FORMALISING THE LOGIC OF SPATIAL QUALIFICATION USING A QUALITATIVE REASONING APPROACH

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FORMALISING THE LOGIC OF SPATIAL QUALIFICATION USING A QUALITATIVE REASONING APPROACH

BY

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B.Sc. Computer Science (UNICAL), M.Sc. Computer Science (Ibadan)

A Thesis in the Department of Computer Science,

Submitted to the Faculty of Science

in partial fulfillment of the requirements for the Degree of

DOCTOR OF PHILOSOPHY

of the

UNIVERSITY OF IBADAN

APRIL 2014

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CERTIFICATION

I certify that this doctoral research titled FORMALISING THE LOGIC OF SPATIAL QUALIFICATION USING A QUALITATIVE REASONING APPROACH was carried out by **Patience Charles BASSEY** with student's identification number, **130162** in the Department of Computer Science, University of Ibadan, Ibadan, Nigeria under my supervision.

……………………………………………………………
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ACKNOWLEDGEMENTS

I return all glory to God Almighty who destined me for this success story recorded today despite all hills and valleys. Through His favour and love, He proved again to be the Alpha and Omega of my life. He used people I would not have imagined to bring this dream true, especially brethren from all over the world. I shall remain grateful to the body of Christ in Glory Tabernacle Ministries, Ibadan who through their teachings of truth have raised men like my supervisor.

My indebted gratitude goes to my supervisor: "*the best supervisor on the planet earth*", Dr. B. O. Akinkunmi, for his personal commitment, initiative, concern and tolerance, through whom God made this research visible. Your availability to give guidance has contributed in granting this work a speedy completion. My gratitude to my supervisor cannot be complete without acknowledging his rib, my sister and my friend, Mrs. Rebecca Akinkunmi, for her prayers, encouragement and care.

I remain grateful to my mentor, Dr. Olufade F. W. Onifade, Computer Science Department, University of Ibadan, Ibadan, for his mentorship spirit, useful contributions, guidance and tolerance which has contributed immensely to the success of this research. And to other lecturers in the department whose criticisms have helped shape this work, to mention but a few are Dr. A.B.C. Robert, Dr. Olumide Longe, Dr. Akinola, Dr. Osunade, Dr. (Mrs.) Yetunde Folajimi. I will not fail to acknowledge a role model whose footsteps I attempt to follow, Prof. Adenike Osofisan, for her positive impartation and encouragement to me as a female student in the department and in the society. I am not forgetting my close friends and colleages such as Mrs. Aderonke Sakpere, Mrs. Ayorinde and Mr. Akinwale who in one way or the other gave my stay meaning.

I want to also express my gratitude to Professor Hussein Suleman and Geoff Nitchske of Computer Science Department, University of Cape Town, South Africa whose

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comments unveiled the most important ingredient in my work during my fellowship programme in that department. I am highly honoured by God to have a loving sister like Mrs. Anietie J. Ekong as my pace setter; Arc. Etuk S. W. Emah, for the fatherly role he played in support of this programme; Pastor and Mrs Emmanuel Joseph of Glory Tabernacle Ministry for being my spiritual parents; and to my sweet mother, Late Mrs. Lucy C. Bassey for her love and prayers before answering God's call to glory. Thanks also go to my friends and love ones whose support and commitment has in one way or the other been an encouragement to the achievement of this goal. They include Mr. Donfack Kana, Mr. Raheem, Mr. Ebenezer John, Dr. Chris Obudu, Dr. Philips Ekpe, Mfon Ntekop, Rev. Chukwuemeka G. Madu, Emmanuel Afolabi, and Rev. and Mrs. Mabadeje and family, Mr. Silas, Mr. Akpabio, Anne Omori, Ola Iheanatu, Fidelis and other Balewites.

I also remain grateful to Professor Akaneren Essien, Professor Akpakpan, Professor Comfort Ekpo whose administration enabled me to embark on this research. Also are Dr. S. A. Sanni, Dr. V.J.U. Ekong, Dr. M. U. Umoren and Dr. U. G. Inyang, all from the University of Uyo community for proof-reading my manuscript. And to all my brothers, sisters and relatives including Ubong Emmanuel, Etimbuk Udom, Mrs. Elizabeth Etuk and family. I will not fail to mention Dr. Tosan Fregene and all the brethren at Bello Quarters Discipleship Centre of Glory Tabernacle Ministry for their prayers and encouragement. May the good Lord bless them all in Jesus Name. Amen.

Bassey, Patience C.

April 2014.

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DEDICATION

This thesis is dedicated to God Almighty, who knows my end from the humble beginning and to members of my family.

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NOTATIONS

AH	-	Awolowo Hall
AI	-	Artificial Intelligence
BH	-	Bello Hall
CCMD	-	Coca Cola Mini Depot
DC	-	DisConnected
EC	-	Externally Connected
EQ	-	Equal
GPS	-	Global Positioning System
IH	-	Independence Hall
JEPD	-	Jointly Exhaustive Pairwise Disjoint
KH	-	Kuti Hall
KRR	-	Knowledge Representation and Reasoning
MH	-	Mellanby Hall
NH	-	New PG Hall
$NTPP^{-}$	-	Inverse of Non-Tangential Proper Part
NTPP	-	Non-Tangential Proper Part
PO	-	Partial Overlap
PTAV	-	Translation pathway along vector
PWS	-	Possible World Semantics
QEH	-	Queen Elizabeth's Hall
QIH	-	Queen Idia Hall
QR	-	Qualitative Reasoning
QSR	-	Qualitative Spatial Reasoning
QTC	-	Qualitative Trajectory Calculus
QTC_B	-	Qualitative Trajectory Calculus - Basic
QTC_C	-	Qualitative Trajectory Calculus – Double Cross
RBG	-	Region-Based Geometry

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RCC-8 - Region Connection Calculus – eight relations

SQ - Spatial Qualification

SQM - Spatial Qualification Model

TAV - Translates along vector

TBH - Tafawa Balewa Hall

TdH - Teddar Hall

TPP^{-} - Inverse of Tangential Proper Part

TPP - Tangential Proper Part

TrH - Trenchad Hall

ZH - Azikiwe Hall

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LIST OF SYMBOLS

$\neg$	-	Negation
$\wedge$, $\vee$	-	Boolean connectives (and, or respectively)
$<$	-	Less than
$>$	-	Greater than
$\exists$	-	there exist
$\forall$	-	for all
$\square$	-	necessarily
$\Diamond$	-	possibly
$\Rightarrow$	-	implies
$\Leftrightarrow$	-	if and only if
φ, ψ	-	formulas
l, l_1, l_2	-	spatial locations
x	-	intelligent agent
t, t_1, t_2, t'	-	time points
KP1, KP2, TP, 4P	-	properties of SQM
S4, S5	-	existing modal systems
K,T,4,B	-	properties of existing modal systems
W	-	*Non-empty set of possible worlds*
R	-	*Accessibility relation*
D	-	*Domain*
I	-	*Interpretation*
V	-	*Valuation function*
$T_{A1}, T_{A2}, T_{A3}, T_{A4}, T_{A5}, T_{A6}, T_{A7}, T_{A8}, T_{A9}, T_{A10}$	-	axioms that make up the SQM logical system.
$Present\_at(x,l,t)$	-	x is present at location l at time t
$Reachable(x,l_1,l_2,(t_1,t_2))$	-	x present at l_1 at time t_1 can reach l_2 at time t_2
$\Diamond Present\_at(x,l,t)$	-	x is possibly present at location l at time t

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ABSTRACT

Spatial qualification problem, an aspect of spatial reasoning, is concerned with the impossibility of knowing an agent's presence at a specific location and time. An agent's location determines its ability to carry out an action given its known spatial antecedents. There are sparse works on the formalisation of this problem. Qualitative reasoning approach is the most widely used approach for spatial reasoning due to its ability to reason with incomplete knowledge or reduced data set. This approach has been applied to spatial concepts, such as, shapes, sizes, distance and orientation but not spatial qualification. Therefore, this work was aimed at formalising a logical theory for reasoning about the spatial qualification of an agent to carry out an action based on prior knowledge using qualitative reasoning approach.

The notions of persistence, discretisation and commutative distance coverage were used as parameters in formalising the concept of spatial qualification. The axioms and derivation rules for the theory were formally represented using quantified modal logic. The formalised theory was compared with standardised systems of axioms: S4 (containing Kripke's minimal system K, axioms T and 4) and S5 (containing K,T,4 and axiom B). The characteristics of the domain of the formalised theory were compared with Barcan's axioms, and its semantics were described using Kripke's Possible World Semantics (PWS) with constant domain across worlds. A proof system for reasoning with the formalised theory was developed using analytical tableau method. The theory was applied to an agent's local distribution planning task with set deadline. Cases with known departure time and routes were considered to determine the possibility of an agent's presence at a location.

From the formalisation, a body of axioms named Spatial Qualification Model (SQM) was obtained. The axioms showed the presence log and reachability of locations as determinants for agent's spatial presence. The properties exhibited by the formalised

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model when examined in light of S4 and S5 systems of axioms were KP1, KP2 (equivalent to axiom K), TP and 4P (equivalent to axioms T and 4 respectively) in an S4 system. The SQM therefore demonstrated the characteristics of an S4 system of axioms but fell short of being an S5 system. Barcan's axiom held, confirming constant domain across possible worlds in the formalised model. Explicating the axioms in the SQM using PWS enabled the understanding of tableau proof rules. Through closed tableaux, the SQM was demonstrably semi-decidable in the sense that the possibility of an agent's presence at a certain location and time was only provable in the affirmative, while its negation was not. Depending on the route, the application of SQM to the product distribution planning domain resulted in agent's feasible availability times, within or outside the set deadline to assess the agent's spatial qualification in agreement with possible cases in the planning task.

The spatial qualification model specified the spatial presence log and reachability axioms required for reasoning about an agent's spatial presence. The model successfully assessed plans of product distribution task from one location to the other for vans' availability.

Keywords: Spatial qualification model, Quantified modal logic, Tableau proof, Possible world semantics.

Word count: 497

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CHAPTER ONE

INTRODUCTION

1.1 Background and Motivation

Spatial qualification problem is a specific type of qualification problem that deals with the impossibility of knowing an intelligent agent's presence at a specific location at a certain time in order to carry out an action or participate in an event given its known location antecedents. The qualification problem (McCarthy, 1986) deals with the impossibility of satisfying all possible preconditions required for a real-world action to take place or have an intended effect. This is a well-known problem in the field of artificial intelligence (AI) (Theilscher, 2001; McCarthy and Hayes, 1969). In spatial qualification, both the knowledge of the antecedents and the uncertain knowledge of possibility are tied to time as well as space. This kind of qualification is an important precondition to spatial actions which has not been considered in most of the formalisms such as temporal reasoning with plans (Allen, 1991) and TRAINS project (Allen and Schubert, 1991). This may possibly be due to the uncertain nature of spatial knowledge or the inadequacy of the existing logical languages for representation of such uncertain knowledge. This involves the use of general commonsense knowledge to tackle the problem and build logics about that particular situation.

The spatial qualification problem is a general problem that is feasible in several application domains such as: Alibi Reasoning, where a person's presence at location, l_1 at time t_1 rules out his/her being present at location, l_2 at time t_1; Homeland Security, for example, *Automated Teller Machine (ATM) Fraud*, to investigate the presence of an account holder at certain locations where multiple transactions occur within questionable time frame; and Planning, example, *a shipping/distribution process,* where a planner needs to work out the feasibility of the delivery process based on its current location.

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Several calculi around spatial domains starting from spatial concepts have been defined and formalised (Freksa, 1992; Randell et al., 1992; Borgo et al., 1996; Carsati and Varsi, 1997; Van de Wedge et al., 2004; Galton and Hood, 2005; Bogaert, 2008). The peculiarity amongst these calculi is the use of qualitative reasoning approach in representing spatial concepts and their relationships. This is due to the vagueness of the spatial knowledge. Qualitative reasoning allows the abstraction away from the quantities of physical domain and enables qualitative mechanisms to be built without resorting to complex methods of calculus. In qualitative reasoning, inferences can be made in the absence of complete knowledge without probabilistic or fuzzy techniques which may rely on arbitrarily assigned probability or membership values (Cohn, 1999). According to Renz and Nebel (2007), when representing knowledge qualitatively, one does not need to depend on specific values and granularities as obtained in quantitative knowledge but uses limited vocabularies to compare two objects, for instance 'a is smaller than b' and 'b is smaller than c'. This is closer to how humans represent and reason about commonsense knowledge. Reasoning is the basis for knowledge and it gives explanation or justification for something.

Due to the incompleteness of existing models in handling everyday knowledge of space and time, spatial and temporal aspects of commonsense knowledge fall under the problematic aspect of everyday knowledge with identified problems of vagueness, uncertainty and granularity (Galton, 2009; Cohn and Renz, 2008). While vagueness and uncertainty affects space, granularity affects both time and space and its combination. Spatial knowledge is vague, incomplete, continuous (that is, it changes with respect to time) thereby paving way for qualitative reasoning. Vagueness and ambiguity in spatial reasoning is as a result of lack of consensus. One of such lack of consensus is the Leibniz's contention that there is no way of identifying a region of space except by referencing what is or could be located or take place at that region against Newtonian's view of space being an individual entity in its own right independent of whatever entities may inhabit it (Casati and Varzi, 1997).

Commonsense reasoning approximates but simplifies the sophisticated mathematical concepts. Asher and Vieu (1995) confirmed that commonsense spatial reasoning tasks do not require the full power of mathematical topology, geometry and analysis but refer to points without measure. Freksa (1992) also added that reasoning based on

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qualitative information is aimed at restricting knowledge processing to that part of the information which is likely to be relevant in the decision process. The required knowledge for solving the spatial qualification problem is one of such information needed for decision making in reality.

Knowledge representation and reasoning (KRR) community tends to restrict itself more to the use of classical logics where only the truth value of a formula is determined instead of non-classical which has to do with the way, mode and state of the truth of a formula (Bennett, 1994). These classical logics can rarely predict the future in uncertain cases. Therefore, there is need to represent and reason with spatial knowledge using a more flexible logical language such as modal logics (Bennett, 1996).

Spatial qualification is an important precondition for any spatially located agent to participate in an action, and this is seen to be missing in early works in the field of KRR, such as that on temporal reasoning with plans (Allen, 1991). There are sparse works on the formalisation of spatial qualification in AI literature. The frequently employed classical logics can rarely predict the future in uncertain cases. The few cases, where the spatial knowledge is represented for reasoning include the one employed to solve the adversarial geospatial abduction problem (Shakarian et al., 2011). This approach used the reward functions to encode the goals. In a stochastic domain (Dean et al., 1993), where space is divided into a grid of locations with four directional states allocated to each of the states, the use of reward functions for efficient planning require the deliberation interval (that is, the time interval between the current time and deadline).

Qualitative reasoning approach, adopted in this work, allows inferences to be made in the absence of complete knowledge without probabilistic or fuzzy techniques which may rely on arbitrarily assigned probability or membership values (Cohn, 1999).

1.2 **Problem Statement**

Qualitative spatial reasoning (QSR) emerged as a sub-field in KRR and is now a full-fledged sub-field in AI towards the development of qualitative calculi that solve most of the reasoning problems (Forbus et al, 1991). Cohn (1997) pointed out that through qualitative reasoning abilities, QSR field got challenged to provide calculi that will

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allow machines to represent and reason with spatial entities of higher dimension, without resorting to the traditional quantitative techniques. Increased researches in QSR have addressed different aspects of spatial concepts including topology (Randell et al., 1992), orientation (Freksa, 1992), shape (Carsati and Varzi, 1997), size (Borgo et al., 1996) and distance (Bogaert, 2008). Attempts to categorize 'place' as it relates with other spatial concepts as neighbourhood, region, district, area and location have also been made (Bennett and Agarwal, 2007).

Although the qualification problem is a well-known problem in AI field, none of these attempts addressed the qualification problem with respect to space. This research therefore addresses the spatial qualification problem by investigating spatial qualification. Thus the problem statement for this research is stated as a question:

> *Given a prior antecedent that an intelligent agent has been present at a certain location, is it possible for the agent to have been present at the scene of incidence at the time of incidence?*

An attempt towards giving an answer to the above question might lead to its refinement to:

> *Is movement of the agent from its last known location to the place of incidence possible within the time the agent was last sighted and the time of incidence?*

1.3 Research Questions

The following research questions are addressed in this thesis.

(i) Are there relations between previous locations of an agent at a certain time and possible agent's location at the current time?

(ii) Can such relations be formalised?

(iii) Can such relations be represented in a language that allows new inferences to be made about the relations?

1.4 Aim and Objectives of the Study

The aim of this work is to formalise the problem of spatial qualification with respect to time. Spatial qualification in this context is the possibility that an agent could be present at a particular place at a certain time given the agent's prior location

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antecedents. The formalism will provide a logical framework for investigating the problem.

The specific objectives are to:

(i) Decide on an appropriate language to be used for the logical theory.

(ii) Use the syntax of the language to formalise the domain of spatial qualification.

(iii) Define the formal semantics of the language.

(iv) Develop a proof system for the formalised logic.

(v) Apply the logical model to a planning distribution domain using case studies of spatial qualification problem for investigation.

1.5 Methodology

The formalism made use of the Quantified Modal Logic (QML), otherwise known as the First-Order Modal Logic (FOML), as its representational language. This language combines the expressivity of First-Order logic with the dynamics of Modal logic as its key feature. The dynamics of Modal Logic has to do with its ability to change over time. In modal logic, a formula (proposition) is necessarily true or possibly true. The *necessarily* and *possibly* symbols are □ and ◊ respectively. Unlike the monotonic reasoning system, any logical system with the modalities of modal logic can reason like humans in real life. The need to make valid conclusions in uncertain domains as new facts (beliefs) are introduced requires the introduction of modalities to make the theory flexible. Modal logic allows for extensions that make sense in the context of possible worlds or alternate universes to be provided to the defined concepts. For example the concept 'X is true' may have extensions. These extensions are known as modalities. Examples of concepts with such extensions include 'X is believed to be true', 'X is known to be true', 'X ought to be true', 'X is eventually true', and 'X is necessarily true'. Hence, modalities can be viewed as a connective that is not truth-functional which takes a formula (or formulas) and produces a new formula with a new meaning.

Again, the structure of the Possible World Semantics (PWS) (Fitting, 2008) is used in the formalism to semantically explicate the logical structure of the formal theory. The possible world structure is useful to formally explain in detail the theory to show its implications. A possible world is defined by Menzel (1990) as a universe in contrast

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with reality. It is also a region indexed with time. This follows from the Kripke model structure which is the basic modal logic model (Zalta, 1995). Kripke structure is a triple M = (W,R,V), where W is the non-empty set of possible worlds (that is states in a computation), $R \subseteq W \times W$ is the accessibility relation (otherwise called transition relation) and V: (Prop × W) → (true, false) is a valuation function (which tells us the properties that is true or false in different worlds) (Goldblatt, 2005).

Figure 1.1 illustrates the Kripke model and the relations: where p and q are elements of the W, which is a set of w_1, w_2 and w_3 called worlds, states or points with w_1Rw_2 meaning w_2 is accessible from w_1 or w_1 sees w_2 or w_2 succeeds w_1.

Another methodology used in this research the analytical Tableau Proof method to further prove the formalised logic for soundness and completeness. Formal axioms in the sound proven logical model are then applied to a product distribution planning domain for reasoning about plans with deadlines.

1.6 Basic Assumptions

In this thesis, the following basic assumptions were made:

(i) That all agents have access to only one mode of transport

(ii) That an agent is restricted to direct paths only.

(iii) That the formalism works with prior location antecedent or given knowledge.

(iv) Two worlds are accessible if there is any route or path between these worlds in existence.

(v) Available paths have known distances, time stamps based on an assumed speed limits assigned to them. So, our intelligent agent has an idea of the existing paths, their distances and their equivalent speed limit. Although, the paths with the shortest distances are often considered, sometimes, these paths might not be the fastest route depending on the state of the road. Oftentimes, routes with shortest distances and at deplorable states might take longer time to traverse than routes with longer distances without obstacle. The use of the given speed limit in determining the time it takes one to traverse these routes makes it adequate for handling cases that fastest routes alone cannot.

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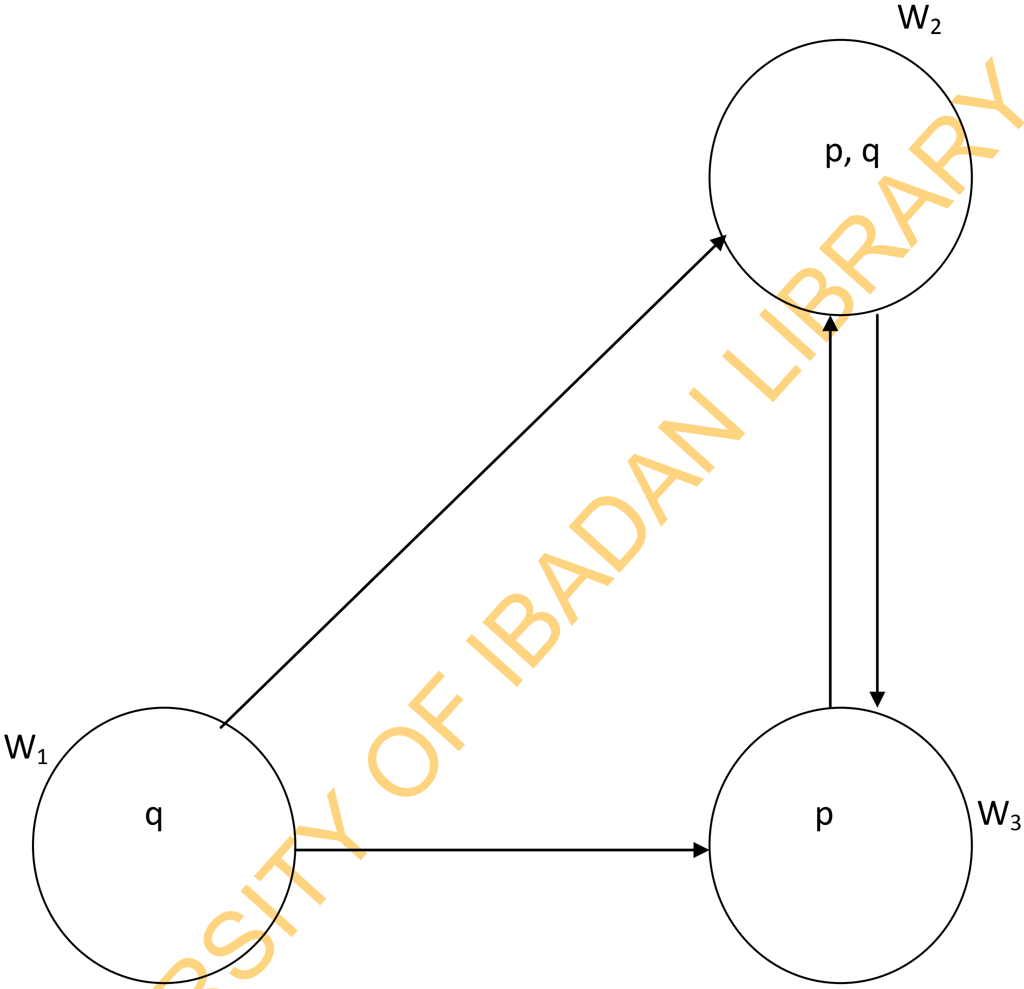

Figure 1.1: The Kripke Model
(Goldblatt, 2005)

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1.7 Organization of the rest of the thesis

The rest of the theses is organised as follows. Chapter two describes in detail the concept of qualitative spatial reasoning, theories for representing and reasoning with these spatial objects and the various logics used. The review of non-classical logic (defeasible and modal logic) pointing out the various forms of modal logic and various semantic structures for logic interpretation such as the possible world semantics for modal logics and situation semantics are also discussed in detail.

In chapter three, the model for the spatial qualification logic is defined showing the syntax and semantics of the logical theory with well described and formal axioms showing how intelligent agents reason.

In Chapter four, the proof system for the logical theory is developed using the analytic tableau proof method.

Chapter five demonstrates how the logical theory is applicable to the planning domain where a distribution plan is assessed and reasoned with. This domain is actually chosen to extend the idea given in the TRAINs project.

Chapter six gives the summary and conclusion, pointing out the contribution of the thesis to knowledge. Also, recommendations of our formal theory for reasoning with spatial entities and trends to further research were clearly pointed out.

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CHAPTER TWO

LITERATURE REVIEW

2.1 General Overview

This chapter highlights the major concepts surrounding this thesis, research carried out around them, the identified gaps and the need to close the gaps. The chapter starts with the introduction of the qualitative reasoning approach, the key principles of this reasoning approach, its impacts on reasoning with domain concepts and its limitations. Highlights of spatial concepts with their reasoning problems and attempts to use qualitative reasoning approach to address them is also discussed here. This is followed by trend of recorded successes and identified gaps in domains that require spatial qualification.

2.3 Qualitative Reasoning (QR)

Before now, mathematical progress in the QR community has been substantially more sophisticated, without link to tasks and explanations. This sophistication led to the proposal of several new reasoning techniques and ontologies that has forged a link between qualitative reasoning and traditional, numeric and analytic techniques. Qualitative reasoning is viewed differently by many researchers. Williams and de Kleer (1991) defined QR as an act of developing computational theories of the core skills underlying engineers, scientists, and just plain folk's ability to hypothesize, test, predict, create, optimize, diagnose and debug physical mechanisms. But the most direct view is that QR allows inferences to be made in the absence of complete knowledge without probabilistic or fuzzy techniques which may rely or arbitrarily assigned probability or membership values (Cohn, 1999). This does not rule out the effect of probabilistic and fuzzy techniques in solving real world problems.

