to mitigate the limitations of the traditional welding flux design approach has been discussed. The following conclusions can be drawn from the study:

- If all the responses defining the quality of a welding flux are related to the same set of predictor variables and regression equations that capture the relationship between the predictor variables and response variables can be established, then the MODM method can be used to determine the best compromise welding flux formulation and also to explore various trade-off options.
- The WSS method is suitable for situations where the WFD is interested in minimizing undesirable responses while at the same time he wants to maximise desirable responses.
- Desirability indices method is suitable when the WFD wants to minimise some responses, maximise some and achieve target values for some simultaneously.

- NGP is suitable for cases where the WFD wants to achieve target values for the responses and the responses are of comparable importance.
- PGP is useful when the responses are in hierarchical order of importance and the WFD wants to achieve lower priority response(s) without sacrificing the achievement of higher priority response.
- CP is useful when the WFD wants a welding flux formulation that is closest to the ideal formulation.

This paper has not exhausted the MODM methods. Many other multi-objective methods such as reference point method and heuristics such as genetic algorithm, particle swarm optimization, tabu search, etc… may also be useful for welding flux formulation.

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