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Qualitative reasoning compares features within an object domain. Qualitatively, one does not need to depend on specific values and granularities as obtained in quantitative knowledge but uses limited vocabularies to compare two objects, for instance 'a is smaller than b' and 'b is smaller than c' (Renz and Nebel, 2007). This is closer to how humans represent and reason about commonsense knowledge. Since the knowledge of the world is necessarily incomplete and unpredictable in detail (Allen and Ferguson, 1994), qualitative reasoning happens to be an appropriate reasoning method that a computer could adopt to make predictions on the basis of certain assumptions (Allen and Ferguson, 1994).

This comparison approach increases the strength of qualitative reasoning, making it seem advantageous over quantitative knowledge with a good number of reasons. These advantages (Freksa, 1991; Freksa, 1992) serve as the motivational properties of QR and they include: having nice properties of its analytical counterparts; being the goal for a reasoning process; serving as frequent input for a reasoning process; being cheaper and less informative in a certain sense; being transparent; being easier and better for human reasoning; and requiring less computational memory. QR helps to push the use of weak, qualitative information as far as it can go, and to use its failure to better understand what additional knowledge is required and how it is best applied.

2.2.1 **Impacts of Qualitative Reasoning**

Qualitative reasoning allows people to draw useful conclusions about physical world without sophisticated mathematical models. It also allows one to work with far less data, than would be required when using traditional, purely quantitative methods. Escrig (2005) pointed out that it is a necessity for robots operating in unconstrained environments and modeling human cognition to require understanding on how this can be done. This need is seen in science and technology generally.

In robotics, qualitative abstraction mechanism can be used to create a representation of space consisting of the circular order of detected landmarks and the relative position of walls towards the agent's moving direction. The use of this representation, said Frommberger (2008a, 2008b), empowers the agent to learn a certain goal-directed navigation strategy faster compared to metrical representations. It also facilitates

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reusing structural knowledge of the world at different locations within the same environment.

2.2.2 Key Principles Governing Qualitative Modeling

Forbus (2008) highlighted the key principles governing qualitative modeling to include:

i. Discretization

Discretization is the quantization of continuous properties into entities for representation and providing a means of abstraction. Erwig et al (1999) noted that continuous properties can be implemented if they are discretized. He concluded that both the abstract level and the discrete model are necessary. The essence of systems qualitative reasoning is to find ways of representing continuous properties of the world by discrete systems of symbols as it will be applied for the modelling of the spatial qualification logic.

ii. Relevance

Qualitative values are constructed to be relevant for some classes of tasks by imposing constraints from the nature of the system and the reasoning to be done. The relevance principle is one way of stating that an idea is useful.

ii. Ambiguity

Predictions (set of qualitative values) resulting from qualitative models (i.e. qualitative arithemetic algebra for exploiting transitivity of the ordering relation) are often ambiguous, making qualitative models ideal complement to traditional mathematical and numerical techniques (Cohn and Hazarika, 2001). The resulting set of qualitative values is the quantity space.

2.2.3 Limitations of Qualitative Reasoning

Despite the above advantages of qualitative reasoning, there are a good number of misconceptions stating what qualitative reasoning is not. These misconceptions are assumed by some to be the failure of QR and perceived by others as the progress in QR community. Qualitative reasoning is not:

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i. the eschewal of quantitative information. This was raised in the poverty conjecture by Forbus, Nielsen and Faltings (Forbus et al., 1987; Forbus et al., 1991) which states that "there is no purely qualitative, general purpose kinematics". This was also pointed out by Cohn (1999) that QSR is potentially useful, and that there may be many domains where QR alone is insufficient. Cohn's point called for the addition of qualitative non-topological information like orientation, distance, size and shapes to the topological relations (Randell et al, 1992). Models that have these combinations were also created (Muller, 1998a, 1998b; Erwig et al, 1999; Bennett et al; 2000).

ii. the eschewal of sophisticated mathematics: Although commonsense reasoning has been the major focus in QR, its intention is not to segregate or exclude more sophisticated mathematical tools, but rather to encourage the most appropriate tools for a particular reasoning task and warns or caveats that a mathematical tool should not be judged better simply because it provides more information. Also, in some instances, one can get away with surprisingly little information and extremely weak inferences, thereby reducing cost and time of acquiring more precise information.

iii. a theory of naivism: There is no advanced mathematics conception which is artless, considered excessively simple without experience and previous experimentation.

iv. the invention of new physics: The claims made by people due to the fact that early researches were tying QR formalisms to existing theories of physics.

v. just event driven simulation: This is just an adequate characterization of the work on Qualitative Simulation - QSIM (Kuipers, 2001; 1994) and some of its successors and not to be confused with the entire goal of QR community. QSIM is an abstraction of the actual behaviour that predicts the set of possible behaviour consistent with a qualitative differential equation of the world.

2.2.4 Domains highlighting the limits of qualitative reasoning

Reasoning about dimensionality is a challenge that lead to the conclusion that RCC-8 and related systems based on C(x,y) are not powerful and therefore impossible without

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imposing a sort of structure that is qualitative (Cohn, 1999). Several attempts to solve this dimensionality problem has been given: first, by introducing two primitives namely the mereological part relation, P(x,y) and a boundary operator, B(x,y) (Galton, 1996; Gotts, 1996).

Another attempt to solve this problem yielded the order of magnitude calculi which introduce measuring scales that allows one quantity to be described as being much larger than another. This requires summing up many of the former quantities in other to surpass the second and much larger quantity. An example is the Delta calculus which introduces a triadic relation, x(>,d)y, meaning x is larger or bigger than y by the amount d (Zimmermann and Freksa, 1996).

Also, the linear quantity spaces used as distances or sizes for measuring representation in qualitative reasoning through relative representations like CanConnect(x,y,z). This primitive allows a metric on the extent of regions to be defined (Cohn and Renz, 2008), with sample domains that require the use of restricted quantification by introducing a sorted predicate given.

The application of qualitative theoretical models developed so far introduces their usage with quantitative imprecise data. The qualitative and quantitative approaches are both integrated with the qualitative theories made to replace the hypothetical approaches (Escrig, 2005), where qualitative description of landmarks of the environment have been obtained.

2.2.5 Successful Applications of Qualitative and Quantitative Approaches

2.2.5.1 Bouncing Ball Domain

The application of qualitative and quantitative approaches in bouncing ball domain features the suggestion that the place vocabulary should be embedded in a more quantitative analog representation (metric diagram) with the results of qualitative spatial reasoning (Forbus, 1981). This came up as Forbus went on to buttress their poverty conjecture. This must be integrated with other knowledge following the way space is broken up in that domain. This is demonstrated in the FROB project which uses quantitative parameter in the qualitative description of motion allowing for different simulation to answer question from the bouncing ball domain (Forbus, 1981).

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Reasons for not using qualitative descriptions only were highlighted to be its difficulty resulting from the use of envisioning approach. Envisioning is the processes of generating all possible categories of behaviours for a system. This can lead to combinatorial search and moreso, they are weak models of space. The combinatorial search for a complex system occurs to an exponential number of qualitative states.

2.2.5.2 **Robotic Applications**

The qualitative/quantitative spatio/temporal models which have been theoretically developed are integrated for application in robotics. This application centres on the navigation in the structured environments of public buildings with two kind of robots: Khepera and Pioneer; the automatic construction of mosaic design by using qualitative shape recognition of ceramic pieces; and the navigation in an environment similar to brain structure with a legged robot (Escrig, 2005).

It is often said that quantitative borrows from qualitative and not vice versa. Hence, this work concludes in support of the poverty conjecture that qualitative reasoning working as a complement with quantitative reasoning makes computation of commonsense properties a reality. Thus, qualitative reasoning does not mean the absence of numbers, rather combining reduced sets of numbers with comparative approach, which means, inferring as much as possible from minimal information.

2.3 Reasoning with Spatial Knowledge

Reasoning with spatial knowledge requires knowing the various components of spatial knowledge. Mennis *et al.*, (2000) highlighted these components in the pyramid framework for spatial knowledge shown in figure 2.1. Spatial knowledge is seen to be made up of two major components: data and knowledge. The knowledge component has to do with the taxonomy and partonomy of the object and explains what the object is while the data component has to do with the theme (location and time) of the object.

Although these components of spatial knowledge are both necessary for drawing inferences in any efficient and workable system, much research has been done concerning them with various controversies about the spatial objects. The usefulness of spatial data where exact information is not available or not easily processed was pointed out by Muller (1998b).

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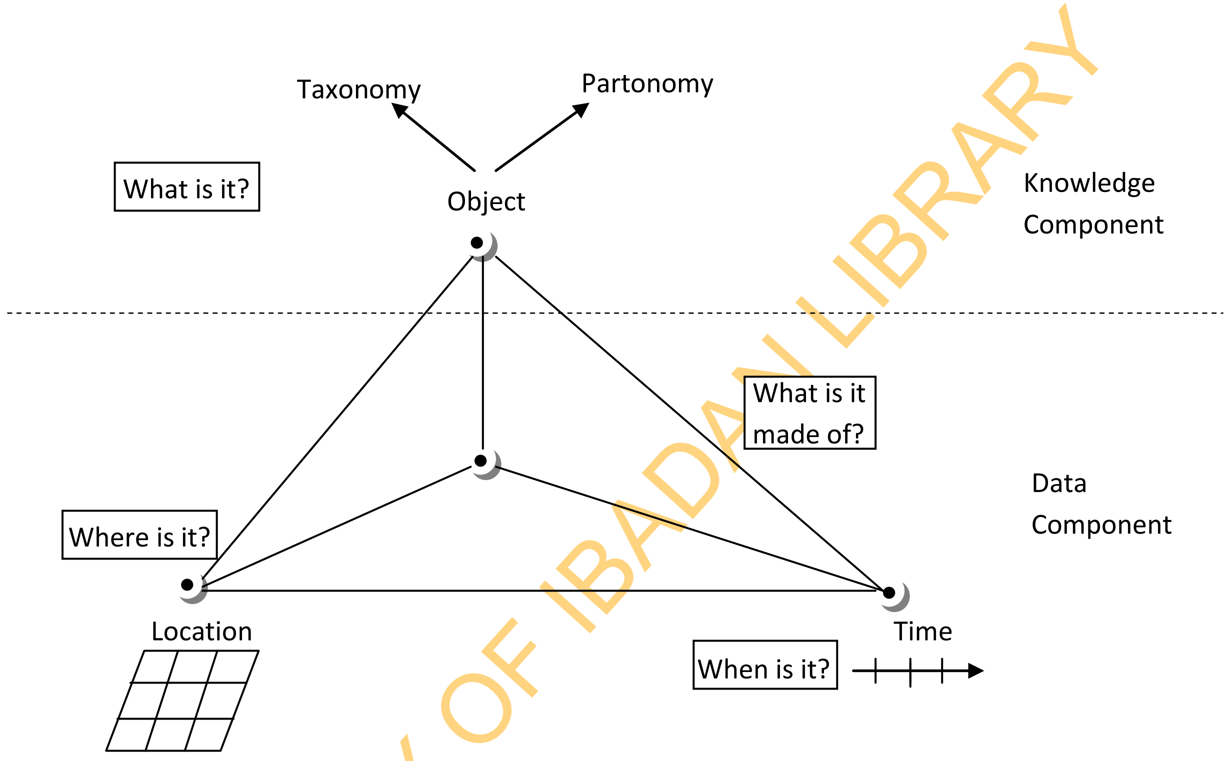

Figure 2.1: A pyramid framework for spatial knowledge
(Mennis et al.. 2000)

Uribe, et al. (2002) also gave a further note that spatial inferences are fundamental to human question answering. Hence, any knowledge based system designed to handle a broad range of questions require spatial reasoning.

Spatial reasoning involving spatial concepts like space and time has brought lack of consensus which has generated a lot of problems over the years. Some of the problems of space and time have been identified to include vagueness, uncertainty and granularity (Galton, 2009; Cohn and Renz, 2008). Therefore, reasoning with such knowledge requires commonsense reasoning since spatial and temporal knowledge are aspects of commonsense knowledge (Cohn, 1999).

One of the lack of consensus was the Leibniz's contention against the Newtonian's view of space (Casati, 1999). Leibniz contended that:

> *"there is no way of identifying a region of space except by referencing what is or could be located or take place at that region"*

against the Newtonian's view of space that:

> *"space is an individual entity in its own right independent of whatever entities may inhabit it."*

In other to investigate spatial qualification, both the space and the spatial objects are seen as two inseparable entities, thereby going with Leibniz's contention but with a time stamp. It is necessary to look into some of the defined spatial concepts and the challenges of spatial reasoning, before moving into the investigation of the spatial qualification.

2.4 Spatial Concepts and Challenges of Spatial Reasoning

Geographic places are regions in space that are categorized according to some commonly agreed upon characteristics. As social entities, places are said to be of interest for individual communities in a certain region and for a particular time span. Places can be referred to by names or descriptions. It is noted by Janowicz (2009) that different people may refer to the same place by various names at different times, because a place can have more than one name in heterogeneous settings. A foundation for incorporating concepts relating to place into an ontology is given by Bennett and Agarwal (2007). Hence, the place terms are categorized in Linguistic and Logical point of view as follows (Bennett and Agarwal, 2007):

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i. Place-Like Count Nouns: These are expressions of natural language that characterize types of objects considered to be places. For instance: room, town, forest, country, etc. These instances are capable of locating other objects, either by hosting or some more complex mode of spatial constraint. The constraints may be topological inclusion, geometrical containment, containment within a concavity, interposition among elements of aggregate, location within or among elements of aggregate, containment within a surface demarcation and support. Place-Like Count Nouns are further categorized into three, namely, substantive, (example, town, cupboard), spatial (example, region, point) and abstract (example, location, position and place).

ii. Locative Property Phrases: These are predicative expressions which characterize the location of an object. Examples include phrases like 'in London', 'on the hill', 'by the sea-side', 'between the church and the oak tree', etc.

iii. Place-Names: This is a case where proper names are applied to places. For example, Leed is a city, John is in London.

iv. Definite Descriptions: These are phrases that function as complex nominal expressions that uniquely identifies a place entity. Examples are 'the library', 'the shed at the end of the garden', etc.

Janowicz (2009) identified three characteristics used for referencing a place to include name, type and spatial footprints which confirms a place to be a social construct that could be modeled using different paradigms. This construct is seen in a semi-formal ontological framework, where the semantics of concepts like habitat and environment, and their relationship with spatial structure of the world are represented (Bennett, 2010).

The need to express location information about objects in space called for the simplification of mathematical concepts by approximately referring to points without measure, that is, without employing the full power of mathematical topology, geometry and analysis (Asher and Vieu, 1995). This approach changed the focus of researchers in the field from holding unto the poverty conjecture promulgated by Forbus, Nielson and Faltings that: "there is no purely qualitative, general purpose

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kinematics" (Forbus, Nielsen and Faltings, 1987; Forbus et al., 1991). The Poverty Conjecture has three arguments:

i. negation by failure, which means failure to find pure qualitative kinematics;
ii. human performance, that is, failure of the use of diagrams on the simplest spatial problems; and
iii. no total order, that is the inability of quantity spaces to work in more than one dimension.

These arguments led to the combination of weak information about spatial properties. An instance of a logical model with such combined information is the Allen's interval logic. Allen's interval logic is weak on its own except when combined together with numbers to provide enormous constraints. Due to the transitivity of both Allen's interval logic and numbers, the suspicion of the sparseness of spatial representation in higher dimensions led to the conclusion that for spatial reasoning, almost nothing weaker than numbers will do. Hence, the use of the combination approach, Metric Diagram/Place Vocabulary - MD/PV model (Forbus, Nielsen and Faltings, 1987). The poverty conjecture sees reasoning with commonsense knowledge to involve qualitative reasoning and some level of quantitative knowledge. They concluded by suspecting that the space of representations in higher dimensions is sparse and for spatial reasoning, nothing less than numbers will do.

Reasoning with space requires categorization of the granularities of space and their relationship. Several attempts to categorize 'place' as it relates with other spatial concepts as neighbourhood, region, district, area and location have been made (Bennett and Agarwal, 2007). Although place itself may not be permanent over time, the objects anchored in such places may be permanent. This made proper categorization of place when modeling spatial objects to become a necessity instead of geo-referencing it as points (Winter et al, 2010). This is because geographic places are seen as abstract entities used to structure knowledge and to ease communication.

Following from the poverty conjecture, Qualitative Spatial Reasoning (QSR) got challenged to provide calculi that will allow a machine to represent and reason with spatial entities of higher dimension, without resorting to the traditional quantitative techniques.

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2.5 Qualitative Spatial Reasoning

Qualitative Spatial Reasoning (QSR) emerged as a sub-field in Knowledge Representation and Reasoning (KRR) and now a full-fledged sub-field in Artificial Intelligence (AI) towards the development of qualitative calculi that solves most of the reasoning problems (Forbus et al., 1991). Qualitative spatial representations are said to be an expressive means of describing relations among features in geometrical space. QSR got challenged to provide calculi that will allow machines to represent and reason with spatial entities of higher dimension, without resorting to the traditional quantitative techniques. Increased researches in QSR, in an attempt to refute the poverty conjecture, has addressed different concepts of space including topology, orientation, shape, size and distances (Randell et al, 1992; Cohn, 1999; Freksa, 1992; Davis, 2006; Davis, 2011).

Central to the efficient production of workable systems are Model-Based Systems (MBS) and Qualitative Reasoning (QR). According to Price et al. (2005), MBS and QR is already a technology with a wide range of applicability in areas including fault detection by model-based prediction, system simulation, process understanding and monitoring, explanation of numerical simulations, compositional model based diagnosis, reusable systems, variant problems, decision making under uncertainty, educational context, etc.

QR represents our everyday commonsense knowledge about the physical world and also the underlying abstracts used by engineers and scientists when they create quantitative models (Cohn, 1999). Frommberger (2008a, 2008b) pointed out that qualitative abstraction mechanism can be used to create a representation of space consisting of the circular order of detected landmarks and the relative position of walls towards the agent's moving direction. Also, the use of this representation empowers the agent to learn a certain goal-directed navigation strategy faster compared to metrical representations, and also facilitates reusing structural knowledge of the world at different locations within the same environment. Frommberger, (2008a) saw this to also work with reinforcement learning where spatial constraints are internally provided by the input representation and do not need to be acquired separately enabling agents

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to develop a generally sensible behavior in space that it can reuse at different locations within the same world or in other environments.

2.5.1 Design approaches in Qualitative Spatial Reasoning

Several approaches are employed in the representation of these spatial models and Egenhofer (2010) described some facets to spatial-relation design to include:

i. Formalization or axiomatic approach

Formalization has to do with the use of a formal language to express concepts and relationships among the concepts. Logical axioms have been proven to be the most expressive formalism. Hence, most formalism makes use logic as the representation language.

ii. Conceptual Neighbourhood graphs

Conceptual neighborhood graphs are used as the primary tool to provide insights about the closeness or similarity of the relations. Two relations are said to be neighbors if a continuous transformation can be performed between the two relations without the need to go through a third relation. This captures for each relation those relations that are conceptually closest to it. When people communicate with information systems, a foundation for the selection of appropriate natural language terminology is provided by conceptual neighborhoods. Pairs of relations connected directly by an edge correspond to transitions that can be obtained by applying topological transformations (translations, rotations or scaling) to one or both objects.

Different types of neighborhood graphs are obtained depending on the type of deformation (movement, rotation and aniosotropic size-neutral, isotropic scaling, anisotropic and anisotropic scaling to some directions). In some types of deformations, the edges are directed, while in others they are not directed, that is, from one relation to the other as the nodes.

iii. Compositions

Compositions are needed where a higher-level inferences about combinations of the relations need to be performed to derive a query response directly from the stored base

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relations and also to assess a more complex query of conjunctions of these relations. The basic inferences over single relations can be made simply based on the properties of the conceptual neighborhood graph, N_8 and the 9-intersection, I_9, matrices. This is typically written as r_i:r_i, where r_i is a relation without the references to the objects involved. The composition table involving a set of n relations captures all n^2 compositions. This means that for the 8-topological relations we have 64 resulting compositions.

Commonsense reasoning about space and things located in space led to the hole trouble (Casati and Varsi, 1997). This trouble and the need for explicit theory are tackled using compositional approach. Several QSR problems such as path consistency problems can be solved based on this composition approach.

Each of the resulting theories tried to solve one problem with the solution unfolding another underlying problem. This led to the conclusion by Casati and Varzi (1997) that with the composition of all the theories, some affinity with common sense (Type I) and a suitable degree of formal specification (Type II) might result.

Problem solving with Qualitative Spatial Reasoning (QSR) involves formalizing one type of spatial relations and discussing their attributes; and composing two or more spatial relations to obtain a previously unknown relation. Since qualitative spatial representations are said to be an expressive means of describing relations among features in geometrical space, our formalism of the logic of spatial qualification problem adopts this composition representation design approach.

2.5.2 **Theories for Spatial Reasoning**

Theories about spatial relations can be traced from the definitions by Tarski (1929) to present day. These theories include ontology, mereology, topology, mereo-topology and mereogeometry.

2.5.2.1 **Ontology**

Guarino (1998) defined ontology to be a logical theory or set of axioms that account for the intended meaning of a formal vocabulary. Ontologies are specification of conceptualization and the corresponding vocabulary used to describe a domain.

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Ontologies correspond to generalized database schemas. However, they can be used to describe the structure of semantics of much more complex objects than common databases and are therefore well-suited for describing heterogeneous distributed and semi-structured information sources such as those found on the semantic web. It is on this note that modern knowledge representation and knowledge engineering advocate the use of explicit ontologies. Different kinds of ontologies exist according to their level of dependence on a particular task or point of view. Guarino (1998) highlighted some type of ontologies as shown in figure 2.2 as follows:

- Top-level ontologies which describe very general concepts like space, time, matter, object, event, action, etc. These concepts are independent of a particular problem or domain.

- Domain ontologies and task ontologies which respectively describe the vocabulary related to a generic domain (like medicine or automobiles) or a generic task or activity (like diagnosing or selling), by specializing the terms introduced in the top-level ontology.

- Application ontologies which describe concepts depending both on a particular domain and task.

According to Forbus (1996), an ontology, whose main goal is to formalize the act of building models of physical systems, is said to be central to qualitative reasoning. Hence, it is concerned with how to carve up the world, that is, the kind of things they are and the sort of relationships that can hold between them. With this, one can interpret the situation or system in terms of the available models. Ontology is seen as the main semantic structure used in annotating metadata in a web page. This is because it represents the meaning of terms in vocabularies and the relationships among those terms and they multiply as one tries to capture more of human reasoning. Hence, any spatial and temporal search conditions in the web require ontology of time and space for proper handling. An instance of such dynamic ontology is that of the process ontology (Forbus, 1996). There is therefore need to increase the accuracy of an ontology. This can be achieved by:

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- Developing richer axiomatisation in such a way that exactly the same models are obtained, that is, by adopting a richer domain and/or a richer set of relevant conceptual relations.

- Adopting a modal logic. This allows one to express constraints across worlds or just reifying the worlds as ordinary objects of the domain.

2.5.2.2 **Mereology**

This is the theory of parthood (Aiello and Ottens, 2007). In this theory, the classical mereology of Lesneiwski is the basis for the axiomatization, where the parthood relations are expressed. An example is the relation 'x overlaps y' expressed as:

$$O(x,y) \equiv_{def} \exists z[P(z,x) \wedge P(z,y)]$$

The parthood relation satisfying the axioms of closed extensional mereology is used to describe mereology.

2.5.2.3 **Topology**

This is a first-order theory that deals with regions in a topological manner. Topology is said to be truly a more basic and more general framework subsuming mereology in its entirety (Casati and Varzi, 1997), where the relation of connection takes over overlapping and parthood as special cases. The subsumption of mereology to topology gives birth to the relation of topological connection ('C'), where one thing is part of another. One thing is said to be part of another, if everything connected to the first is also connected to the second:

$$P(x,y) =_{df} \forall z\, (C(z,x) \rightarrow C(z,y))$$

Topological theories may be boundary-tolerant or boundary-free. It can also be via n-intersections. Topology is the theory of how things are connected, that is, how a set of entities might interact with one another. There are two topological models (9-intersection and region-connection calculus – RCC) that yield the same topological relations when considering any two simple regions. The 9-intersection defines binary topological relations between two simple regions, A and B with their interiors, boundaries and exteriors to be A^0, ∂A, A^- respectively for A and B^0, ∂B, B^-

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respectively for B (Egenhofer, 2010). The intersection of these six object parts describe a topological relation and can be concisely represented by a 3 × 3 matrix equation below, called the 9-intersection.

$$I_9 = \begin{pmatrix} A^0 \cap B^0 & A^0 \cap \partial B & A^0 \cap B^- \\ \partial A \cap B^0 & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^0 & A^- \cap \partial B & A^- \cap B^- \end{pmatrix}$$

By considering the values empty (ϕ) and non-empty (¬ϕ) for each of the nine intersections, $2^9 = 512$ binary topological relations can be distinguished. Eight of these 512 relations can be realized between two regions embedded in $\Re^2$ and subsequently referred to as the $\Re^2$ – relations. The primitive binary relations for the Region Connection Calculus are given in the next section. This 9-intersection and the region-connection calculus give rise to the eight topological relations between two regions in $\Re^2$ as shown in figure 2.3.

RCC-8 is seen to play very important role in spatial representation and reasoning (Wolter and Zakharyaschev, 2000b; 2002). An ontologically well founded logical language for describing spatial, temporal and material properties of the physical world is also presented (Bennett, 2001a). Current models for topological relations fall primarily into the two major categories those based on connection and those based on intersection (Egenhofer, 2010).

2.5.2.4 **Mereotopology**

Mereotopology is the study of part-whole and topological relationships for describing qualitative aspects of region connection. This is the combination of a spatial theory as a topological base along with a temporal theory to formulate a spatiotemporal interaction between the two theories. Mereotopological concepts are incorporated into calculus of relations between regions. This theory follows a particular syntax: spatiotemporal operators precede the regions they operate upon (prefix notation) while temporal operators come in-between the regions they operate upon (infix notation).

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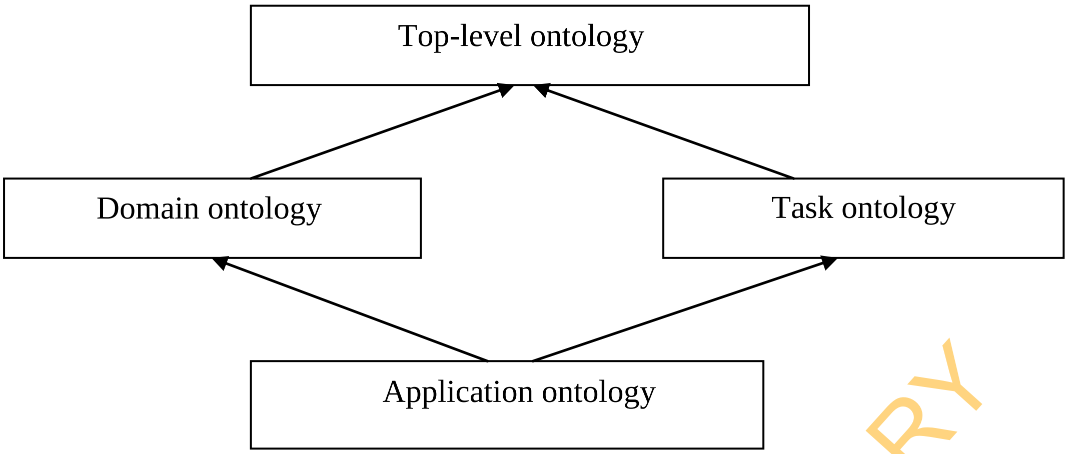

Figure 2.2: Types of ontology

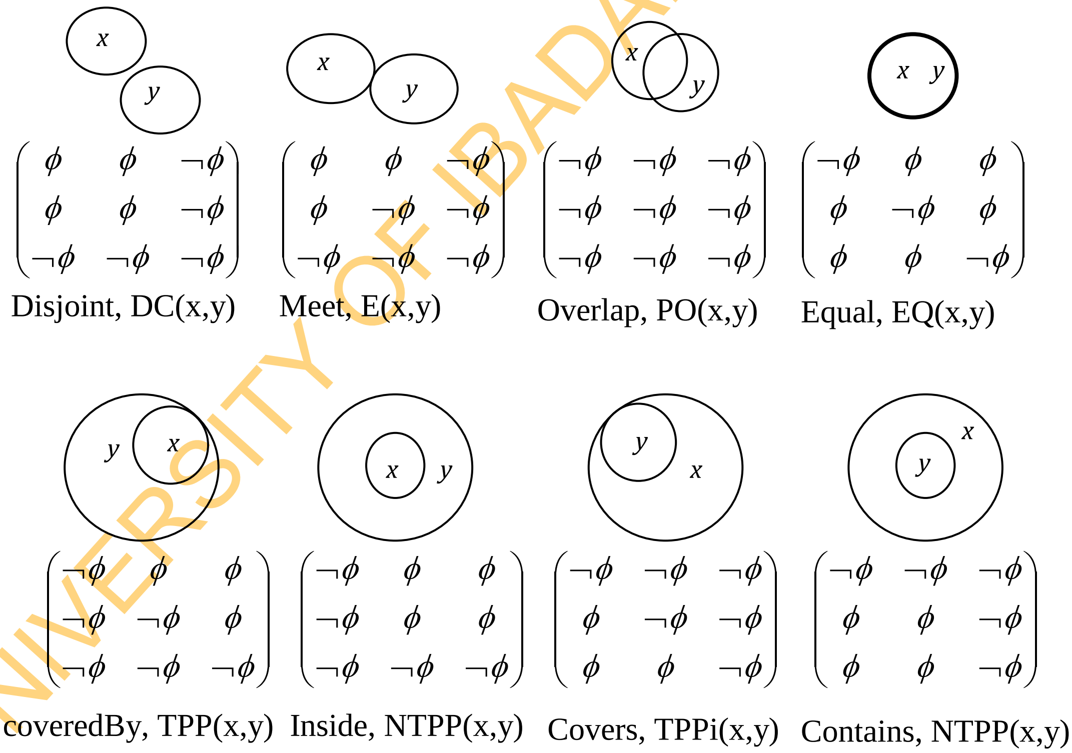

Figure 2.3: Eight topological relations between two regions in $\Re^2$

Although the absence of boundary entity has been considered by many as a serious flaw in mereotopology (Muller, 1998a), it does not always entail problems in most

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theories as it preserves certain homogeneity of the formulations of problems tackled by those theories.

The relation of strong connection between regions is used to describe topology by means of a "simple region" predicated "congruence" primitive whose axioms exploit Tarski's analogy between points and spheres describes morphology. This gives the three distinct primitives used in describing mereology, topology and morphological properties of the logical theory of space with tridimensional regions (Borgo et al., 1996). Also, spheres are defined and they make it possible to analyse Tarski's mereomorphological theory within their framework as shown below:

$$SPH_x =_{df} SR_x \wedge \forall y(CG_{xy} \wedge PO_{xy} \rightarrow SR(x-y)).$$

The notion of an egg-yolk is used to present a calculus for representing and reasoning about the location of rigid objects which may move within some regions. An axiomatization for congruence has both a mereological primitive and a morphological one.

2.5.2.5 **Mereogeometry**

This theory combines geometry and mereology in a simpler way and expresses it in First-Order axioms. It was developed to provide a secure ontological foundation for theories of spatial information and is directly inspired by Tarski's Geometric of Solids. The theory of parthood and the concept of spheres are taken as primitive for Tarski's geometry of solid (Bennett, 2001b). This theory builds on Lesneiwski's mereology following Tarski to introduce the sphere predicate, S(x) while defining the relations of external tangency (ET), internal tangency (IT), external diametricity (ED), internal diametricity (ID) and concentricity (x ⊚y) (Bennett et al., 2000).

Due to the availability of a categorical interpretation in terms of Cartesian fields of R, more traditional representations that employ this classical model of space are readily compatible. The limitation of region based geometry - RBG is that it can only deal with a domain of entities having a given fixed dimension. The second-order nature poses several problems for automated reasoning and requires more computationally effective representations to function in some applications.

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2.5.3 **Aspects of Qualitative Spatial Reasoning**

Qualitative spatial reasoning is applicable for reasoning in two major aspects of the physical world namely: temporal and spatial.

2.5.3.1 **Temporal Reasoning**

Many applications involving automated reasoning see time as a very crucial concept (Russel and Norvig, 2003). Since the world is dynamic (that is, constantly changing), the need to reason about time arises as events occur at different states of the world. These needs are seen in several AI domains such as that of question answering and also in task explanation and prediction (Vila, 1994). The efficacy of time has been explored as the theory of recurrence in time is defined by Koomen (1989) and the temporal properties of repetitive entities are also stated and proven (Akinkunmi and Osofisan, 2004).

Most of the systems involving temporal reasoning today follow existing formalism with well-defined semantics like the Allen's interval logic (Allen, 1984). Temporal reasoning owes so much to the work of McCarthy and Hayes on situation calculus (McCarthy and Hayes, 1969) which is actually its starting point. Situation calculus is a point-based temporal logic with a branching time model. Situation calculus sees only one agent at a time in the world without any external intuition about actions and events (Allen and Ferguson, 1994). The concept of time have been analysed by many. McDermott (1982) viewed time as an infinite collection of states or time points. Temporal entities were divided into facts and events where facts are true over a single point and events true over a time interval. Despite the fact that it was challenged by Galton (1994) by showing the existence of events that are instantaneous in the domain of bodies moving in space, was partially supported by Allen who considered time intervals to be the primitive time units in his trichotomy of temporal entities (that is, properties, events and processes).

Temporal logic is needed to describe any system of rules and symbolism for representing and reasoning about propositions qualified in terms of times (McDermott, 1982), for example, "I am always hungry", "I will eventually be hungry", "I will be

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hungry until I eat something." Temporal Logic has ability to reason about time line: Single or Multiple time lines.

Reasoning about multiple time lines or acting unpredictably is what led to the branching logic. For example, representing statements like "there is a possibility that he will stay hungry forever" or "there is possibility that eventually I am no longer hungry" will require branching-logic. Attempts to provide temporal calculi are based on these time structures: time point or time interval.

2.5.3.1.1 **A point-based system**

A typical point-based time structure is an ordering (P, $\preccurlyeq$), where P is a set of points, and $\preccurlyeq$ is a relation that (partially or totally) orders P. Ma and Knight (1994) pointed out that time points are needed for both theoretical and practical modeling of temporal phenomena. From point-based systems, interval may be defined as derived temporal object, either as sets of points or as ordered pair of points (Ladkin, 1987). Defining intervals as objects derived from points may lead to the so called dividing instant problem (Villa, 1994; Ma and Knight, 2003).

2.5.3.1.2 **Interval-based system**

This system is believed by many researchers to have been more suitable for representing commonsense temporal knowledge, notably in the domain of linguistics and AI. Allen's temporal theory is a representative of the interval-based system which posits a set of intervals as the primitive temporal entities. Allen introduced the 13 binary relations between intervals (equal, meets, starts, ends, contains, overlaps, before and their inverses) (Allen, 1983). These jointly exhaustive pairwise disjoint (JEPD) base relations over intervals have been employed in most of the spatio-temporal models.

This allows for non-trivial interpretations, which may involve linear flow of time, at every moment where the future is fixed or at different evolution of history. Talking about history H, formulas are evaluated relative to pairs (h,w) consisting of an actual history, $h \in H$ and a time point $w \in h$. Temporal operators are interpreted along actual history, h as in a linear time framework, with the modal operators quantifying over the

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set of all possible histories H(w)={h′∈H: w∈h′} coming through the time point, w, where w is said to be the branching-time (Wolter and Zakharyaschev, 2002). Several branching-time logics have been defined: Linear Time Logic (LTL), Computational Tree Logic (CTL) and the extended Computational Tree Logic (CTL*) (Katoen, 2007). CTL is branching-time logic with a tree-like structure of the model of time in which the future is not determined or has many possibilities. CTL* is the superset of CTL and LTL and combines path quantifiers and temporal operators. Its formal semantics is defined with respect to a given Kripke structure.

2.5.3.2 **Qualitative Spatial Calculi**

Based on the theories discussed in section 2.5.2, several calculi have been constructed to handle spatial concepts such as shapes, size, distance, orientation, topology and time. Qualitative spatial calculi are well-suited to bridge between quantitative scene information observable by an agent and linguistic descriptions of object configurations (Galton, 1994). This is because qualitative spatial calculi abstract from metrical data by summarizing similar quantitative states into one qualitative characterization, thereby, revealing the relative nature of spatial information, that is, properties of objects are compared to one another rather than comparing the properties to some external scale. (Dylla et al, 2007). Some of these calculi due to their relevance and common usage are discussed below.

2.5.3.2.1 **Region Connection Calculus (RCC):**

The Region Connection Calculus (RCC) is a topological approach to qualitative spatial representation and reasoning where spatial regions are non-empty regular subsets of a topological space. Relationships between spatial regions are defined in terms of the relation *C(a. b)*, read as "*a* connects with *b*". In the standard interpretation of the RCC theory, the relation *C(a, b)* is true if and only if the closure of region *a* is connected to the closure of region *b*, i.e., if the closures of the two regions share a common point.

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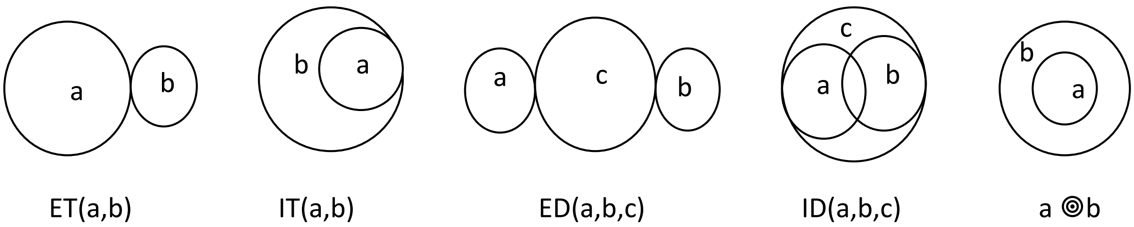

Figure 2.4: Relations among spheres defined by Tarski (Bennett, 2001b)

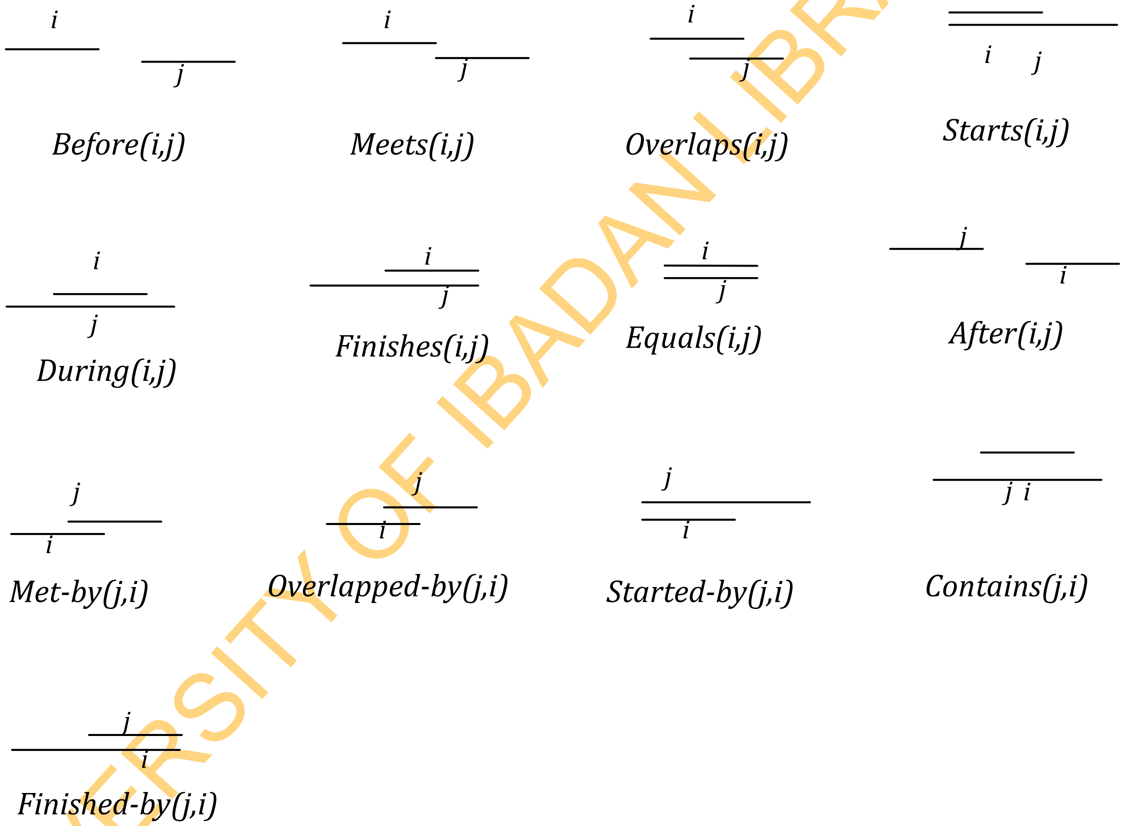

Figure 2.5: The Thirteen Allen's Interval Relations

Regions themselves do not have to be internally connected, i.e., a region may consist of different disconnected parts, and regions are allowed to have holes. The domain of

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spatial variables (denoted as *X. Y, Z*) is the set of all spatial regions of the topological space (Egenhofer and Franzosa, 1991).

Randell, Cui and Cohn introduced the Region Connection Calculus (Randell et all, 1992) with the aim of reasoning based on the primitive binary relation ‘x connects y’, C(x,y). RCC relations are defined in terms of the connection, C. The RCC theory is formulated in first-order predicate calculus. RCC-8 is a set of eight jointly exhaustive and pair wise disjoint (JEPD) relations, called base relations, definable in the RCC theory, denoted as DC, EC, PO, EQ, TPP, NTPP, TPP-^{1}, and NTPP-^{1}, with their meaning as *DisConnected, Externally Connected, Partial Overlap, EQual, Tangential Proper Part, Non-Tangential Proper Part*, and their converses. Exactly one of these relations holds between any two spatial regions. These relations can be given a straightforward topological interpretation in terms of point-set topology. Examples for the RCC-8 relations are shown in Table 2.1.

RCC-5 is a set of five JEPD relations definable in the RCC theory on a coarser level of granularity than RCC-8. For RCC-5, the boundary of a region is not taken into account, i.e., one does not distinguish between DC and EC and between TPP and NTPP. These relations were combined in the RCC-5 base relations with DR for *DiscRete* and PP for *Proper Part*, respectively. Thus, RCC-5 contains the five base relations DR, PO, PP, PP-’, and EQ. From this primitive binary relation, several other binary relations (Bogaert, 2008) sprang up as shown in Table 2.1.

This set of relations is also known as RCC-8. There are other sets of extended RCC relations resulting from the combination of RCC-8 and the earlier version, RCC-5. Another extension of the RCC-8 relations is the Boolean Region Connection Calculus (BRCC-8) which combines the region variables using the Boolean operators ∪, ∩ and ¬. BRCC-8 behaves like the RCC-s computationally with BRCC-8 considering the union, intersection and negation of regions which RCC did not.

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Table 2.1: Relations Defining the Region Connection Calculus (RCC) (Randell et al, 1992; Bogaert, 2008)

Relation	Condition	Symbol	Meaning
x is disconnected from y	$\neg C(x,y)$	DC(x,y)	x and y are not connected
x is a part of y	$\forall z[X(z,x) \rightarrow C(z,y)]$	P(x,y)	Every region connected to x is connected to y
x is proper part of y	$P(x,y) \wedge \neg P(y,x)$	PP(x,y)	x is part of y but not equal to it
x is identical with y	$P(x,y)=P(y,x)$	EQ(x,y)	Each of x and y is part of the other
x overlaps y	$\exists z[P(z,x) \wedge P(z,y)]$	O(x,y)	Some region is part of both x and y
x is discrete from y	$\neg O(x,y)$	DR(x,y)	x does not overlap y
x partially overlaps y	$O(x,y) \wedge \neg P(x,y) \wedge \neg O(y,x)$	PO(x,y)	x overlaps y but neither is part of the other
x is externally connected to y	$C(x,y) \wedge \neg O(x,y)$	EC(x,y)	x and y are connected but do not overlap
x is tangential proper part of y	$PP(x,y) \wedge \exists z[EC(z,x) \wedge EC(z,y)]$	TPP(x,p)	x is a proper part of y and some region is EC to both
x is a nontagential proper part of y	$PP(x,y) \wedge \neg \exists z[EC(z,x) \wedge EC(z,y)]$	NTPP(x,p)	x is a proper part of y but not TPP

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However, such an encoding does not lead to efficient decision procedures. In order to overcome this problem, Bennett used different encoding of RCC-8 in modal logic. Based on this encoding, the fact that reasoning is NP-complete in general and identifies a maximal tractable subset of the relations in RCC-8 that contains all base relations is proven. Furthermore, path consistency is shown as being sufficient for deciding consistency for this subset (Renz and Nebel, 1999).

2.5.3.2.2 **Anchoring Relations**

Several approaches have been adopted in the qualitative spatial reasoning field to represent vague or uncertain information concerning spatial location. Amongst these is the anchoring relations proposed by Galton and Hood (2005) to express location information about objects in information space. These relations enable one to state exactly what is known about the spatial location of an object without forcing people to identify the fuzzy sets.

Anchoring according to Galton and Hood means referencing spatial information without assignment of precise coordinates to its location. This does not eliminate vagueness forcefully by approximating precise regions. Galton and Hood pointed out that other approaches are based on faulty assumptions that vague objects can be associated with regions representing their spatial extent. Some of these approaches include the rough/fuzzy sets, egg-yolk and super valuation (Cohn et al., 1997). All these establish the nature of the relationship between information space and precise space.

Two sets of relationship between an entity in information space and an exact spatial location in the ontology of anchoring are allowed, including:

- That which represents information space: The information space contains all the various geographical objects and phenomena of interest.
- That which represents precise space: The precise space contains points and various kinds of points which can stand on their own as precise spatial location.

The object in information space may be anchored to the locations in precise space using the following relations:

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(i) Anchored in

(ii) Anchored over

(iii) Anchored outside

(iv) Anchored alongside

Through these relations location information about objects in information space can be expressed without being embarrassed by the uncertainty or vagueness that unavoidably attends much of our day-to-day information. With these anchoring relations (Galton & Hood, 2005), one can state exactly what is known regarding the spatial location of an object without forcing people to identify that location with either a region in precise space or any mathematical construct from such regions.

2.5.3.2.3 **Direction Calculus**

Bogaert pointed out that despite the qualitative description of directions used by people in their day-to-day communications (such as 'west of', 'behind', 'on top of', etc.), the directional relations of an object to another object can be defined in terms of three basic concepts, namely: a primary object, a reference object and a certain frame of reference (Bogaert, 2008; Clementini et al, 1997).

These concepts are equally applicable to our location information prior our spatial qualification as will be seen in the later sections of this thesis. Hence, most directional relations are ternary due to the introduction of the frame of reference unlike the topological relations (Cohn and Renz, 2007) earlier discussed. A frame of reference may be extrinsic, intrinsic and deictic (Bogaert, 2008; Retz-Schmidt, 1988), but our emphasis in this work will be on the deictic frame where the system is imposed by the point of view from which the reference object is seen.

Examples of directional relations include the cardinal direction calculi or relations which in most cases are extended by a qualitative value'0' for closed points or directional representation. This may be cone-based directional relations or projection based directional relations.

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Directions in geographic domains, according to Egenhofer (2010b), are referred to as cardinal directions, a triple <A,d,B> where A and B are reference and target objects respectively and d is a non-empty subset of nine symbols {N,S,E,W,NE,SE,SW,NW,O} for north, north-east, east, south-east, south, south-west, west, north-west, equator/origin respectively, with semantic motivated by a compass rose. Another example of the directional calculus is the double cross calculus (Freksa, 1992).

Two different methods for determining the different sectors corresponding to the single directions include the cone-based method and the projection-based method (Frank, 1991). The projection-based approach allows us to represent the nine different relations (n, ne, e, se, s, sw, w, nw, eq) in terms of the point algebra by specifying a point algebraic relation for each of the two axes separately. Renz and Mitra (2004) proposed and analysed a more generalized calculi that is based on a number of n lines: the star calculus shown in figure 2.6 where multiple granularities are allowed over an intrinsic reference frame. Star calculus can be used for representing and reasoning about qualitative directions of arbitrary granularity. Freksa (1992) further developed the point-based approach referred to as the double-cross calculus, which defines the direction of a located point to a reference point with respect to a perspective point.

2.5.3.2.5 **Distance Calculus**

Distance relations can be relative or absolute. In other words, distances can be named and compared using relative distance relations or absolute distance relation. While relative distance relations are purely qualitative, absolute distance relations can be both qualitative and quantitative. Looking closely at the qualitative absolute distance relations, are they truly qualitative? The relative distance relations, can they really hold without prior knowledge? This will be expounded in the coming section of this thesis.

Renz and Nebel (1999) pointed out situations where directional qualitative distances can lead to difficulties despite the fact that reasoning in distance calculi is often based on points rather than regions or lines. This led to the combination of directional and distance information referred to as positional/location information (Hazarika, 2005).

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2.5.3.2.6 **Positional Calculus**

Positional calculus is employed in reasoning. It combines direction calculus and distance calculus. An instance of positional calculus are seen where information given by cardinal directions and two distance relations are combined (Clementini, 1997). Isli and Moratz (1999) also combined relative directions with relative distance.

2.5.3.2.7 **Qualitative Trajectory Calculus (QTC)**

Qualitative trajectory calculus came into describe the level of disjointness between two moving objects (Van de Wedge, 2004) and is used for the representation of and reasoning about movements of objects in a qualitative framework. QTC features the exploration of trajectories of moving (point-like) objects with concentration on the shortest path of spatially disjoint object (assuming the object doesn't change form). Objects are viewed as points with the direction and orientation relation determining the positional information of the object. Two types of a moving object, namely: objects with free trajectory and objects with constrained trajectory were considered as all traffic movements are bounded by a network.

Apart from the movement of the object, relative speed of the moving object is another function that qualitatively represents the movement or transition of the object in consideration. The movement of two objects restricted to a line, be it straight or otherwise is viewed and the assumption that R1 and R2 only differ in one character that cannot change continuously between both states without passing through an intermediate qualitative value, and then the conceptual distance between R1 and R2 composing of sub-distances. This follows the assumption that R1 and R2 differ in many characters, and then the conceptual distance is the sum of the sub-distances determined for each individual character.

Also, considering two objects that can move together freely in a plane, the landmark to describe the qualitative relations as seen in Qualitative trajectory Calculus (QTC_B) is the distance at time t between the two objects (Bogaert, 2008). The following parameters were considered necessary: the position of an object x at time t; the distance between two positions u and v; and the speed of x at time t. Despite the use of these parameters, the need for the logic that can tell the possibility of an object

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reaching another location from former was not seen or constructed. The qualitative trajectory calculus (QTC) enables comparisons between positions of objects at different time points to be defined with distance and speed constraints as the base primitives (Van De Wedge et al., 2006). QTC relations simplify continuous movements of objects in the real world to be:

(i) Only objects in a disjoint relation
(ii) Definition at an exact movement in time with duration (i.e. no time points)
(iii) Only relations between two spatial entities with respect to a certain frame of reference
(iv) Generalized objects into points.

The above abstractions simplify complex motion problems without having significant disadvantages (Van de Wedge, 2004).

QTC are of two types, the Qualitative Trajectory Calculus – Basic (QTC_B) and the Qualitative Trajectory Calculus – Double Cross (QTC_C) depending on the level of detail and the number of spatial dimensions (Van de Wedge, 2006; Van de Wedge et al, 2005). In their work, two objects are considered: one is fixed in time and the other varies over time. Motion is defined with relations including: move toward, move away from and stable. This formalism did not give general motion relations, thereby still having the need for a general motion axiom.

In QTC_C, objects have a tangential trajectory and combines distance and orientation information. This is otherwise referred to as the positional calculus while QTC_B is a pure distance calculus.

2.5.3.3 **Spatiotemporal Representations and Reasoning**

In other to see space and time as closely connected spatial concepts due to occurrence of change, the combined reasoning approach where spatial and temporal information are used sprang up as a subfield in Qualitative Reasoning. According to Bogaert (2008), change can be discontinuous or continuous. An instantaneous alteration in the value of a property of an object from one value to another depicts a discontinuous change. On the other hand where the property of an object varies as a function of time we refer to this as a continuous change, e.g. temperature change during the day.

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Two types of spatial entities are identified to be, life of a spatial entity and the position and geometric form of a spatial entity. The former can appear, split, merge or disappear while the later can move or appear to move while or while not simultaneously changing its form. On close examination one may say that the first type acts under natural influences while the second is time dependent.

2.5.3.3.1 **Qualitative Spatial Change**

Substantial progress has been made in QSR about motion (Forbus et al., 1991; Muller, 1998a; Muller, 1998b). From an absolutist view, motion is a change of value of a location function, usually assumed to be continuous. This assumption is integrated in the different qualitative calculi (Cohn and Hazarika, 2001; Muller, 1998b, Gerevini and Nebel, 2002; Cole and Hornsby, 2005; Hornsby and Cole, 2007). Planning motions in the presence of uncertainty for any kind of rigid or articulated object capable of controlling its motions within a workspace had been a major problem. Whenever an object occupies different positions in space at different times, then the phenomenon of movement arises (Galton, 1995). Instances are as seen in a manipulator arm, a multi-joint multi-finger hand, a wheeled vehicle, or a free-flying vehicle which requires control of its motion. However, in practice, the complexity of the motion planning problem augments exponentially with the number of degrees of freedom of the robot system. Motion planning in the presence of uncertainty is one of the important problems that need to be solved in order to create autonomous robots, that is, robots that can execute tasks in the physical workspace without human intervention (automatic) and those that accept high level task descriptions (taskable). This uncertainty cannot be handled by some of the model with motion command like M=(CS,TC) (Latombe, 1988), which is quantitative and only feature the control statement specifying the trajectory along which the controller executing the command has to move and the termination condition upon which the controller should terminate the motion (Latombe, 1991). This led to the shift from quantitative to qualitative approach (Egenhofer, 2010)

Qualitative attempts that have been made are categorized into two: that dealing with change (environmental change) and that dealing with moving objects. Most of these models are spatiotemporal and are set to deal with imprecise or incomplete information

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in AI systems (Ibrahim and Tawfik, 1998; Muller, 1998a; Muller, 1998b). Some of the approaches used in describing motion in the literature include the following:

a. **Translation and Rotation Approach**

Qualitative kinematics capable of describing the possible movements of systems of rigid objects in a far more general way were formulated within the framework of region based geometry (RBG). This was done to show their expressive nature by Bennett et al. (2000). These formulations give qualitative description of rigid body motions within constraining environments, where simple motions of linear translation and rotation about the centre point of some spheres are specified. The relation, TAV (Translates Along Vector) is defined, with

$$TAV(x1, x2, d1, d2) \equiv \exists d[d \circledcirc d2 \wedge CG(x1,d1,x2,d)]$$

meaning that the translation of a region x1 to the congruent region x2 along a vector is defined by the points of two discs d1 and d2.

Also, the PTAV (translation pathway along a vector) is defined as

$$PTAV(x1, x2, d, d2) \equiv \exists\ d[B(d1,d,d2)] \wedge CG(x1, d1, x2, d)$$

and the rotation of a region about a centre point is generally defined as:

$$Rot(x,y,s) \equiv S(s) \wedge CG(x,s; y,s)$$

The rotation of an object within some confining environment is defined as

$$\begin{aligned} RotOrd(a,c,c,s) \equiv\ & Rot(a,b,s) \wedge Rot(b,c,s) \wedge \exists a',b',c'[CG\{a',b',c',s\} \wedge EC(a',s) \\ & \wedge EC(b',s) \wedge EC(c',s) \wedge CG(a,a'; b,b') \wedge CG(b,b'; c,c') \\ & \wedge \exists^{\circ}t[EC(t,s) \wedge IT(b',t) \wedge O(t,a) \wedge O(t,c)]] \end{aligned}$$

Using the defined theories, a model of physical environments useful for reasoning about motions of rigid objects was built. Bennett et al. (2000) viewed the following questions in their model:

(i) Can a rigid body move between two locations within a confining environment?
(ii) If yes, what is a possible path between the two locations?

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They defined the linear translation within (LTW) relation to be:

$$LTW(x1,x2,y) \equiv \exists d1d2[TAV(x1,x2,d1,d2) \wedge \forall x\ [PTAV(x1,x,d1,d2) \rightarrow P(x,y)]]$$

This model is limited to robotic applications, as continuous motions cannot be reduced to a series of linear translations and rotations.

b. **Historical Examination Approach**

Muller (1998a, 1998b), in his work, combined RCC and Allen's Interval Logic to define motion by examining the relationships between histories at two consecutive time intervals and identifies classes of motion that may be true as regions move accordingly. He did this by presenting the study of motion from the point of view of qualitative theory and the representation of human spatial knowledge in a computational perspective.

Muller saw spatial data as being useful in context where exact information are not available or not easily processed. This further shows QSR as a subfield of AI that emerged to focus on problems arising from quantitative data, which are remote to human cognition and experience but obstacle to human-computer communication. The work formally models relations between moving entities which are the properties of space and time. Several categories of motion that can be useful in a quantitative context are expressed in the model. The characterization of motion described by the verbs considered in this model shows the isolation of three main features, namely: a polarity, a phase of the motion which the verb semantics focuses; the topological relation between two entities related by the motion event during the phase defined by the polarity of motion; and the change of this topological relation during the motion. The combination of these features yielded six non-empty classes of motion verbs: internal/initial, internal/final, contact/final, internal/median, medians with change, and non-topological medians.

Based on some of the defined spatio-temporal concepts, Muller defined six classes of motion to be: LEAVE, REACH, HIT, CROSS, INTERNAL and EXTERNAL. Their formal representations are as follows:

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(i) $REACHzxy = TEMP\_IN \wedge FINISH$

(ii) $LEAVE = TEMP\_IN \wedge START$

(iii) $INTERNAL = PP$

(iv) $HIT = EC \wedge \forall x1,y1\ [(P_{x1} \wedge P_{y1} \wedge EC_{x1y1}) \rightarrow (FINISHx1z \wedge FINISHy1z)]$

(v) $EXTERNAL = \neg Cxy$

(vi) $CROSSzxy = \exists z1,z2\ (z=z1+z2 \wedge MEETSz1z2 \wedge REACHz1xy \wedge LEAVEz2xy)$

c. **Oriented Curves**

Eschenbach et al. (1999) also reported the application of oriented curves (that is, geometric specification of arrows of maps or diagrams, or any other linear and directed device in diagrammatic reasoning) on the course of motion or a given trajectory of an object. An oriented curve represents both the collection of the positions occupied by the moving object and the order of occupation of the places.

Eschenebach et al (1999) saw trajectory objects to exhibit several spatial properties: they are connected, they have shape, do not branch and are directed. The successive positions occupied by the objects in the course of its motion are represented as points. However, this cannot represent walking in contrast to running, which is the reason representations of trajectories of moving objects are traditionally based on mappings from time to space. Simultaneity is the only temporal notion needed to reason about the possibilities of moving objects meeting, but not explicitly represented.

Some of the reviewed qualitative spatial and temporal calculi, their relations with examples are summarised in table 2.2 with corresponding citations.

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Table 2.2: Qualitative Spatial and temporal Calculi Reviewed

Citations	Calculi	Relations	Examples
(Freksa, 1992)	Star, Double cross, cardinal direction	direction	left, above,…
(Bogaert, 2008)	Point, relative distance	distance	far, near,…
(Borgo et al., 1996)	Size	size	large, tiny,…
(Carsati and Varzi, 1997)	Shape	shape	oval, convex,…
(Randell et al. 1992)	RCC-5, RCC-8	topology	touch, inside,…
(Van de Wedge et al., 2004; Muller, 1998a)	QTC	Motion	reach, leave, hit,…
(Allen, 1983)	Allen Interval logic	Interval rel.	before, after, …
(Galton and Hood, 2005)	Anchoring relations	Anchoring	around, within

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2.6 Qualitative Spatial Reasoning: Gaps and Way Forward

The capabilities of qualitative reasoning in solving most commonsense reasoning problems with partial or incomplete information were addressed (Cohn and Hazarika, 2005; Uribe et al, 2002; Forbus, 1995). Most of this intuitive and informal knowledge are qualitatively formalized or modeled (Frommberger, 2008a, 2008b; Erwig et al, 1999).

While most of the literatures argue for qualitative reasoning, others who desired qualitative approach to handle the shortcomings of quantitative reasoning keep experiencing challenges that make them think that qualitative reasoning alone cannot actually model all the commonsense problems in all domains. Forbus (2008) stated that qualitative reasoning (QR) does not only represent commonsense knowledge but the underlying abstractions used by engineers and scientists when they create quantitative models. QR utilizes discrete quantity spaces and this discretization is relevant to the behavior being modeled. This means that very little quantity space can be useful to reason qualitatively. An example is a finite quantity space, which is a totally ordered set of symbolic landmark values (zero and positive and negative infinity) representing qualitatively important values (Kuipers, 2001).

Several aspects of Qualitative spatial reasoning depends on the angle at which it is viewed, that is, the way one has chosen to describe the relationship between the spatial entities (Cohn and Renz, 2008; Egenhofer, 2010). These aspects of qualitative spatial reasoning include ontology (Guarino, 1998), topology (Cohn et al, 1997), orientation (Freksa, 1991), distance (Van de Wedge et al 2004; Van de Wedge et al 2006), size and shapes (Bennett et al., 2000). Hence, the proposed logical theory will also adopt the use of qualitative approach and it does not also mean the avoidance of quantitative approach since discrete quantities such as distances and time stamps will still be very useful in the logical model. In the next section, this research is taking a walk around the existing logical theories in literatures in other to know the most suitable for spatial qualification problems.

2.7 Logical Theories

Generally, logic is accepted to be the formal method of analyzing and representing any valid argument type using its language (grammar and symbolism). The use of any

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logical language to represent sentences of any form of argument makes its content usable in formal inference. Formalization means producing the logical forms of argument by translating English sentences into the language of the logic. These sentences are referred to as axioms and/or formal theories and their combination gives what is referred to as the logical system or formal theory.

Through the set of axioms, a resolution-based module has proven that a question logically follows from the answer. An instance of this proof is given in the first order representation and reasoning for natural language as it applies to question answering (Uribe et al, 2002). The use of logic is further seen while showing the relationship between tokens and types and their usage in highlighting the expressive limits of reified theories (Akinkunmi, 2000). The role and place of logic is as shown in the static and dynamic aspects of a semantic web layer cake given in figure 2.7.

2.7.1 Reasons for Using Logical Theories

Pan (2007) reported in his work that logic specifically provides a sound computational basis for the verifiability, inference, and expressiveness requirements. Logic helps to characterize the difference between valid and invalid arguments. This means that, logic allows one to distinguish correct reasoning from poor reasoning, thereby, aiding one's correct reasoning. Without correct reasoning, one does not have a viable means for knowing the truth or arriving at sound beliefs. No wonder why it plays an important role in the semantic web (Pan, 2005; Pan and Hobbs, 2006; Pan et al., 2006a, 2006b, 2007). A logical system for a language is a set of axioms and rules designed to prove exactly the valid statements in the language. Several formal languages/logical representational languages have been used for reasoning within the spatial domains. Some of the logical languages relevant to this study are discussed in the following section.

2.8 Logical/Formal Languages

Logical languages are particularly suitable for use as knowledge representation languages for AI because they provide precise means for determining what conclusions followed from available facts and rules (Ramsay, 1989). Examples of these logical languages as it applies to the study amongst the numerous ones are discussed in the following sub-sections.

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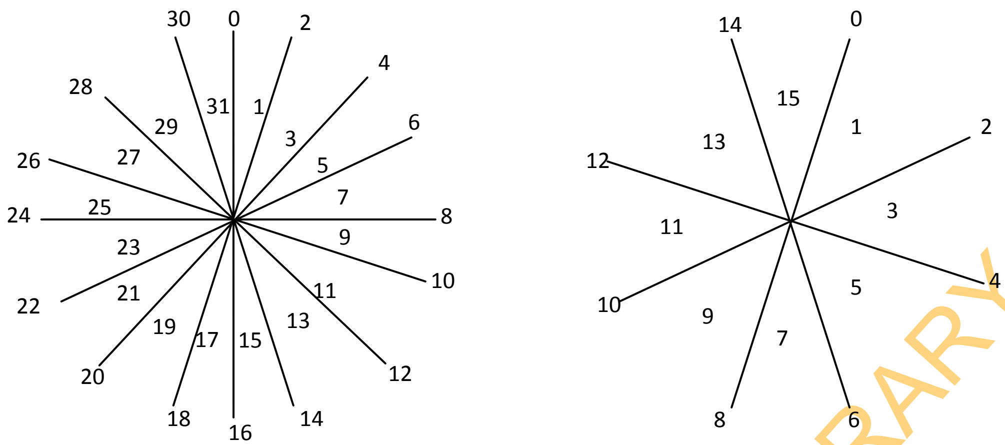

Figure 2.6: Star calculi showing relation formation

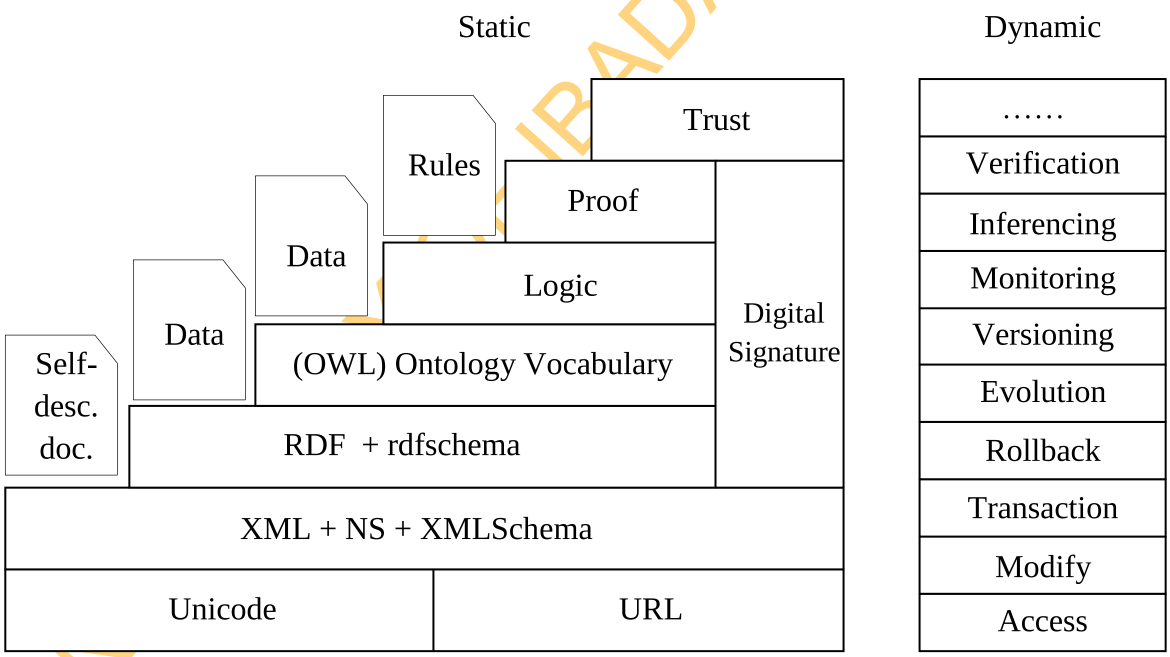

Figure 2.7: Static and Dynamic aspects of the Semantic Web Layer cake
(Source: Oberle et al., 2003)

2.8.1 **First Order Logic (FOL)**

First Order Logic amidst other representational languages has a flexible structure which permits accurate representation of natural language reasonably well. Also, it offers a formal approach to reasoning that has a sound theoretic foundation. First Order Logic is an expressive formal language that allows for powerful reasoning and can manage incomplete knowledge. Models in first order logic (FOL) require total functions, that is, there must be a value for every input tuple.

2.8.1.1 **Syntax of First Order Logic**

In the context of this language, symbols stand for objects, relations and functions in the universe. Symbols could be constants which represent the objects, predicates representing the relations and the functions. First order logic (FOL) follows the following syntax (Russel and Norvig, 2003):

Sentence	→	Atomic Sentence | (Sentence Connective Sentence)
		| Quantifier Variable Sentence | ¬ Sentence
Atomic Sentence	→	Predicate (Term, …) | Term = Term
Term	→	Function (Term, …) | Constant | Variable
Connective	→	⇒|∧|∨|⇔
Quantifier	→	∀|∃
Constant	→	A|X_1|John|…
Variable	→	a|x|s|…
Predicate	→	Before|HasColor|Raining|…
Function	→	Mother | Leftleg …

From the syntax, it could be noticed that there are two standard quantifiers, namely, *for all* (∀) and *there exist* quantifiers (∃). A term is a logical expression that refers to an object. Constant symbols are terms. Formally, $f(t_1, \ldots, t_n)$ is considered as a term, the function symbol *f* refers to some functions in the model. Atomic sentences are formed from the predicate symbol followed by a parenthesized list of terms. An atomic sentence is true in a given model, under a given interpretation if the relation referred to

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by the predicate symbol holds among the objects referred by the arguments. Complex sentences use logical connectives to construct more complex sentences.

This gives rise to the following formal definitions:

Term:

(i) Constants, variables are terms

(ii) If f is a function of k variables and $t_1, \ldots, t_k$ are terms then $f(t_1, \ldots, t_k)$ is a term.

Formula:

(i) If P is a k-ary predicate and $t_1, \ldots, t_k$ are terms, it implies that $P(t_1, t_2, \ldots, t_k)$ is a formula

(ii) If A, B are formulae then $A \vee B$, $A \wedge B$, $\neg A$ are formulae.

2.8.1.2 **Semantics of First Order Logic**

According to Paulson (2002), an interpretation of a language maps its function symbols to actual functions, and its relation symbols to actual relations. For example, the predicate symbol 'student' could be mapped to the set of all students currently enrolled at the University.

Let L be a first-order language. An *interpretation* I of L is a pair (D, I). Here D is a nonempty set, the *domain* or *universe*. The operation I maps symbols to individuals, functions or sets:

(i) if c is a constant symbol (of L) then $I[c] \in D$

(ii) if f is an n-place function symbol then $I[f] \in Dn \rightarrow D$ (which means $I[f]$ is an n-place function on D)

(iii) if P is an n-place relation symbol then $I[P] \subseteq Dn$ (which means $I[P]$ is an n-place relation on D).

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He highlighted various ways of talking about the values of variables under an interpretation. One way is to 'invent' a constant symbol for every element of *D*. More natural is to represent the values of variables using an environment, known as a valuation. A valuation *V* of *L* over *D* is a function from the variables of *L* into *D*. Writing *IV* [*t*] for the value of *t* with respect to I and *V*, it is defined by

$IV\,[x]$ def = $V(x)$ if x is a variable

$IV\,[c]$ def = $I\,[c]$

$IV\,[\,f\,(t1, \ldots, tn)]$ def = $I\,[\,f\,](IV\,[t1], \ldots, IV\,[tn])$

Writing $V\{a/x\}$ for the valuation that maps *x* to *a* is otherwise the same as *V*. Typically, a valuation is modified one variable at a time. This gives a semantic analogue of substitution for the variable *x*.

2.8.2 **Modal Logic**

Modal logic (ML) is a better tool for talking about topologies. This is because it allows for change, hence, a tool for reasoning about time, beliefs, computational systems, necessity and possibility, and much else besides topologies. Modal logic is a non-classical logic which has modality or several modalities in it that allows for reasoning about uncertainty (Coppin, 2004). According to Coppin, ML allows a logical system without the law of excluded middle, it only holds due to time, and is called a contingent. Modality is also a connective which takes a formula/s and produces a new formula with a new meaning. Modal means qualification over the truth of claim. Examples of Modal logic (Ballarin, 2010) include:

(i) Alethic Logic - Necessarily, Possibly

(ii) Temporal Logic - Will be, Was, Has been, Will have been

(iii) Deontic Logic - May, Can, Must

(iv) Epistemic Logic – Certainly, Probably, Perhaps, Surely.

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Most applications employing key ideas (like flow of time, relation between epistemic alternatives, transitions between computational states, networks of possible worlds) can be represented in graph-like structures.

Modal logic has been widely employed to formalize mental attitudes of intensional artifacts such as beliefs and desires, although some choose to stick to classical predicated logic. Typical issues are how to deal with motivational attitudes such as intensions and with informational attitudes such as beliefs. Another issue is on how to formalize commonsense reasoning (the way human reason in formal sciences such as mathematics and logic), the reasoning patterns connected with default (rules of thumb) and counterfactuals. This include non-monotonic reasoning and belief revision (dealing with how the belief of a reasoner change when new information becomes available and is incorporated) applicable in companion robots. The interesting aspect of Modal logic is its dynamics, that is, the ability to change over time (Dosen, 1992).

In other to proof the decidability of RCC-8, modal logic was introduced and used as a spatial logic: a logic of necessity and possibility. Although Orlov, Lewis and Godel first introduced Modal Logic (Zalta, 1995), they did that without any intention to reason about space but to interpret intuitionistic logic in classical logic. Lewis went further to give the modal schemata of S4 to be:

$$\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$$

$$\Box\varphi \rightarrow \varphi$$

$$\Box\varphi \rightarrow \Box\Box\varphi$$

with possibility operator defined to be

$$\Diamond\varphi = \neg\Box\neg\varphi$$

This S4 modal schemata is interpreted in topological spaces. Also, modalities, when added to temporal language serve as quantifiers over possible histories (Wolter and Zakharyaschev, 2002). This further qualifies ML as a suitable language for reasoning with spatial and temporal concepts.

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2.8.2.1 **Syntax of Modal Logic**

A basic modal formula consists of a proposition symbol, a Boolean constant, a Boolean combination of basic modal formula, or a formular prefixed by a diamond. According to (MacFarlane, 2011) a formula (a proposition) could be

(i) necessarily / possibly true
(ii) true today / tomorrow
(iii) believed / known
(iv) true before / after an action / the execution of a program.

There are three types of modal logic, namely: Basic Modal Logic, Normal Modal Logic and Multi-Modal Logic.

1. **Basic Modal Logic (BML)**

The language ζ_{PML} of BML is that of propositional logic with two extra connectives, $\Box$ and $\Diamond$. The alphabets of ζ_{PML} consist of:

(i) Enumerable set of propositional variables: $\phi = p_0, p_1, \ldots, q_0, q_1, \ldots,$
(ii) The logical constraints: T(true) and $\bot$(false)
(iii) The Boolean connectives: $\wedge$(and), $\vee$(or), $\Rightarrow$(implies), $\Leftrightarrow$(if and only if) and $\neg$(not).
(iv) The modal operators: $\Box$(it is necessary) and $\Diamond$(it is possible).

The language is defined by the following Backus Naur Form(BNF):

$$\zeta_{PML} ::= \bot \mid T \mid p_i \mid \neg\varphi \mid (\varphi\wedge\psi) \mid (\varphi\vee\psi) \mid (\varphi\Rightarrow\psi) \mid (\varphi\Leftrightarrow\psi) \mid \Box\varphi \mid \Diamond\varphi,$$

The propositional variables (p_0, p_1, …) and constants ($\bot$, T) denote atomic formulas whereas φ, ψ denotes sets of formulas. If φ and ψ are ζ_{PML} formulas, then so are $(\varphi\wedge\psi)$,$(\varphi\vee\psi)$, etc. following standard tradition. Examples of formulas of the BML are

$$(p_0\wedge\Diamond(p_0\Rightarrow\Box\neg p_3))$$

and

$$\Box((\Diamond p_1\wedge\neg p_3)\Rightarrow\Box p_0)).$$

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Although many of such formulas can be constructed using the basic modal language, ζ_{PML}, not all of them constitute a model system. The basic idea of constructing a modal system is to single out and describe those formulas that represent true propositions no matter what true values are assigned to the variables. The set of formulas (usually referred to as axioms) together with a set of rules is denoted by Λ and is often called the logic of the corresponding system.

2.8.2.2 **Semantics of Modal Logic**

Formulas in a modal system should satisfy certain truth conditions. Note that, the falsity and truth of variables in classical propositional logic is with respect to states only whereas in the case of modal logic one has to think of the truth and falsity of a variable with respect to states and certain relations between the states. Giving semantic to our logic, mean, interpreting our modal language as a way of talking about relational structures. Modal logic can be given different interpretations: algebraic semantics, topological semantics and relational/Kripke semantics. The Kripke semantics is commonly used to explicate the logical structure of a modal system (Kripke, 1963). This means to explain in a detailed and formal way (i.e. developing an idea or theory and show its implications). There are two ways of doing this, namely: model level and frame level shown in figure 2.8 (a and b respectively). Both levels support the key notions of satisfaction and validity.

Kripke Frame: A Kripke frame is defined as a pair, F= (W,R) where W is a set called the set of worlds and R is a binary relation called an accessibility relation. A frame is a set of points and the relations between them.

Kripke Model: A Kripke model is a triple $\mathfrak{R}$ = (F,v,w), where $v:\phi \rightarrow 2^W$. From this definition, a model is based on the frame F, or F is the underlying frame of $\mathfrak{R}$. Elements of W are called worlds, states or points and x_1Rx_2 means x_2 is accessible from x_1 or x_1 sees x_2 or x_2 succeeds x_1.

A frame is more like a directed graph and has no information about the atomic formulas at various points. But models evaluate the truth of propositions with the notion of giving satisfaction in a world.

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The basic modal logic model is the Kripke structures / Possible World structures. Kripke structure is a triple $M = (W,R,V)$, where W is the non-empty set of possible worlds (that is states in a computation), $R \subseteq W \times W$ is the accessibility relation (otherwise called transition relation) and $V: (Prop \times W) \rightarrow (true, false)$ is a valuation function (which tells us the properties that is true and of which state).

This model is also viewed as a graph (W, R) with a function V that tells which proposition variables are true at which vertices (Alechina, 2003).

PWS gives the different variations on the concept of truth or satisfaction to include:

(i) truth of a formula at a world of a model

(ii) truth of a formula in a model

(iii) truth of a formula in a frame

(iv) truth of a formula in a class of frames

$R,w \models \Diamond\varphi$ iff for some $w' \in W$ we have wRw' and $R,w' \models \varphi$. Other logical connectives can be defined thus.

Inductively,

$R,w \models p_0$ iff $c \in v(p_0)$

$R,w \models \neg\varphi\wedge\psi$ iff $R,w \not\models \varphi$

$R,w \models \varphi\wedge\psi$ iff $R,w \models \varphi$ and $R,w \models \psi$.

$R,w \models \Box\varphi$ iff for all $w' \in W$ with wRw' implies $R,w' \models \varphi$.

From here,

Modal Logic follows several Possible World Semantics (PWS) like the Kripke semantics (K), T, S4 and S5 (Cohn and Hazarika, 2001). Familiar logics in modal family are constructed from the weak logic, K, the foundational logic for any modal system.

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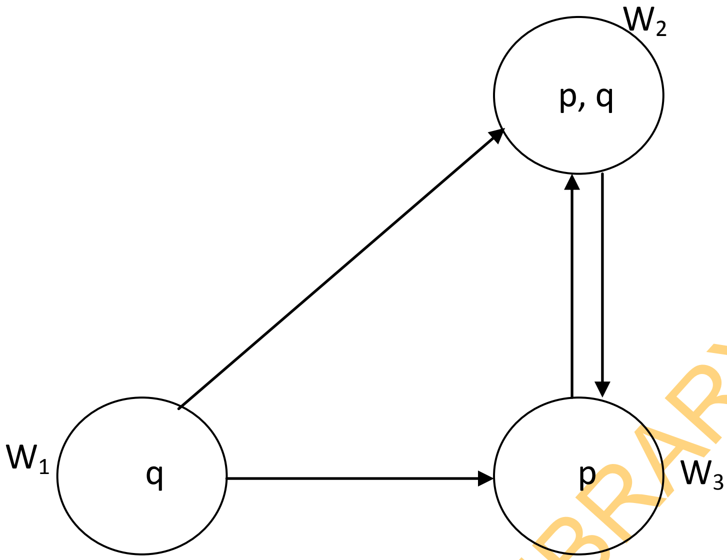
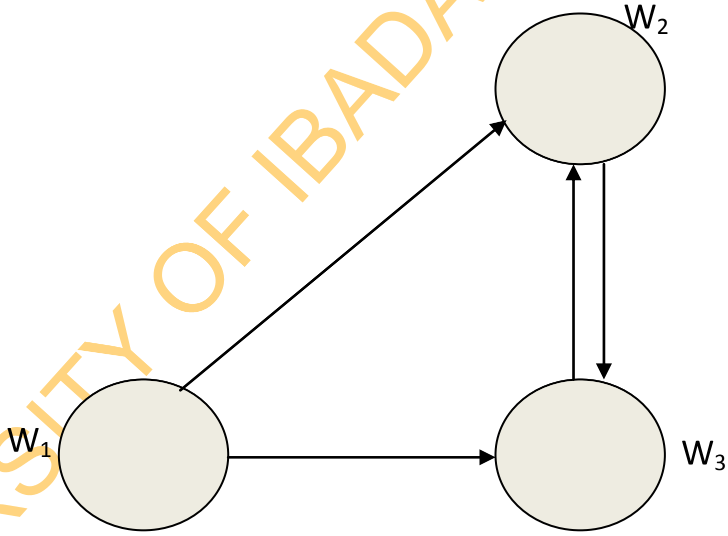


(a) Kripke Model



(b) Kripke Frame

Figure 2.8: Kripke Frame versus Kripke Model

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2. Normal Modal Logic (NML)

Normal modal logic is defined by the set of modal formulas that contains:

(i) all propositional tautaulogies

(ii) axiom k: $\Box(p\Rightarrow q) \Rightarrow (\Box p\Rightarrow\Box q)$

(iii) $\Diamond p \Leftrightarrow \neg\Box\neg p$

and that is closed under necessitation, that is if $|\text{-}_\Lambda\ \Box\varphi$ where $|\text{-}_\Lambda\ \varphi$ is a theorem, modus ponens and uniform substitution.

Modus ponens: if φ and $\varphi\Rightarrow\psi$ are theorems so is ψ

Necessitation: if φ is a theorem, so is $\Box\varphi$

Uniform substitution: given formula $\psi(p_1, \ldots, p_n)$ in φ, it respectively gives the derived formula $\varphi(\psi|p_1), \ldots,\varphi(\psi|p_n)$.

The above definition of NML is the smallest form. The extension of NML with a set $\sum$ of axioms gives other forms of NML and is denoted as Λ=K$\oplus\sum$.

3. Multi-Modal Logic (MML)

This is a generalized modal logic that allows more than one modal operator to appear in a formula. This is suitable to reason in a multi-agent environment, to model several agents and to represent group properties like knowledge, beliefs and flow of time. The syntax of the MML consists of K-axiom and the generalization rule formulated for each of the boxes $\Box_1, \ldots, \Box_n$.

$\Box_I (p_0\Rightarrow p_1) \Rightarrow (\Box_i\, p_0 \Rightarrow\Box_i\, p_1)$ (K)

given φ, derive $\Box_i\varphi$ (NEC)

Hence, the smallest (minimal) n-modal logic (K_n) is defined by a set of Λ_{PMLn} formulas that contains all propositional tautologies and (K), for $1\leq i\leq n$, and is closed under the rules of Modus Ponens, substitution and necessitation, for all i=1, …, n.

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Examples of multi-modal logics include:

$K4_n \quad K_n \oplus \{\Box_i\, p_0 \Rightarrow \Box_i \Box_i\, p_1 | 1 \le i \le n\}$

$T_n \quad K_n \oplus \{\Box_i\, p_0 \Rightarrow p_0 | 1 \le i \le n\}$

$S4_n \quad K4_n \oplus \{\Box_i\, p_0 \Rightarrow p_0 | 1 \le i \le n\}$

$KD45_n\ K4_n \oplus \{\Box_i\, p_0 \Rightarrow \Diamond_i\, p_0,\ \Diamond_i\, p_0 \Rightarrow \Box_i \Diamond_i\, p_0\ | 1 \le i \le n\}$

$S5_n \quad S4_n \oplus \{\Diamond_i\, p_0 \Rightarrow \Box_i \Diamond_i\, p_0\ | 1 \le i \le n\}$.

This means that the knowledge that whatever is necessarily true is the case, M: $\Box A \rightarrow A$, when added to K produces another system, T. And since M is still seen as being weak, the suggestions to strengthen it yielded other systems. Adding 4: $\Box A \rightarrow \Box\Box A$ to T produces S4 system; adding 5: $\Diamond A \rightarrow \Box\Diamond A$ to T produces S5 and adding B: $A \rightarrow \Box\Diamond A$ to M gives system B, hence adding B to S4 yields S5. We are saying that if A is an axiom of (classical) non-modal propositional logic, PL then $\Box A$ is an axiom of each modal propositional logic, K defined by

$$\Box(P \rightarrow Q) \rightarrow (\Box P \rightarrow \Box Q),$$

the addition of the primitive transformation rules of uniform substitution, modus ponens and necessitation of theorems in order to obtain the system **K**. The various systems **T**, **B**, **S4** and **S5** are specified by adding certain further axioms to the base of **K** and summarized in Table 2.3 with their features.

Nair (2003), in his work, gave an overview of how the mental states of an agent (in particular its beliefs, desires and intensions (BDI) states/model can be represented using a modal language. ζ_{BDI} is a propositional modal language with 3 families of modal operators namely: BEL_i, DES_i, INT_i, I = A (agents) and BEL, DES and INT is beliefs, desires and intensions of the agent respectively.

The semantics (Kripke structure) of their logic is defined as a tuple

$$M=(W,\{S_w{:}w \in W\}, \{R_w{:}w \in W\}, v,B,G,I)$$

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where, W = set of possible worlds

S_w = set of states in each world, W

R_w = binary relation i.e. $R_w \subseteq S_w \times S_w$

v = truth assignment to the primitive propositions of ϕ for each world $w \in W$ at each state, $s \in S_w$, (i.e. $v(w,s):\phi \rightarrow \{true,false\}$

B,G,I = relations on the worlds, W and states, S (i.e. $B \subseteq W \times S \times W$).

2.8.3 Quantified (First-Order) Modal Logic

This logic combines features of two logical languages, namely, first order logic and modal logic. Over the years, multiplicity of versions and inadequate syntax had been the reason for non-standardization of first order modal logics as a tool in many disciplines. The syntax and semantics of first order modal logic given by Fitting (1998) shows how it easily copes with several familiar problems such as our problem of spatial qualification. This leads to our choice of first order modal logic as the representational language for our logic. Quantified ML combines adequate expressibility of first order logic with the dynamics of modal logic as seen in the discussion of the syntax and semantics of both languages in previous sections.

2.9 Standard Formal Semantics

Formal logics follow several standard interpretations (semantics) such as situation semantics and possible world semantics.

2.9.1 Situation Semantics

Although situation semantics was originally conceived as an alternative to extensional model theory and possible world semantics (Devlin, 2004), it has the central ideas including partiality, realism and the relational theory of meaning (Perry, 1998). Partiality is concerned with the limited parts of reality that we in-fact perceive which was seen as partial first order models (Barwise, 1983), reason about, and live in. This is because what goes on in a particular situation will determine answers to some issues, but not all.

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Table 2.3: Standard Logical Systems and their features

System	Axioms added to K	Distinguishing feature of R
K	None	None
T	□P → P	Reflexivity
B	T plus (B*) P →□◊P	Reflexivity and symmetry
S4	T plus □P →□□P	Reflexivity and transitivity
S5	T plus ◊P →□◊P	Reflexivity, symmetry and transitivity (i.e. equivalence)

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The emphasis on partiality according to Devlin (2006), was that it brings situation semantics in contrast with possible world semantics. Instead of the generalized approach (that is, the potentially infinite models of the real world or possible world) by Montague, Barwise and Perry adopted the finite situations as their basis (Janson, 2012).

Early versions of situation semantics followed the standard set theory and/or the concepts of constraints (Barwise, 1983). McCarthy described an aspect of the world in association with a situation together with the intended meaning that the property holds in that situation (Koomen, 1989). For example, a cup C is full in situation S, is stated as

Full(C,S).

Actions in situation calculus are function that produces a new situation from existing one together with arguments through rules (constraints).

Situations are real, actual parts of the world that deal with properties and relations that are real uniformities across the real/actual parts of the worlds. Situation semantics is also a method for analyzing semantic phenomena, that is, it provides a relational theory of meaning. This theory distinguishes three notions that are often treated as if they are somewhat interchangeable. These notions are information, representation and proposition.

(i) Situation semantics distinguishes the abstract meaning (general meaning) of a word/phrase

(ii) Utterance situation: This is the immediate context (environment) but not necessarily. For example, a man is at the door, where the man is the object and the door is the part of the situation linked to the object.

(iii) Resource situation: This can be exploited in various ways namely:

a. As perceived by speaker
b. As the object of some common knowledge about the world
c. As the way the world is
d. As built up by previous discourse.

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Example: The man I saw running yesterday is at the door.

(iv) Focal situation: This is the described situation. Simply put, focal situation is the part of the world the utterance is about.

Situation semantics distinguishes the abstract meaning (general meaning) of a word/phrase/sentence from the meaning in use (as used in a particular context or instance). This means that the meaning in use is induced by the abstract meaning. In situation semantics, analysis is done in three ways:

Situation semantics use complexes of objects and properties to directly or indirectly classify parts and aspects of reality, or situations (Schubert, 2000). Classification is done by what goes on in the situations, that is, the objects' properties and the relations they stand in to one another in virtue of the events that comprise the situation. From one type of situation, another one is involved, meaning that a situation determines the next state of affairs or possibilities (whether or not another situation is possible). These states of affairs are constraints. Considering an example of two situations:

(i) A dog breaks a leg

(ii) A dog doesn't run,

a state of affairs occur where (i) involves (ii), that is, dogs with broken legs don't run. This becomes a constraint giving rise to the possibility of indirect classification. By indirect classification, it does not follow the supported state of affairs but the types of situations involved, relative to some constraint. This is by what they mean or by their contents (factual or fictional). These situations may be local connections between objects in the situation and other objects (subject matter), remote (the content) and the constraint according to type combination. Examples:

(i) The veterinary said that Jackie had a broken leg

(ii) The xray shows that Jackie has a broken leg

(iii) Jackie has a broken leg.

with (i) and (ii) as indirect classification and (iii) is direct classification.

Semantic Web is based on formal logic for which one can assert facts that are unambiguously certain. Context meaning concept of type or proposition was first

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presented as formal objects in logic using existential graphs by a philosopher named C. S. Peirce. Ontologies are ways that one can implement the situation awareness of the logical reasoning about situations and context.

2.9.1.1 **Advantages of Situation Semantic**

Situation semantics, as identified by Perry, allows us to see how different information can be gleaned from the same 'signal' given different starting points. The fact that a situation involves another, allows one to determine the next state of affairs or possibilities, thereby knowing whether or not another situation is possible. Also, the introduction of parameters allows the crucial information links to actual/specific entities to be tracked. This is made possible through the use of a function called an anchor.

2.9.2 **Possible World Semantics**

Possible World Semantics (PWS) is viewed by many as a family of methods that have been used to analyze a wide variety of intensional phenomena (properties), including modality, conditionals, tense and temporal adverbs, obligation and reports of informational and cognitive content (Star, 2008).

Over time, applications of logic in Computer Science and Artificial Intelligence only featured extensional (set) semantics where the extension of a singular term is the object it formally describes, and the extension of a sentence is a truth-value. Predicate calculus is really adequate for important work in mathematical logic. The success recorded introduced the intensional phenomena of which the extensional constructions are not suitable for tackling them. In the quest to deriving intensional construction, Lewis, whose disliked to extensional treatment of "if…then…" led him to "strict implication", considered modal logic, the logic of necessity and possibility. He maintains that possible worlds are alternative concrete realities, they are actual for their inhabitants, as ours is for us. Inhabitants of other worlds are not identical with the inhabitants of the actual world.

Leibniz idea that necessary truth was truth in all possible worlds was recruited by Carnap to the task of building an intensional semantics. This recruitment serves as the

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guiding idea of PWS since others rely on linguistics representations of possible world called "state-descriptions".

PWS supplies only one necessary proposition, the set of all worlds, and only one contradictory proposition, the null set. This makes it more problematic for mathematical knowledge to be dealt with. This, he said, can be resolved if mathematical knowledge is viewed by seeing a linguistic element in the knowledge of mathematical truth. Intensional logic combines modal, temporal and other operators.

Guarino (1998) stated that a standard way to represent intensions (and conceptual relations) is to see them as functions from possible worlds into sets. Despite some of its disadvantages, it works fairly well for our purposes. Here, conceptual relations are defined on a domain space, <*D,W*>, that is, *D* is a domain and *W* is a set of maximal states of affairs of such domain (also called possible worlds). The structure <*D,W*> refers to a particular world (or state of affairs), called world structure. Conceptualization contains many of such world structures called intended world structures.

The interplay between semantic structures and logical systems involved in these investigations constitute a development in logic comparable to the move in geometry away from Euclidean Geometry, conceived as the one true system to geometry as the study of alternative axiom systems for spaces with diverse properties (Perry, 1998).

2.10 **Theorem proofing and Analytic Tableau Proof Method**

The conclusions reached from the descriptions using logical languages need to be proven. Theorem proofing can be done using several proof methods such as unification, resolution and tableau proof method (Ramsey, 1989). Classical logics such as first order logic can best be handled using resolution since it is in clausal form. The most widely used proof method for modal logics is the analytic/semantic tableau proof method (Ramsey, 1989). The use of analytic tableau proof method reduces the burden of transforming sentences such as possibility to its clausal form. Semantic tableau is a proof system used to prove the validity of a formula, or if a formula is a logical consequence of a set of formulas and/or prove of satisfiability of a set of formula (Sabri, 2009).

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A tableau is a tree-like representation of a formula or set of formulae in logic (Sheremet et al, 2010). Tableau calculus consists of finite collection or set of rules. Rules specify how to break down one logical connective into its constituent parts. In tableaux, if any branch leads to an evident contradiction, the branch closes. If all branches close, the proof is complete and the original formula is said to be a logical truth. A proof procedure on the other hand is a policy of application of the rules. The objective of tableaux is to show that the negation of a formula is not satisfied.

A signed tableau is an expression TX or FX. Where formula X is unsigned, it is called TX where X is true, FX where X is false. Also a signed formula is called FX if X is true and TX if X is false. Hence, a tableau is a rooted dyadic tree where each node carried a signed formula. The application of a tableau rule following a finite path causes an immediate extension of the formula. A path of a tableau is said to be closed if it contains a conjugate pair of formulas, that is, a pair such as TA, FA. On the contrary, it is said to be open if it is not closed. A tableau is closed if each of its paths is closed.

Tableau can be used to prove a formulae as follows:

(i) To test a formula A for validity, a signed tableau starting with FA is formed. If the tableau closes off then A is logically valid.

(ii) To test whether formula B is a logical consequence of $A_1...A_k$, a signed tableau starting with TA_1 ... TA_k , FB is formed. If it closes off, then B is in deed a logical consequence of $A_1...A_k$.

(iii) To test $A_1...A_k$ for satisfiability, a signed tableau with $TA_1...TA_k$ is formed. If it closes off then $A_1...A_k$ is not satisfied. If the tableau does not close off then $A_1...A_k$ is satisfied, and from any open path, one can read off an assignment satisfying $A_1...A_k$.

The basic rules for constructing the tableau stems from that of propositional logic, extends to rules that cope with the universal and existential quantifiers in first-order logic and then to rules that cope with *possibly* and *necessarily* modalities of modal logics. Since our logic is a quantified modal logic, we shall combine all the rules for propositional, first-order and modal logics as described in the chapter addressing the development of the proof system. In most cases, first-order logic is said to be

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undecidable and thereby produces no correct or complete proof. This research will demonstrate the effectiveness of modal logics.

2.11 Spatial Qualification Problem in AI Domains

Spatial qualification problem is prominent in domains such as planning, which has remained a domain of concern in the AI field. Several projects like the TRAINS project (Allen and Schubert, 1991; Traum et. al., 1994) helped to unveil the need to spatially qualify objects required for problem-solving in planning domain. Planning problems such as prediction (assumption of the world) and planning (assumption about the future world) require reasoning.

Although action viewed as state change is the predominant approach to modeling action in AI and Computer Science, attempts have also been made by researchers in the field to build planning model that considered past knowledge before its decision on the next action to take. This view underlies the state-based planning systems, formal models of planning and works in dynamic logic for the semantics of programs. These researches help in plan monitoring and execution (Georgeff and Lanskey, 1987; Allen et. al., 2004). An example of such model is that of planning as temporal reasoning by Allen (1991) for solving the planning of the execution of the door latch problem. In this door latch problem, the preconditions for pulling the door is that the agent is holding the lock open.

Considering the Door latch problem, other preconditions were identified by Allen without considering the spatial qualification problem. There is need for the intelligent agent to be spatially qualified before assessing the identified preconditions to carry out the plan. Hence, this study was motivated about the planning as its application domain.

2.12 Challenges and the way forward

The use of qualitative reasoning approach in the logic further showcases the strengths of the approach over purely quantitative reasoning approaches where highly sophisticated models and complete quantification of domain knowledge are used in commonsense reasoning.

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RCC is a first-order theory that deals with regions in a topological manner and is appropriate because aside from being expressive, it captures basic distinctions of objects and the regions such as, whether or not two glyphs are disjoint, touching, or inside one another. Also, domain-specific inference rules (Forbus et al, 2004) can use these relationships when needed, such as inferring containment of physical objects depicted based on one glyph being inside another. Much of the work on RCC8 and other qualitative topological algebras have focused on using transitivity for efficient inference.

Importantly, also in modal logic, the way, mode, state of truth of a formula is also important rather than only seeking to know whether a formula is true as it is the case in classical logic. Hence our use of the first order (quantified) modal logic combines features of both logical languages for appropriate representation in our spatial qualification domain.

Even though concepts in the world are not total situations but partial, and can be better handled using the Situation Semantics (Schubert, 2000) due to its outstanding features which includes talking about incomplete world, Possible World Semantics offers the most appropriate semantics approach when considering intensions. In this thesis, the formal semantics of our logic will be defined with respect to the Possible World Semantics/Kripke structure where each partial world is seen as a possible world due to its dynamics according to time.

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CHAPTER THREE

LOGICAL THEORY OF SPATIAL QUALIFICATION

3.1 Introduction

This chapter describes the conceptual framework of the SQM and its formal specification. The use of qualitative reasoning approach in SQM to reason about an agent's spatial qualification is diagrammatically and formally defined and represented. Describing the model following the Possible World Semantics/Kripke structure shows its properties. Further comparison of the logical model with existing standard modal systems to further ascertain its satisfiability is carried out. Examples of application domains where the model can be applicable are also highlighted.

3.2 The Conceptual Framework of Spatial Qualification Model (SQM)

Before the logical theory of Spatial Qualification (SQ) is defined, several parameters for its qualification as shown in the model structure of the Kripke model in figure 2.8(a) are considered. In spatial reasoning field (Allen and Ferguson, 1994), it is stated that knowledge of the world is necessarily incomplete and unpredictable in detail, thereby causing predictions to be done only on the basis of certain assumptions. Hence, SQM assumes that the distances and the speed limits on all possible routes are available for use when needed. This assumption is based on successes recorded in the area of geographic information systems where the GPS in mobile devices can conveniently constitute the database required as input for SQM.

The choice of formal language used for the representation of the logical model to be Quantified (First Order) Modal Logic is due to its expressivity and ability to represent incomplete and unpredictable information about agents in a possible world. The conceptual framework of SQM as presented in figure 3.1 has spatial concepts and

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calculi. Attempts to model spatial concepts which include distance, shape, size, direction, time and motion in the *known* section have been made. Spatiotemporal calculi such as distance, direction, anchoring, RCC, QTC and motion resulted from these models. Spatial qualification in the *unknown* section intends to use known *distances* and *time* in figure 3.1 as quantitative measures and the *locations*, *presence log* and *reachability* relation from one location to the other of an intelligent agent as qualitative measures. The *presence log* is the carrier of the *locations* of the agent at certain *times* while the *reachability* relation is the determinant of the distances between these locations. These parameters are necessary to determine the possibility of agents' presence in n possible locations to participate in any action.

With the existing calculi and the newly defined qualitative relations, spatial qualification logic, a new calculus as the basis of the SQM model is obtained.

3.3 Formal Specification of the Spatial Qualification

Problem solving with Qualitative Spatial Reasoning (QSR) involves formalizing one type of spatial relations and discussing their attributes; and composing two or more spatial relations to obtain a previously unknown relation. Formalization has to do with the use of a logical language to express concepts and relationships among the concepts. Logical axioms have proven to be the most expressive formalism. Hence, most of these formalisms make use of logic as the representation language. In the formal logic, modalities are introduced into First order logic. The syntax of the combined logic is given below.

The vocabulary of the spatial qualification logic includes:

Spatial locations: $L = \{ \ldots, l_i, \ldots \}$
Predicates: *Present_at*, *Occupy*
Time points: $T = \{\ldots, t_i, \ldots\}$
The Boolean connectives: $\wedge$(and), $\vee$(or), $\Rightarrow$ (implies), $\Leftrightarrow$ (if and only if)
and $\neg$(not)
Quantifiers: $\forall$ (Universal) and $\exists$ (Existential)
Modal operators: □(necessity) and ◊(possibility)
The logical constraints: T(true) and F(false).

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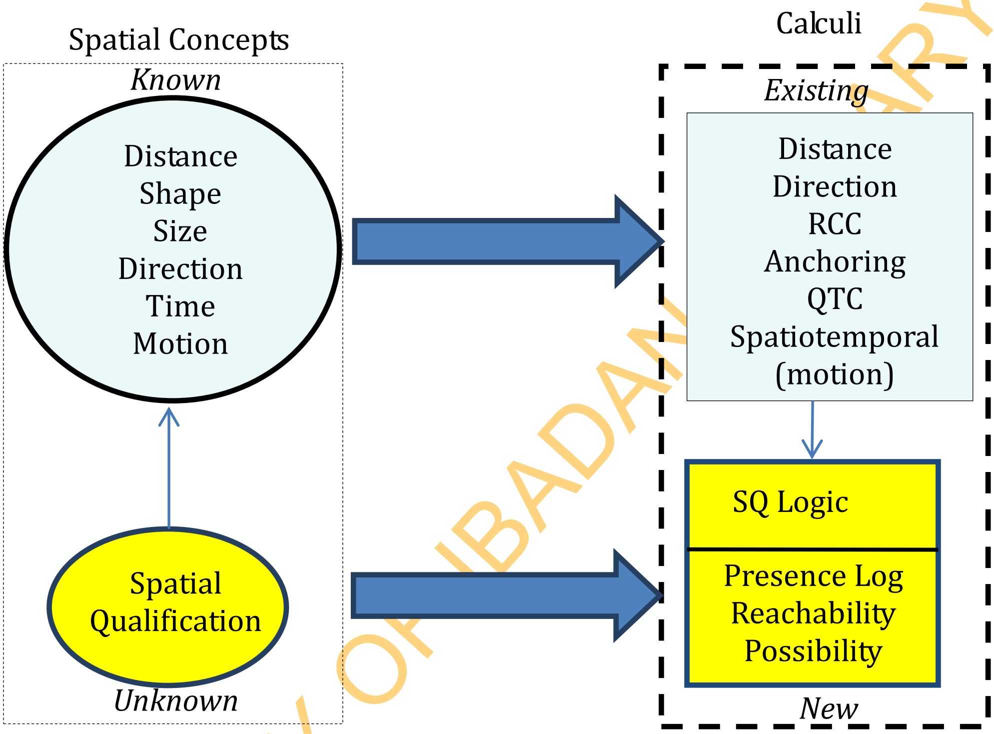

Figure 3.1: Conceptual Framework of SQM

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The language of the SQ logic is a many sorted quantified (first order) modal logic. In SQ logic, the formalism assumes that constants definitely refer to known agents in the world, unlike in Fitting's quantified modal logic (Fitting, 1998) where constant referents may not refer to a definite agent. As such, basic formulae in our logic take the form: $P(t_1, t_2, t_3 \ldots t_n)$ where P is an n-nary predicate symbol and $t_1, t_2, \ldots, t_n$ are terms. Each term can either be a constant symbol or variable symbols.

There are three basic sorts of constants in the language. These are *Agents*, *Locations* and *Time points*. Locations in our logic denote the notion of regions generally used in spatial logic. Apart from the predicates denoting the standard spatial relations from RCC, the major predicate is ***Present_at*** with the following signature.

Present_at : Agents × Location × Time point → Boolean

Each proposition formed with *Present-at* is called a presence log. We can think of the fact that x is *Present-at* location *l* at time *t* as defined by the fact that an agent occupies a region which is within the location *l*. The definition is presented thus:

$$\forall x, l, t.\ Present\_at(x, l, t)$$
$$\Leftrightarrow \exists r.\ Occupy(x, r, t) \wedge (NTPP(l, l_1) \vee TPP(l, l_1) \vee l_1 = l)$$

where *Occupy* is a relation between individuals or object and the exact 2-dimensional space they occupy at a certain time. If an object or individual occupies a space, that object does not occupy any larger region containing that region

$$\forall x,l,t.\ Occupy(x, l, t) \wedge (NTPP(l, l_1) \vee TPP(l, l_1)) \Rightarrow \neg Occupy(x, l_1, t)$$

Following the standard tradition of first order modal logics, if φ and ψ are formulas, then so are $(\varphi\wedge\psi)$, $(\varphi\vee\psi)$, $\neg\varphi$, $\varphi\Rightarrow\psi$, $\varphi\Leftrightarrow\psi$, $\forall x.\varphi$, $\exists x.\ \psi$, $\Box\varphi$ and $\Diamond\varphi$. The scope of variables in quantification is the formula following the dot after it. The meanings of the classical logic operators are as given in the model semantics for first order predicate logic. The modal operators in this formal logic have meaning attributed to them from the standard possible world semantics. The proposition $\Box\phi$ means ϕ is true in all possible worlds accessible from the current world, while $\Diamond\phi$ means ϕ is true in some world accessible from the current world.

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3.4 Qualitative Model for Spatial Qualification

The qualitative model for spatial qualification gives rise to the spatial qualification reasoner with its input, processes and output described in the diagrammatic representation of the SQ reasoning process shown in figure 3.2. The basic input to the reasoner is the prior knowledge which includes an agent's past location and time as well as the location and time of incidence. The basic assumption is that the SQ reasoner has this knowledge.

The role of the SQ reasoner is to investigate the reachability of the locations in the prior knowledge within the recorded times. The result of this investigation is the determinant of the possibility and otherwise the impossibility of the presence of the agent in the location of incidence to be a participant. The possibility of presence of the agent which is the uncertain knowledge is therefore the output from SQ reasoner.

Considering an agent or object that was present at place p and at a time t. Is it possible for the same agent to be present at a different place p_1 at a subsequent time t_1, given what was known? This problem may be reduced in a sense to the problem of determining whether or not the agent can travel between place p to place p_1 between time t and time t_1. A human reasoning agent confronted with this problem would reason using the distance between place p and p_1, and the rate at which the agent could travel. Most human agents are able to estimate how long it takes to complete a journey on a certain highway (or path). As can be affirmed by most people, this kind of reasoning is commonsense reasoning because it can be answered experientially by anyone who has traversed the highway or estimated by anyone who knows the length of the highway. The agent having known the locations will use some prior knowledge of the distance and the speed limit allowed on the road. This knowledge can then be used to determine the time it will take simply by dividing the distance by the speed. It is obvious that the distance and the speed limit of the road to traverse have to be known in other to determine the minimum time it will take to traverse the road.

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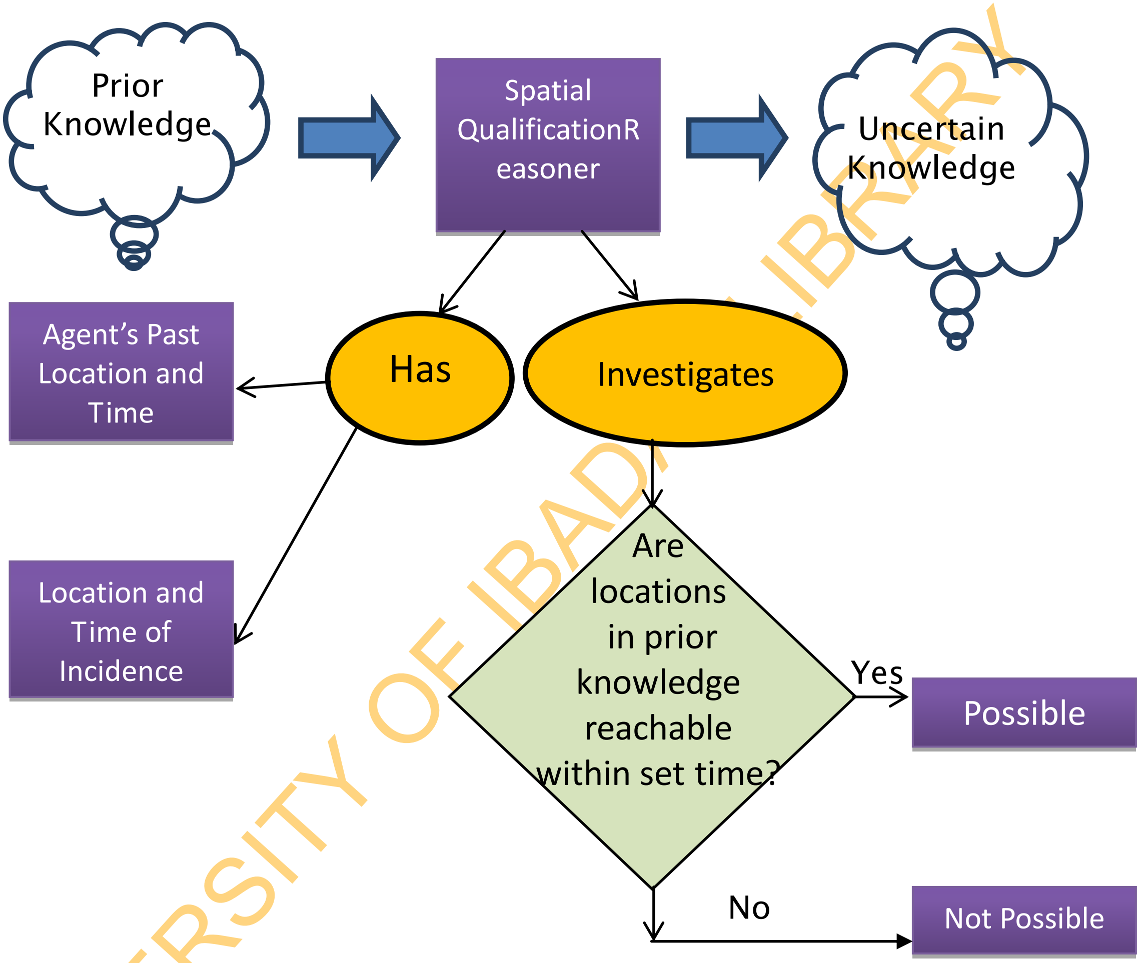

Figure 3.2: Diagrammatic Representation of the Spatial Qualification Reasoning Process

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This approach to solving this problem was based on qualitative modeling. Intelligent agents can use qualitative models to reason about quantities without having to resort to the nitty-gritty of mathematics and calculus. A particular approach that is powerful in this regard is that of discretization. In discretization, quantities are divided into chunks, and the solutions to our problems can be deduced from the solutions to the smaller versions of the problem. For example, if an agent's presence at location l_1 at time t implies he/she can be in location l_2 at a later time t_1, and an agent being at location l_2 at time t_1 implies he can be at location l_3 at a later time t_2 and l_3 is farther from l_1 than l_2 is, then x being present at l_1 at time t implies x can be present at l_3 at time t_2. The lemmas and basic definitions that make up our qualitative logic for spatial qualification is presented in section 3.4.1.

3.4.1 Basic Definitions for spatial qualification

Let l be a location (region) in space and l_1 a different location (region) in space too. Following from the definitions of the eight disjoint pair of relations - RCC-8 (Wolter and Zakharyaschev, 2000a, 2000b, 2002; Randell et al, 1992), which is based on the region connection relation, C, we have the following definitions.

$\forall l\ C(l,l)$

$\forall l,l_1\ (C(l,l_1) \rightarrow C(l_1,l))$

$DC(l,l_1) \equiv \neg C(l,l_1)$

$P(l,l_1) \equiv \forall z\ (C(x,l) \rightarrow C(z,l_1)$

$EQ(l,l_1) \equiv P(l,l_1) \wedge P(l_1,l)$

$O(l,l_1) \equiv \exists z(P(z,l) \wedge P(z,l_1)$

$PO(l,l_1) \equiv O(l,l_1) \wedge \neg P(l,l_1) \wedge \neg P(l_1,l)$

$EC(l,l_1) \equiv C(l,l_1) \wedge \neg O(l,l_1)$

$PP(l,l_1) \equiv P(l,l_1) \wedge \neg P(l_1,l)$

$TPP(l,l_1) \equiv PP(l,l_1) \wedge \exists z\ (EC(z,l) \wedge EC(z,l_1))$

$NTPP(l,l_1) \equiv PP(l,l_1) \wedge \neg\exists z\ (EC(z,l) \wedge EC(z,l_1))$

Hence, the definition of *Regionally_part_of* and the *Regionally_disjoint* relations for SQ logic are *Def1* and *Def2* as shown in figure 3.3.

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3.4.2 **Logic of Presence**

Theorems $T_{A1} - T_{A9}$ below constitute the spatial qualification logic of presence. Note that in the logic, a world's accessibility is determined by its reachability from another world and not the reverse.

3.4.2.1 **Persistence of Truth**

Our logic treats any known fact as something that remains permanently true. As such if we know that an agent is present at a location *l* at time *t*, then that fact is always true. For every agent *x* present at location *l* at time *t*, it implies that it is necessarily true that every agent *x* is present at location *l* at a certain time *t*.

$$T_{A1}: \quad \forall x, l, t. \quad Present\_at(x,l,t) \Rightarrow \Box Present\_at(x,l,t)$$

3.4.2.2 **Possibility of Location Persistence**

For every agent *x* present at location *l* at some time *t*, it implies that it is possible that the same agent is present at that location at a later time t_1.

$$T_{A2}: \quad \forall x, l, t.\ Present\_at(x,l,t) \Rightarrow (\forall t_1.\ t < t_1 \Rightarrow \Diamond Present\_at(x,l,t_1))$$

Possibility can also be defined based on the reachability of spatial locations. So there is need to first define reachability as follows.

3.4.2.3 **Definition of *Reachability***

Now, defining what it means for an agent *x* to be able to reach location l_2 from l_1 in the interval (t_1, t_2) is given thus.

$$T_{A3}: \quad \forall x, l_1, l_2, t_1, t_2.$$
$$Reachable(x, l_1, l_2, (t_1, t_2)) \Leftrightarrow (t_1 < t_2 \wedge$$
$$(Present\_at(x, l_1, t_1) \Rightarrow \Diamond Present\_at(x, l_2, t_2)))$$

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3.4.2.4 **Reachability is Reflexive**

A location is reachable from itself for any agent within any interval of time no matter how small. This means the location is self-accessible or that an agent can remain in the same location till a later time.

$$T_{A4}: \quad \forall x,l_1,l_2,t_1,t_2.\ l_1=l_2 \wedge t_1<t_2 \Rightarrow Reachable(x,l_1,l_2,(t_1,t_2))$$

3.4.2.5 **Reachability is Commutative**

Generally, if one can reach l_2 from l_1 in a time interval, then it is possible to achieve a reverse of that feat within the same interval.

$$T_{A5}: \quad \forall x,\ l_1,\ l_2,\ t_1,\ t_2.$$
$$Reachable(x,\ l_1,\ l_2,\ (t_1,\ t_2)) \Leftrightarrow Reachable(x,\ l_2,\ l_1,\ (t_1,\ t_2))$$

Note that the reachability two possible worlds can apply to 'n' possible worlds accessible from a nearby node.

3.4.2.6 **Reachability depends on duration of time interval**

Here is the definition of a property for the notion of being reachable. If it is possible for an agent to reach one location from another, it should still be possible for the same agent to perform the same feat within any interval of about same or different duration.

For instance, with justifiable reason, one can go from Uyo to Eket within 2 hours and return through the same route within an hour or more.

$$T_{A6}: \quad \forall x,\ l_1,\ l_2,\ t_1,\ t_2.$$
$$Reachable(x,\ l_1,\ l_2,\ (t_1,\ t_2)) \wedge \forall t_3,\ t_4.\ t_3 < t_4 \wedge$$
$$(t_4 - t_3) \geq (t_2 - t_1) \Rightarrow Reachable(x,\ l_1,\ l_2,\ (t_3,\ t_4))$$

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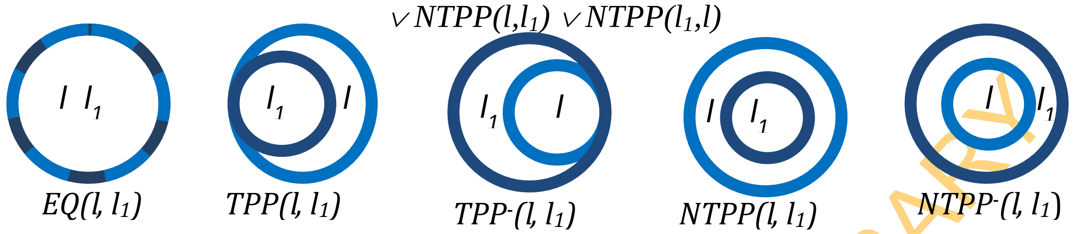
Def1: $\forall l,l_1$ *Regionally_part_of*$(l,l_1) \equiv EQ(l,l_1) \vee TPP(l,l_1) \vee TPP(l_1,l)$

$\vee\, NTPP(l,l_1) \vee NTPP(l_1,l)$



Def2: $\forall l,l_1$ *Regionally_disjoint*$(l,l_1) \equiv DC(l,l_1) \vee EC(l,l_1) \vee PO(l,l_1)$

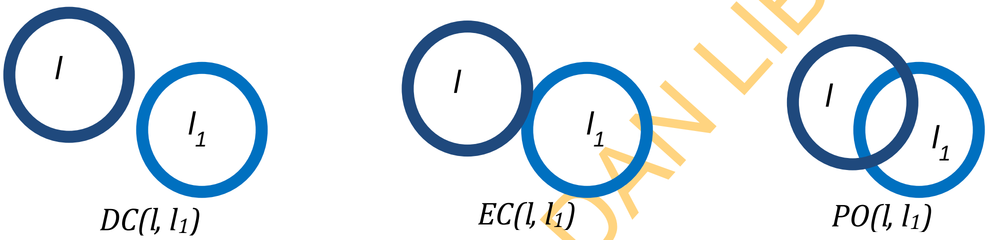


Figure 3.3: Basic definitions using the RCC-8 relations

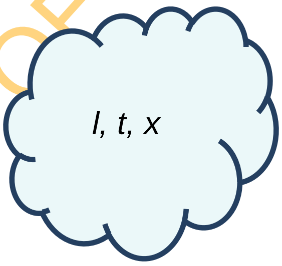


Figure 3.4: A Possible World

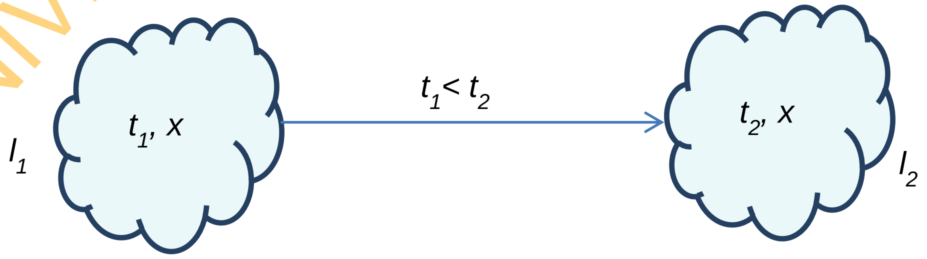


Figure 3.5: Definition of the accessibility of two possible worlds

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3.4.2.7 **Possibility of presence in regions at same time**

The possibility of an agent to be present at two different locations at the same time can be determined by the topological relationship between the two locations. For every agent x said to be in location l at time t and also at location l_1 at the same time and the locations are regionally part of each other, it then implies that it is the case that the agent is present at both locations at the same time.

$$T_{A7}: \quad \forall x, l, l_1, t. \ (Present\_at(x, l, t) \wedge Regionally\_part\_of(l, l_1)) \Rightarrow Present\_at(x, l_1, t)$$

Representations in figure 3.3 depicting *Def1* clearly describes this logical theorem.

3.4.2.8 **Persistence within regions**

If an agent is at a certain location then for some time afterwards, the agent will be within some region surrounding the location.

$$T_{A8}: \quad \forall x, l, t. \ Present\_at(x, l, t) \Rightarrow \exists r, t'. \ NTPP(l, r) \wedge Present\_at(x, r, t+t')$$

3.4.2.9 **Exclusivity of Presence**

For every agent x said to be in location l at time t and also at location l_1 at the same time and the locations are regionally disjoint, it then implies that it is not possible for the agent to be present at both locations at the same time.

$$T_{A9}: \quad \forall x, l, l_1, t. \ (Present\_at(x, l, t) \wedge Regionally\_disjoint(l, l_1)) \Rightarrow \neg\Diamond(Present\_at(x, l_1, t)$$

Representations in figure 3.3 depicting *Def2* clearly describes this logical theorem.

3.4.2.10 **Reachability is transitive**

For every agent x present at location l_1 at time t_1, it is possible for it to be at location l_2 at another time t_2. Also, being at location l_2 means it is possible for it to be at another location l_3 at time t_3 and the distance between l_1 and l_2 is smaller than the distance

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between l_1 and l_3 and t_1 is also less than t_2 then it implies that it is possibly true that the agent at location l_1 at time t_1 is at location l_3 at time t_3.

$$T_{A10}: \quad \forall x, l_1, l_2, l_3, t, t_2, t_3.$$

$$Reachable(x, l_1, l_2, (t, t_2)) \wedge \quad Reachable(x, l_2, l_3, (t_2, t_3))$$

$$\Rightarrow Reachable(x, l_1, l_3, (t, t_3)).$$

The axioms presented here are able to infer reachability when it is true. Otherwise they are not able to make the inference. In other words reachability is only semi-decidable. In order to make it decidable, we need a closure for the reachability concept.

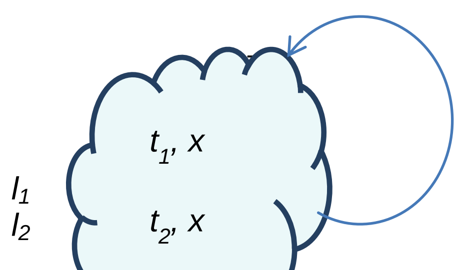
Logic of spatial qualification must be able to reason about the presence of agents at different locations. It is possible to view the problem of spatial qualification as the problem of reasoning about the accessibility of worlds. Each world contains a log of "who is at what location and when?"

3.5 Formal Semantics of the SQM

To further explicate this logic to give its actual meaning as it applies in reality, the concepts are defined with respect to the model structure of the Possible World Semantics (PWS) or Kripke structure. The semantics of the language is formally defined with respect to the Possible World Semantics (PWS), a triple $M = (W,R,V)$ which is defined as:

$$M = \langle W, R, V \rangle \qquad (3.1)$$

Where W is the non-empty set of possible worlds (that is states in a computation), $R \subseteq W \times W$ is the accessibility relation (otherwise called transition relation) and $V: (Prop \times W) \rightarrow (true, false)$ is a valuation function (which tells us the properties that are true and of which state).



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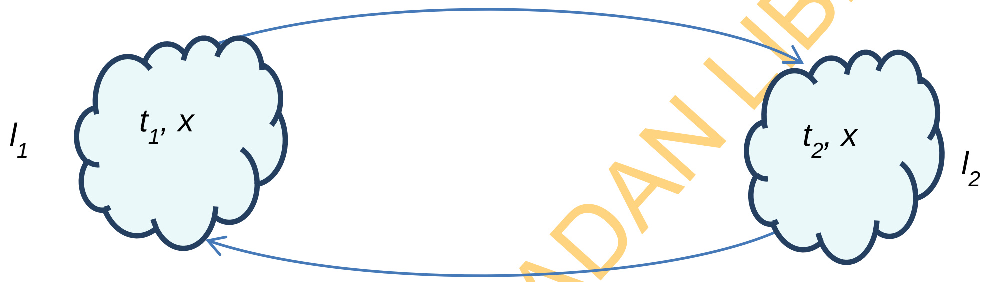

Figure 3.7: Two possible worlds accessible from each other

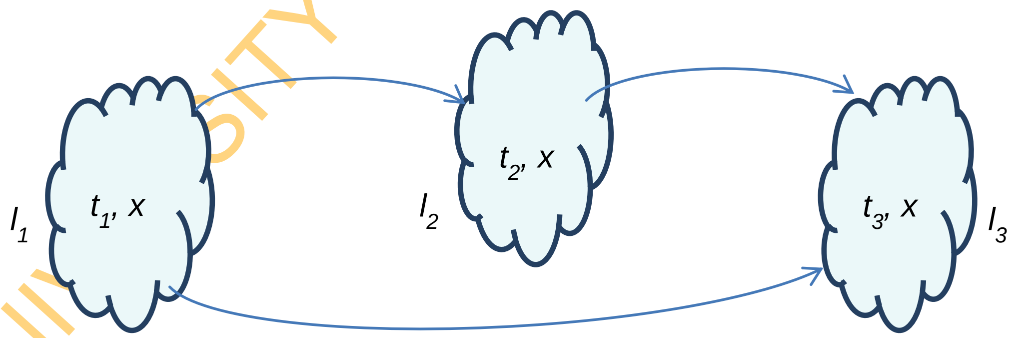

Figure 3.8: Accessibility of possible worlds by discretisation

In our problem domain, the possibility of an agent to be present at a particular place at a certain time is viewed as a possible world. The set of possible worlds in our case here is as defined in equation (3.2).

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$$W = S \times T \qquad (3.2)$$

and $W \neq \{\ \}$ *or* ϕ, *that is* W is not an empty set with S being states indexed with time and defined by

$$S = \{\langle O, L \rangle\} \qquad (3.3)$$

where O is a set of objects in space and L is the actual locations where the objects are and

$$T = \langle t, \prec \rangle \qquad (3.4)$$

where t is the set of possible flow of time.

Considering the definitions in *(3.3)* and *(3.4)* above, T and S are respectively stated in *(3.5)* and *(3.6)* as follows:

$$T = \{t_1, t_2, t_3, \ldots, t_n\} \qquad (3.5)$$

and

$$S = \{\langle O_1, L_1 \rangle, \langle O_2, L_2 \rangle, \langle O_3, L_3 \rangle, \ldots \langle O_n, L_n \rangle\} \qquad (3.6)$$

If the pair $\langle O_i, L_i \rangle$ is denoted by s_i, then the set of possible worlds, W, which is defined as the Cartesian product of S and T can be derived.

$s_1t_1 \ldots s_nt_n$ are therefore the possible worlds contained in the set of possible worlds, W.

From the model structure of the PWS in *(3.1)*, the accessibility relation, R, is defined to be

$$R \subseteq W \times W \qquad (3.7)$$

This means that there is transition between some or all the possible worlds in the set, W. Some of these transitions may be true while some may be false. The interest in this research is on the set of all the possibilities. These set of possibilities, are determined by certain properties. In the logical model, there are properties that may

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lead to the truth or falsity of a transition and the state on which it holds. These are handled by the valuation function.

The Boolean function, F of the accessibility relations can evaluate to V, which may be 0 or 1 in the set of zeroes and ones. That is

$$F(R(s_1t_1, s_1t_2),V) \rightarrow \{\text{true, false}\}$$

if a valuation, V of a function, F is a set of impossibilities then they are denoted with zeroes; if it is a set of possibilities then we denote with ones. That means:

$$F: W \times W \rightarrow V \qquad (3.8)$$

and $V = \{0,1\}$.

If the transition from state, s_1 at time, t_1 to state s_1 at time t_2 is possible, then the valuation function returns true. If the transition is not possible, the valuation function returns false. Sometimes the truth value of a transition may not be decidable due to lack of prior knowledge, which is suspended in our model for further studies. SQM model depends so much on prior antecedent for most conclusions about n possibilities to be made.

Therefore our set of prior antecedents is also a non-empty set of possible worlds which is now a thing of the past (that is a set of history).

The SQM is built around a Kripke modal frame which is the triple $<W, R, D>$ where W is a set of possible worlds, R is the accessible relation between pairs of worlds, and D is a definite domain from which agents in the worlds are drawn. Note the use of D in the adapted Kripke modal frame to replace V, the valuation function. SQM logic contrasts with Fitting's quantified modal logic (Fitting, 1998), in which there is a domain function D associated with the modal frame such that the function D is defined for each world and returns a unique domain associated with that world. One may treat our modal frame as a special case of Fitting's modal frame, in which the domain function D is a constant function.

This research assumes the existence of an Interpretation function I which interprets constant and predicate symbols for each world. The function I maps each constant

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symbols to specific individuals in some specific world. The expression $I[c, w_1]$ denotes the application of the interpretation function I on the constant symbol c in the world w_1. All constant symbols are interpreted uniformly in all worlds so that for any two worlds w_1 and w_2 from W: $I[c, w_1] = I[c, w_2]$, the function I also maps each n-ary predicate symbols to an appropriate n-ary relation in some appropriate world. For example the interpretation of *Present_at* $I[Present\_at, w_1]$ refers to the actual ternary relation that the predicate *Present_at* refers to in the world w_1. It is important to note that in any world $w \in W$:

$$I[Present\_at, w] \subseteq A \times L \times T$$

where A is the set of all agents, L is the set of all locations and T is time points.

Thus we have a model M which is a 4-tuple $<W, R, D, I>$ which comprises of the modal structure <W, R, D> adapted from the Kripke model <W, R, V> and the interpretation function, I. Let us as usual denote by $M,w \models \varphi$, the fact that formula φ is true in a world w of the model M. Thus the following statements hold for *Present_at* as well as for any other predicate.

$M, w \models Present\_at(Paul, Airport, Noon)$ *if and only if*

$(I[Paul, w], I[Airport, w], I[Noon, w]) \in I[Present\_at, w]$

$M, w \models \Diamond Present\_at(Paul, Airport, Noon)$ *if and only if*

For some w_1 *such that* $(w, w_1) \in R$ *it is the case that:*

$(I[Paul, w_1], I[Airport, w_1], I[Noon, w_1]) \in I[Present\_at, w_1]$

$M, w \models \Box Present\_at(Paul, Airport, Noon)$ *if and only if*

For every w_1 *such that* $(w,w_1) \in R$ *it is the case that:*

$(I[Paul, w_1], I[Airport, w_1], I[Noon, w_1]) \in I[Present\_at, w_1]$

$M, w \models \neg Present\_at(Paul, Airport, Noon)$ *if and only if*

$(I[Paul, w], I[Airport, w], I[Noon, w]) \notin I[Present\_at, w]$

$M, w \models Present\_at(Paul, Airport, Noon) \wedge$

$Present\_at(Paul, Swimming\text{-}pool, Noon)$ *if and only if*

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(I[Paul, w], I[Airport, w], I[Noon, w]) $\in$ *I[Present_at, w] and*

(I[Paul, w], I[Swimming-pool, w], I[Noon, w]) $\in$ *I[Present_at, w]*

M, w $\models$ *Present_at(Paul, Airport, Noon)* $\vee$

Present_at(Paul, Swimming-pool, Noon) if and only if either

(I[Paul, w], I[Airport, w], I[Noon, w]) $\in$ *I[Present_at, w] or*

(I[Paul, w], I[Swimming-pool, w], I[Noon, w]) $\in$ *I[Present_at, w]*

In order to be able to interpret variables we need a valuation function such that v has the signature:

$v: V \rightarrow D$

where *v* is the set of all variables and *D* is our domain of agents. It is important to note here that valuations do not depend on the world. Thus in order to strengthen the interpretation function to deal with variables, we redefine the interpretation function as *Iv* so that for any item *t*:

$$Iv[t] = \begin{cases} \text{v(t)} & \text{if t is a variable} \\ \text{I(t)} & \text{otherwise} \end{cases}$$

In that case, the model is now redefined as a <*W, R, D, Iv*> where <*W, R, D*> is the Kripke frame defined for SQM. In that case, the model can redefine what it means for propositions to be true in a world under our model for different terms *x, l,* l_1 and *t*:

M, w $\models$ *Present_at(x, l, t) if and only if*

(Iv[x, w], Iv[l, w], Iv[t, w]) $\in$ *Iv[Present_at, w]*

M, w $\models$ $\Diamond$ *Present_at(x, l, t) if and only if*

For some w_1 *such that* $(w,w_1) \in R$ *it is the case that:*

$(Iv[x, w_1], Iv[l, w_1], Iv[t, w_1]) \in Iv[Present\_at, w_1]$

M, w $\models$ $\Box$ *Present_at(x, l, t) if and only if*

For every w_1 *such that* $(w,w_1) \in R$ *it is the case that:*

$(Iv[x, w_1], Iv[l, w_1], Iv[t, w_1]) \in Iv[Present\_at, w_1]$

M, w $\models$ $\neg$ *Present_at(x, l, t) if and only if*

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$(Iv[x, w], Iv[l, w], Iv[t, w]) \notin Iv[Present\_at, w]$

$M, w \models Present\_at(x, l, t) \wedge Present\_at(x, l_1, t)$

if and only if $(Iv[x, w], Iv[l, w], Iv[t, w])$ *and*

$(Iv[x, w], Iv[l_1, w], Iv[t, w]) \in Iv[Present\_at, w]$

$M, w \models Present\_at(x, l, t) \vee Present\_at(x, l_1, t)$ *if and only if either*

$(Iv[x, w], Iv[l, w], Iv[t, w])$ *or*

$(Iv[x, w], Iv[l_1, w], Iv[t, w]) \in I[Present\_at, w]$

Finally the interpretation of the quantifiers is presented. The universal quantifier is interpreted such that variables can take values from the worlds.

$M, w \models \forall x. P(x)$ *if and only if for every possible valuation that can be given to x in the world w through Iv, it is the case that* $(Iv[x, w]) \in Iv[P]$

Similarly, the existential quantifier is interpreted thus:

$M, w \models \exists x. P(x)$ *if and only if there is a possible valuation such that can be given to x in the world w through Iv, it is the case that* $(Iv[x, w]) \in Iv[P]$

Note that the model can be applicable to different terms x, l_{n-1}, l_n, t_{n-1} and t_n with varying locations and time.

It is important to emphasize that SQM is based on worlds in which the domains remain constant as opposed to worlds in which domains increase or decrease. As such the following Barcan' s axioms (Fitting, 1997) hold

$\Box \forall x. P(x) \leftrightarrow \forall x. \Box P(x)$.

3.6 Modal Properties of the Spatial Qualification Model

The logic of presence exhibits the basic property of Kripke's minimal system, **K** along with every other property of the standard **S4** system: These properties are:

K: $\Box(\phi \Rightarrow \psi) \Rightarrow (\Box\phi \Rightarrow \Box\psi)$

T: $\Box\phi \Rightarrow \phi$

4: $\Box\psi \Rightarrow \Box\Box\psi$

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However it falls short of being an S5 system because it does not satisfy the following property:

B: $\Diamond\Box\phi \Rightarrow \phi$

If this research considers the propositions formed from the *Present-at* relations, then one can argue that axioms K, T and 4 hold. For example, note that it is the case that if l is regionally part of l_1, then any agent that is present at the location l is also present at location l_1.

$$\forall x, l, l_1, t. (NTPP(l, l_1) \vee TPP(l, l_1) \vee l_1 = l) \Leftrightarrow$$
$$(Present\text{-}at(x, l, t) \Rightarrow Present\text{-}at(x, l_1, t))$$

Thus the following clearly hold:

KP1 $$\forall x, l, l_1, t. \Box(Present\_at(x, l, t) \Rightarrow Present\_at(x, l_1, t)) \Rightarrow$$
$$(\Box Present\_at(x, l, t) \Rightarrow \Box Present\_at(x, l_1, t))$$

Similarly note that x_1 is always collocated with x if and only if x_1 is part of x. That axiom is stated as:

$$\forall x, x_1. \; Part\text{-}of(x_1, x) \Leftrightarrow \forall l, t \, (Present\text{-}at(x, l, t) \Rightarrow Present\_at(x_1, l, t))$$

Therefore, it is the case that:

KP2 $$\forall x, x_1, l, t. \Box(Present\_at(x, l, t) \Rightarrow Present\_at(x_1, l, t)) \Rightarrow$$
$$(\Box Present\_at(x, l, t) \Rightarrow \Box Present\_at(x_1, l, t))$$

In another vein, the fact that a body is in a certain location l at time t can imply that the same body is in a different location at a later time, if the body is in some kind of constant and predictable motion such as the case of planetary bodies. That is, if its trajectory is fixed, then:

$$\forall x, x_1, l, l_1, t. \; Fixed\_Trajectory(x)$$
$$\Leftrightarrow \; Not\_PP(l, l_1) \wedge Not\_PP(l_1, l) \wedge$$
$$Present\_at(x, l, t) \Rightarrow Present\_at(x, l_1, t)$$

As such, if a body x is always in a fixed trajectory, it must be the case that:

$$\forall x, x_1, l, l_1, t. \Box(Present\_at(x, l, t) \Rightarrow Present\_at(x, l_1, t)) \Rightarrow$$
$$(\Box Present\_at(x, l, t) \Rightarrow \Box Present\_at(x_1, l, t))$$

Axioms *KP1* and *KP2* show that our system, under examination in light of the S4 and S5 systems of axioms, conforms to the properties of the Kripke minimal system.

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In another vein, the only way a particular presence log, that is, the fact that *x* is present at a location *l* at time *t*, can occur in all possible worlds reachable from the current world if that presence log already occurs in the current world.

TP $\forall x, l, t.\ \Box Present\_at(x, l, t) \Rightarrow Present\_at(x, l, t)$

Similarly, the fact that a presence log holds in all the worlds accessible from the current world implies it will be true in all worlds accessible from those worlds accessible from the current world.

4P $\forall x, l, t.\ \Box Present\_at(x, l, t) \Rightarrow \Box\Box Present\_at(x, l, t)$

Axioms *KP1*, *KP2*, *TP* and *4P* all show that the logic of presence we describe here constitute an **S4** system of axioms.

The comparison of SQM with S4 and S5 systems is as summarised in table 3.1.

S5 system contains K, T and 4 of S4 and also axiom B: $\Diamond\Box\phi \Rightarrow \phi$. From B, $\Diamond\Box$Present_at(x,l,t) $\Rightarrow$ Present_at(x,l,t) does not hold in SQM. Since S4 has been proven to be sound and complete, it can also be concluded that SQM is sound and complete

The model of time used in this research is a branching model of time. It is linear in the past and branches into the future. Within each world there is a linear model of time. That line branches into the different accessible worlds in the future.

Since the logical model will be a first order modal logic, the model theoretic nature of the logic needs to be clarified. Particularly, it is important to know the variability or otherwise of the domains in the world as the model moves from world to world. Another issue to be resolved is the nature of the constants in the logic. It is important to clarify whether they are definite constants or referents.

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Table 3.1: Comparison of SQM with S4 and S5 Modal System

SQM		S4	
Axiom Name	Detail	Axiom Name	Detail
KP	$\forall x, l, t.\ \Box(\text{Present\_at}(x,l,t) \Rightarrow \text{Present\_at}(x,l,t)) \Rightarrow (\Box\text{Present\_at}(x,l,t) \Rightarrow \Box\text{Present\_at}(x,l,t))$	K	$\Box(\phi \Rightarrow \psi) \Rightarrow (\Box\phi \Rightarrow \Box\psi)$
TP	$\forall x, l, t.\ \Box\text{Present\_at}(x,l,t) \Rightarrow \text{Present\_at}(x,l,t)$	T	$\Box\phi \Rightarrow \phi$
4P	$\forall x, l, t.\ \Box\text{Present\_at}(x,l,t) \Rightarrow \Box\Box\text{Present\_at}(x,l,t)$	4	$\Box\psi \Rightarrow \Box\Box\psi$

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3.7 Possible application domains for the Spatial Qualification Logic

With the question "is it possible for an account holder who made cash transaction using his/her debit card at First Bank Automated Teller Machine (ATM), University of Ibadabn (UI) branch, at 12 noon to be present at UBA ATM machine, Lagos, for another cash transaction at 12.15?" Using commonsense, to be able to tell whether it is possible or impossible, one must have known that land transportation is the fastest available means from Ibadan to Lagos and the minimum time that can be used by this means on that route is 2 hours. Since the duration between the transaction at Ibadan and the one at Lagos is 15 minutes, one can easily tell that it is not possible for the account holder to have arrived Lagos in 15 minutes to have embarked on another transaction.

Without prior knowledge of how long it can take for one to be in Lagos and the time the account holder made the last transaction at Ibadan, it will be difficult for one to tell the possibility or impossibility of being present at Lagos. The possibility of being present at a location remains valid even when a state at some time points, seen to be possibly true is not actually true. The anchoring relations of Galton are somewhat embedded in this logical model. This means that an object who is said to be possibly present at another known state at a certain time might as well be anchoring around the previous state without getting to the designated state.

Another domain where SQM could be applied to is the alibi reasoning domain which often features in the court during trials. An instance of such court case has to do with investigating the actual location of the accused person at the time of incidence from prior spatial knowledge. All possible worlds with their reachability can be determined using the spatial qualification model.

Also, planning, that involves a transportation process is another domain where SQM can be applied to. A typical case of this problem is seen in the TRAINS project (Traum et al., 1991). An application to the planning domain with deadlines is considered in subsequent chapters. A proof system for the formalised logical model of the spatial qualification (SQM) is developed in chapter four.

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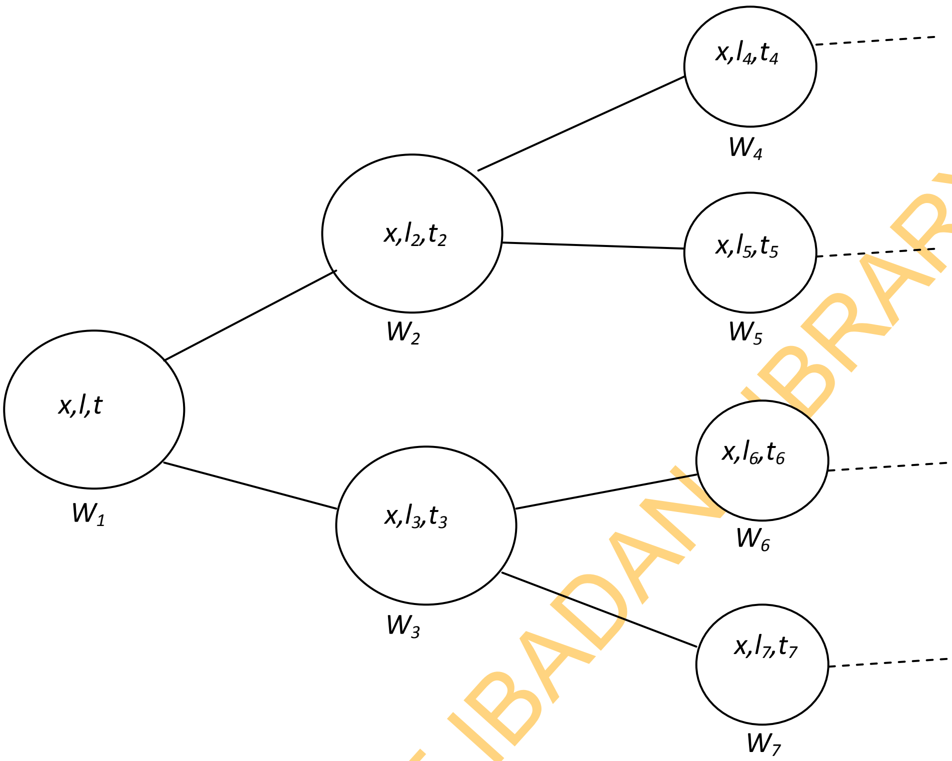

Figure 3.9: Possible World branching into different accessible worlds in the future

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CHAPTER FOUR

PROOF SYSTEM OF THE SPATIAL QUALIFICATION LOGIC

4.1 Introduction

The tableau proof rules for the quantified modal logic are listed as they are applicable to the proof system of SQM. This is followed by some stated lemmas and the corresponding proofs. The decidability as well as the soundness and completeness proof of the SQM system is also described in this chapter.

4.2 Tableau Proof Rules

Since SQM follows the syntax of quantified modal logic which combines features of first-order and that of modal logic, our proof rules therefore will combine the tableau rules in propositional, first-order and modal logic. Hence, the following tableau rules are applicable to the proof system of SQM.

(i) Negation rules

$T\ \neg A$ | $F\ A$

$F\ \neg A$ | $T\ A$

(ii) Conjunctive rules

$T\ (A \wedge B)$ | $T\ A$, $T\ B$

$T\ (A \vee B)$: $T\ A$ / $T\ B$

$T\ (A \Rightarrow B)$: $F\ A$ / $T\ B$

$T\ (A \Leftrightarrow B)$: $T\ A$, $T\ B$ / $F\ B$, $F\ A$

(iii) Disjunctive rules

$F\ (A \wedge B)$: $F\ A$ / $F\ B$

$F\ (A \vee B)$ | $T\ A$, $T\ B$

$F(A \Rightarrow B)$ | $T\ A$, $F\ B$

$F\ (A \Leftrightarrow B)$: $T\ A$, $F\ B$ / $F\ A$, $T\ B$

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(iv) Universal rules

$$\frac{T\ \forall x\ A(x)}{T\ A(t)} \qquad \frac{F\ \exists x\ A(x)}{F\ A(t)}$$

for any term t in the language.

(v) Existential rules

$$\frac{T\ \exists x\ A(x)}{T\ A(c)} \qquad \frac{F\ \forall x\ A(x)}{F\ A(c)}$$

for a new constant c.

(vi) Necessity rules

$$\frac{T\ \Box A}{T_k\ A} \qquad \frac{F\ \Diamond A}{F_k\ A}$$

(vii) Possibility rules

$$\frac{T\ \Diamond A}{T_k\ A} \qquad \frac{F\ \Box A}{F_k\ A}$$

4.3 Tableau proofs for satisfiability of SQM

To prove that a formula, B is a logical consequence of a set of formula, $A_1 \ldots A_k$, the following lemmas are hereby stated as they make up the proof system for the spatial qualification logic.

Lemma 4.1:

Given that:

$$\{Present\_at(x,l_1,t_1),\ \forall x,l_1,l_2,t_1,t_2.\ l_1=l_2 \wedge t_1<t_2 \Rightarrow Reachable(x,l_1,l_2,(t_1,t_2)),\ l_1=l_2,\ t_1<t_2\} \vdash \Diamond Present\_at(x,l_2,t_2)$$

Proof:

To proof by contradiction that the above lemma is true, we start by saying that the set of axioms entails $\neg\Diamond Present\_at(x,l_2,t_2)$. Including the negated axiom to the set and proving using tableau rules is as analysed in figure 4.1 and completed in figure 4.2.

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Since both branches of the above tableau do not lead to a closure, we look for a way of extending the branch that is still open.

From the system of axioms in SQM, axiom T_{A3} defines *Reachable* to be

$$Reachable(x,l_1,l_2,(t_1,t_2))\Leftrightarrow(t_1<t_2\wedge(Present\_at(x,l_1,t_1)\Rightarrow\Diamond Present\_at(x,l_2,t_2)).$$

By equivalence, we have that

$$Reachable(x,l_1,l_2,(t_1,t_2)) = (t_1<t_2\wedge (Present\_at(x,l_1,t_1)\Rightarrow\Diamond Present\_at(x,l_2,t_2)).$$

By substitution rule, $(t_1<t_2\wedge(Present\_at(x,l_1,t_1)\Rightarrow\Diamond Present\_at(x,l_2,t_2))$ replaces $Reachable(x,l_1,l_2,(t_1,t_2))$ in the above tableau and thus extends the branch further in other to check for its closure as shown in the tableau in figure 4.2.

From the closed tableau in figure 4.2, the set of axioms in node (1) is expanded to have nodes (2), (3), (4), (5) and (6). By applying necessity rule from (6), we have item (7). Node (8) is from node (3) by universal rule. By conjunctive rule, node (8) opens into two branches with nodes (9) and (10). Nodes (9) also opens into two branches with nodes (11) and (12) and nodes (11) and (5) closes as well as nodes (12) and (4) since there is a contradiction.

Extending node (10) as explained in the tableau in figure 4.1, we have node (10) replaced as shown in the tableau in figure 4.2. By substitution rule, we have node (13) from (10). Again by conjunctive rule from node (13), we have nodes (14) and (15). Node (15) opens into two branches with nodes (16) and (17) by conjunctive rule. By possibility rule from (17), we have node (18). All branches in the tableau lead to closure as nodes (17) and (2) close and also nodes (18) and (7) close.

The closure of the tableau of the contradiction shows that the proof of the original set of formulas is complete and that SQM logic is satisfiable with the given statement in lemma 4.1.

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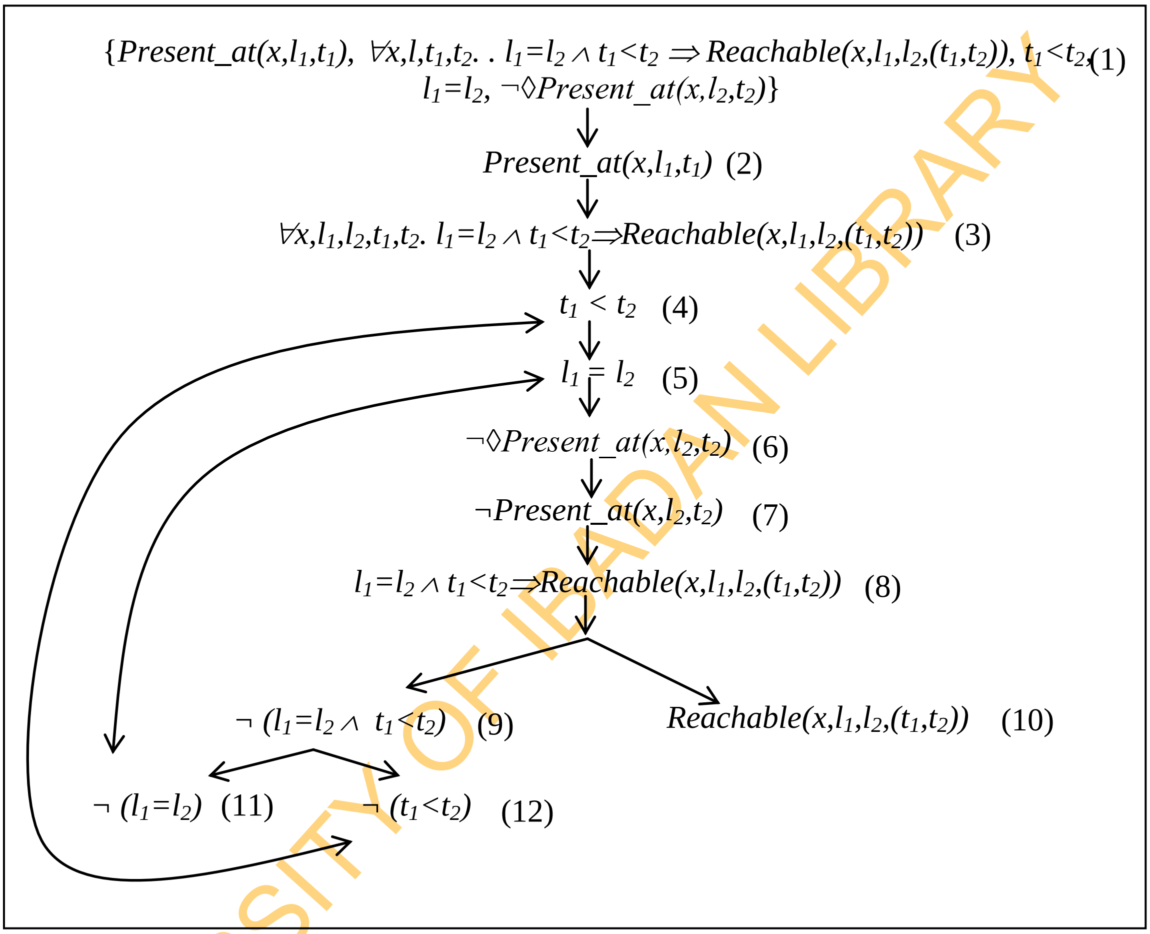

Figure 4.1: Tableau Proof of Axiom T_{A4} (open)

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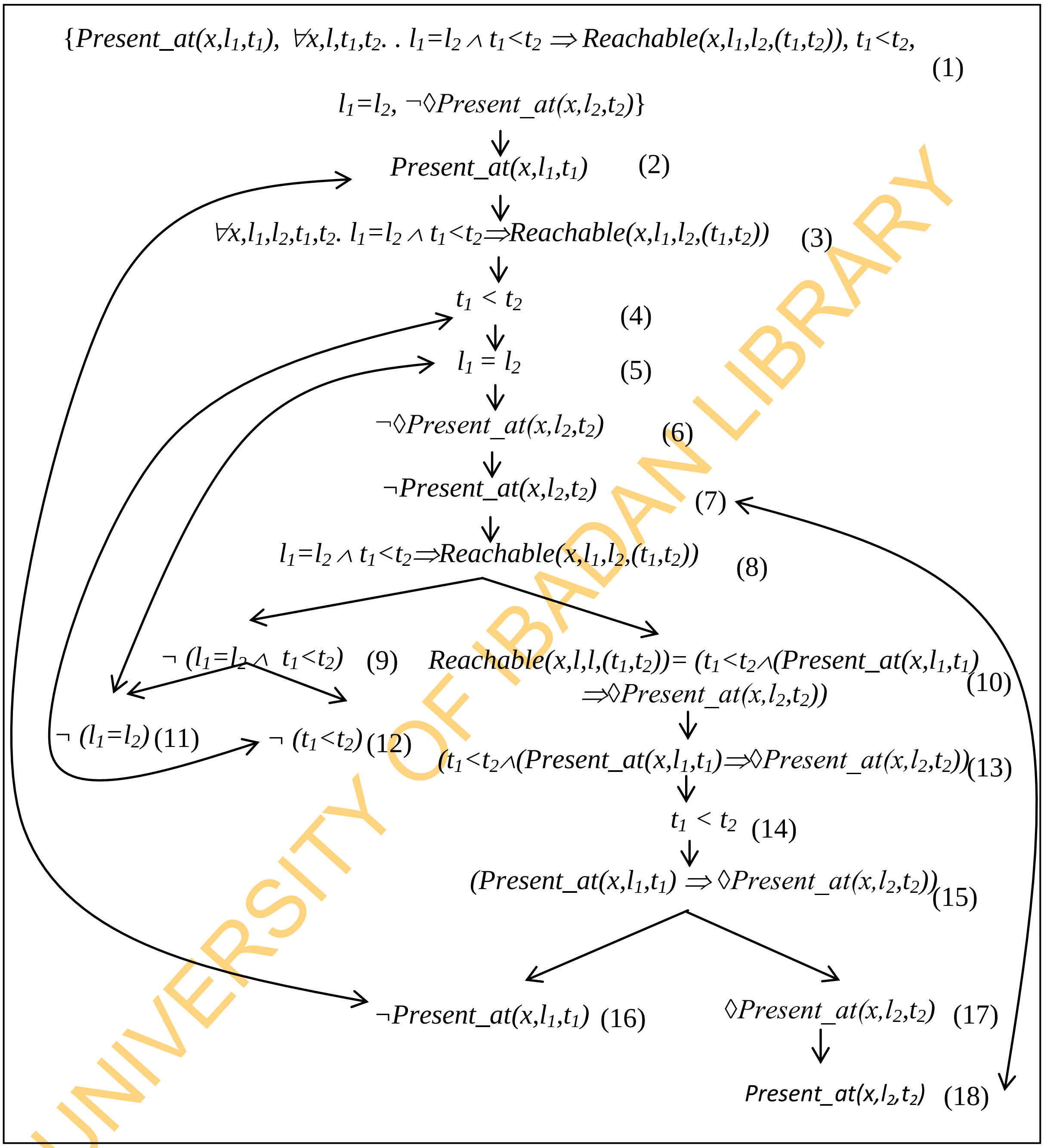

Figure 4.2: Tableau Proof of Axiom T_{A4} (closed)

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Lemma 4.2:

Given that:

$\{Present\_at(x,l_1,t_1), Reachable(x,l_1,l_2,(t_1,t_2)), Reachable(x,l_2,l_3,(t_2,t_3)),\ t_1<t_2<t_3, Reachable(x,l_1,l_2,(t_1,t_2)) \wedge Reachable(x,l_2,l_3,(t_2,t_3)) \Rightarrow Reachable(x,l_1,l_3,(t_1,t_3))\} \vdash \Diamond Present\_at(x,l_3,t_3)$

Proof:

The proof is as shown on figure 4.3 and completed in figure 4.4

Lemma 4.3:

Given that:

$\{Present\_at(x,l_1,t_1), \forall x,l_1,l_2,t_1,t_2.\ l_1=l_2 \wedge t_1<t_2 \Rightarrow Reachable(x,l_1,l_2,(t_1,t_2)), t_1<t_2\} \nvdash \Diamond Present\_at(x,l_2,t_2)$

Proof:

To proof by contradiction that lemma 4.3 is true, we start by saying that the set of axioms entails $\neg(\neg\Diamond Present\_at(x,l_2,t_2))$. Including to the set and proofing using tableau rules is as shown in figure 4.5 and completed in figure 4.6.

By employing substitution rule as seen in the proof of lemma 4.1 and replacing, $Reachable(x,l_1,l_2,(t_1,t_2))$ of node (11) in figure 4.5 with $(l_1=l_2 \wedge t1<t_2 \wedge (Present\_at(x,l_1,t_1) \Rightarrow \Diamond Present\_at(x,l_2,t_2))$, and thus extending the branch further leading to closure as shown in figure 4.6.

From tableau in figure 4.6, the set of axioms in node (1) is expanded to have nodes (2), (3), (4), (5) and (6). Node (7) is obtained by applying double negation rule on node (6). And by necessity rule from node (7), we have node (8). Node (9) is from node (3) by universal rule. By conjunctive rule, node (9) opens into two branches with nodes (10) and (11) and by disjunctive rule node (10) extends to nodes (12) and (13). Nodes (12) and node (5) close and nodes (13) and (4) also close.

As seen in figure 4.6, we have node (11) replaced as shown in the tableau in figure 4.4. By substitution rule, we have node (14) from node (11). Again, by conjunctive rule from node (14), we have nodes (15) and (16). Node (16) opens into two branches with nodes (17) and (18) by conjunctive rule. By possibility rule from (18), we have node (19). Nodes (17) and (2) close but node (19) is open without a contradiction.

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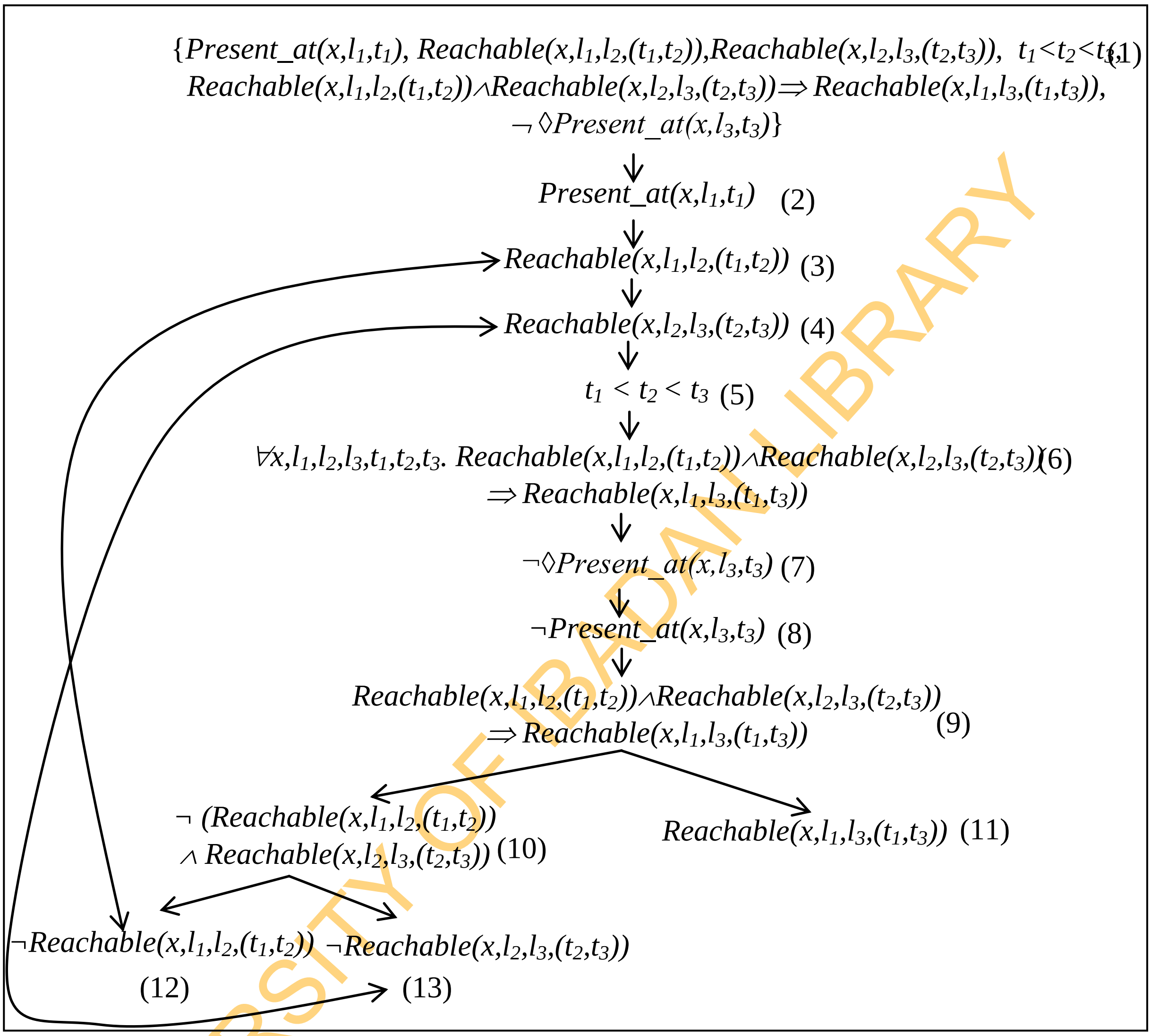

Figure 4.3: Tableau Proof of Axiom T_{A10} (open)

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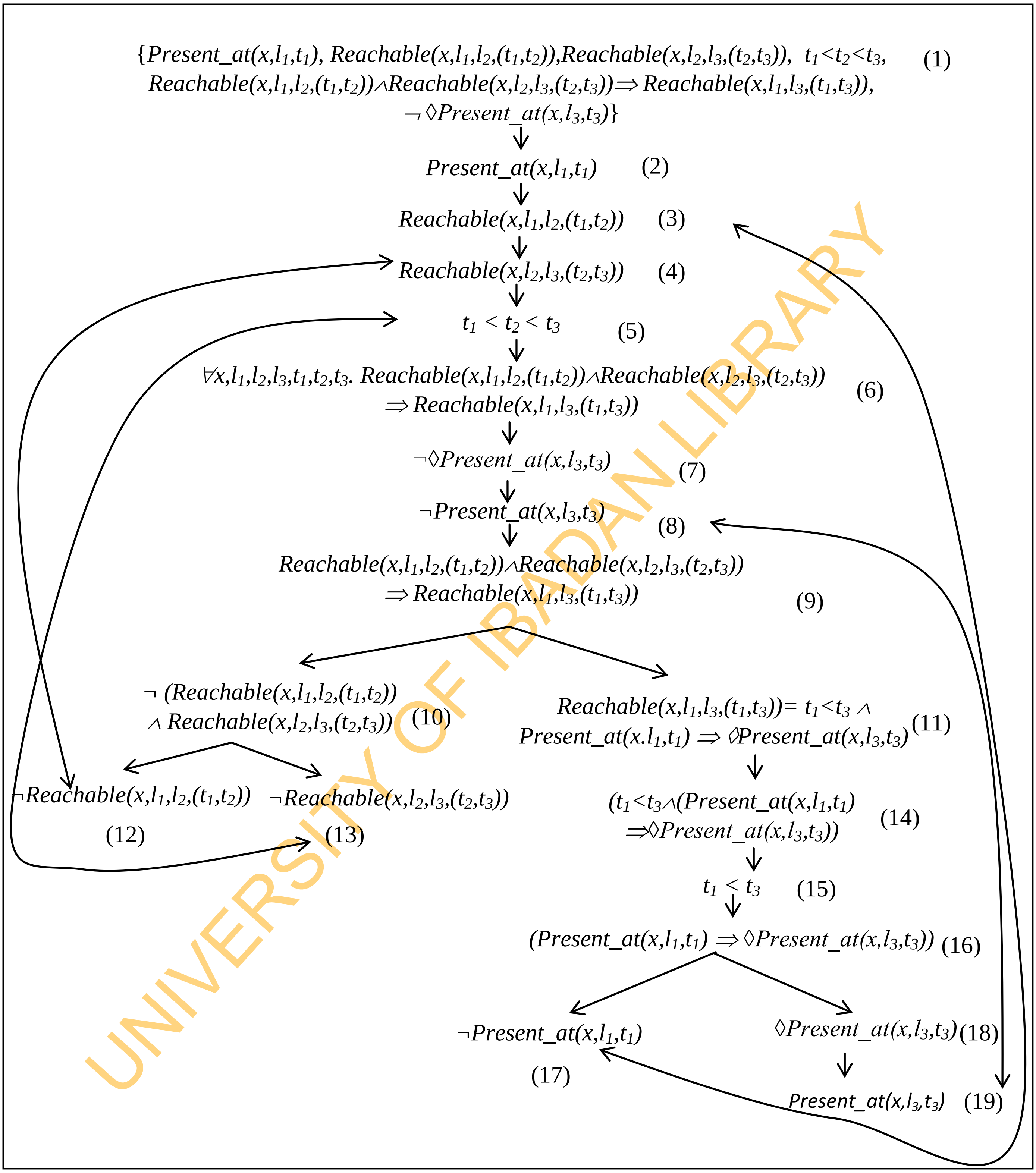

Figure 4.4: Tableau Proof of Axiom T_{A10} (closed)

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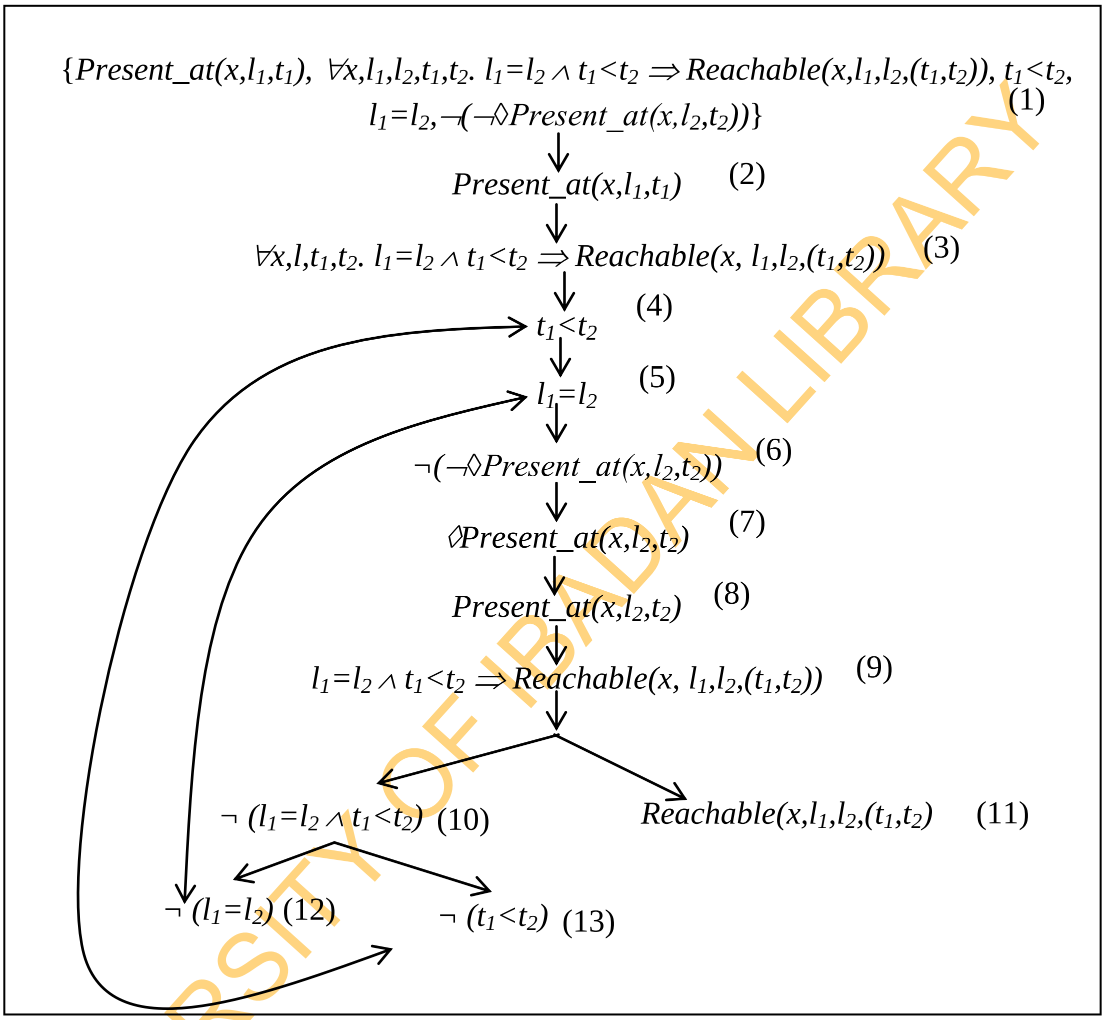

Figure 4.5: Tableau Proof of the Negation of Axiom T_{A4} (open)

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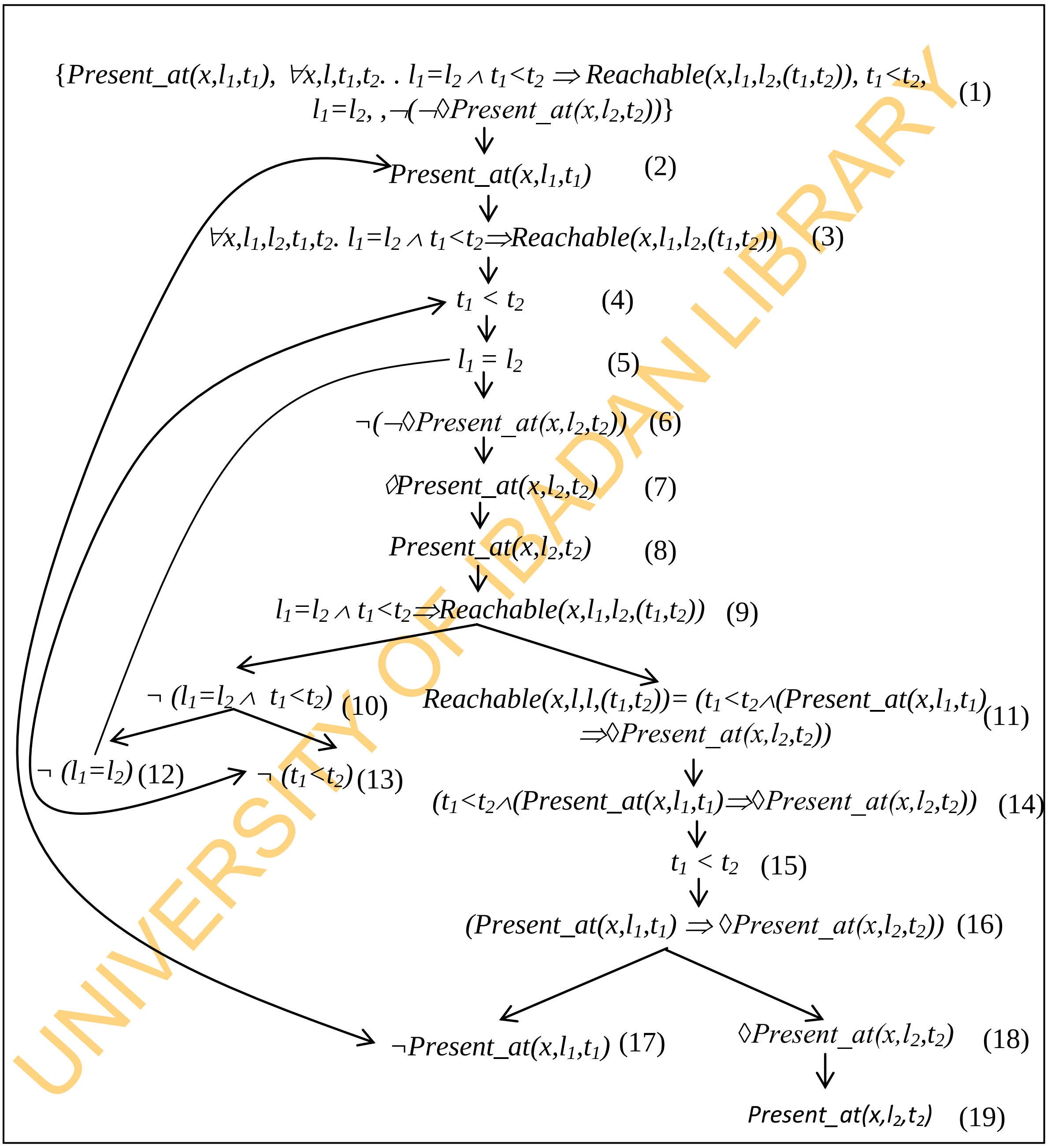

Figure 4.6: Tableau Proof of the negation of Axiom T_{A4} (open)

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Since the branches in the tableau of figure 4.6 do not all lead to closure, the proof is therefore incomplete showing that the negation of the assertion is not provable. This then shows that the original axiom is invalid since its contradiction did not terminate with a closure on all the branches.

4.4 Decidability of the SQM system

The closures of some of the tableaus give the validity of the original axioms whose contradictions were considered. Also, the tableau showing the non-closure, on the other hand, gives the non-validity of the original axiom. The non-determination of the validity of all the axioms in the SQ system of axioms demonstrates the semi-decidability of SQ logic in SQM. Hence, Quantified modal logic is semi-decidable.

The decidability of SQ logic is clearly described from the tableau proof system in figures 4.1 to 4.6. The decision procedure shows that tableau proof method does not work for some axioms in the logic. This is obvious since the logic uses the quantified modal logic which is the hybridization of the known undecidable first-order logic and modal logic.

From this proof system, it is possible to decide the possibility of an agent being present at a location at a certain time, if it is possible for that agent to be present at that location at the time, t given the antecedents. This shows that the possibility of an agent's presence at a certain location and time was only provable in the affirmative, while its negation was not. However, it is not possible to infer the fact that it is not possible for an agent to be present at a certain location and at a certain time. The reason is that, most of our axioms are implications and not equivalence.

The next chapter gives a more detailed case study with a successful application of the SQM in planning as a pilot test for validity of our model in solving real world problems.

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4.5 Soundness of SQM Proof System

To further prove that the SQM proof system (SQM_p) is sound, the satisfiability of the set of formulae or sentences is checked in line with the closure of a tableau with contradiction forming part of the set of sentences. This showed that the set is not satisfiable as earlier pointed out.

This corresponds with the soundness theorem stated by Sabri (2009) as follows:

> Let $\{S_1,\ldots,S_n\}$ be a finite set of (signed and unsigned) sentences with parameters. If there exist a finite closed tableau starting with $\{S_1,\ldots,S_n\}$, then $\{S_1,\ldots,S_n\}$ is not satisfiable.

Proof:

Let τ be a closed tableau starting with $\{S_1,\ldots,S_n\}$ with $\tau = \tau_0; \tau_1,\ldots, \tau_m$ finite sequence of tableaux such that: $\tau_0 = \{S_1,\ldots,S_n\}$ and each τ_{k+1} is an intermediate extension of τ_k. That is, τ_{k+1} is obtained from τ_k by applying a tableau rule to a path of τ. If τ_k is satisfiable, then τ_{k+1} is satisfiable.

Considering the SQM_p, we have some representative cases for each tableau rule as follows:

Case 1: Suppose that τ_{k+1} is obtained from τ_k by applying the necessity rule

:
□A
:
A

to a path in τ_k. It follows that τ_{k+1} is satisfiable since □A = A= T is satisfiable in at least one path in t_k.

Case 2: Suppose that τ_{k+1} is obtained fron τ_k by applying the possibly rule

:
◊A
:
A

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to a path in τ_k. It follows that τ_{k+1} is satisfiable since ◊A = A= T is satisfiable in at least one path in τ_k.

More of the representational cases as they apply to First-Order logic were described by Sabri (2009).

Suppose $\{S_1,\ldots,S_n\}$ is satisfiable, then τ_0 is satisfiable, and by induction on k, it follows that all of the τ_k are satisfiable. In particular, $\tau_m=\tau$ is satisfiable, but since τ is closed, this is impossible.

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CHAPTER FIVE

SPATIAL QUALIFICATION REASONING IN PLANNING

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5.1 Introduction

This chapter gives an overview of planning and highlights related projects in planning domain. The overview clearly describes the need for the spatial qualification logic in the domain of discourse. An instance showing the application of SQ logic on product distribution from known locations within University of Ibadan is given with detailed case studies using the SQ logic. Results from the analysis of these case studies are also shown and discussed.

5.2 Planning and the Spatial Qualification Logic

There are several preconditions to actions and the impossibility of knowing all is referred to as the qualification problem in AI (Thielscher, 2001). One of such preconditions is to determine whether or not an agent is spatially qualified to carry out an action. As the need to reason about plans ahead of their execution increases, the need to represent knowledge about spatial domains increases as well. A plan is defined as an argument that the execution of its actions will result in the achievement of its goals given the assumptions on which it is based (Ferguson, 1995). This brings about the need to reason about an agent's spatial qualification to carry out actions in this domain. In planning domain, the planner must be able to assess a plan's potentials to succeed in terms of the various qualifications for actions that make up the plan. While the relationship between planning and temporal reasoning has been addressed (Allen, 1991), the connection between planning and spatial reasoning has not been explored.

At the University of Rochester, in the 1990s, researchers undertook the building of a conversationally proficient intelligent planning assistant as part of the TRAINS project (Allen et al., 1991). The TRAINS project domain "is a transportation world with cities, rail links between them, engines, boxcars and the like to move things around and a variety of commodities" (Ferguson, 1995). This TRAINS project proposed a solution to the problems of the domain through mixed initiative planning that involved using dialogues to carry out various kinds of plan inference.

To get an idea of the kind of plans that the TRAINS system needs to reason with, this research presents an abridged form of a dialog from the TRAINS domain:

> *We better ship a boxcar of oranges to Bath by 8 am. There are some oranges at Corning and a boxcar at Danville. So we need an engine to move the boxcar. So we should move the engine at Avon, Engine e1, to Danville to pick up the boxcar there and move it from Danville to Corning, Load up some oranges into the boxcar and then move it on to Bath.*

The kind of plans in this domain involves both the need to cover distances as well as do so within a certain deadline. Not only should the boxcar of oranges arrive at Bath, it should arrive by 8 am. In order for the engine moving the boxcar full of oranges to arrive bath by 8am, it must be fully loaded with oranges, ready to leave Corning by 7am if it takes an hour to drive between Corning and Bath. Similarly if it takes fifteen minutes to load a boxcar of oranges, the empty boxcar driven by engine e1, must arrive Corning from Danville by 6:45 am. It is obvious that one of the major keys to reasoning about the feasibility or otherwise of these plans is to be able to reason about the engine to make each of its assigned journeys within a time limit.

Planning is a well-known problem in the field of AI. As such reasoning about plans is an equally important task. Attempts to address this problem in the literature uses preferences and time-dependent continuous costs (Benton et al., 2012); time windows (Braysy and Gendreau, 2005); and adversarial abduction problems (Shakarian et al., 2011). Plans are dynamic and require monitoring and re-planning.

In this chapter, the problem of spatial qualification of agents is considered in a planning domain requiring deadlines where the planner has the knowledge of the agents' current location and time. Given the location of agents, the spatial qualification

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reasoner introduced in this work will help the agent reason about the possibility of being present at another location at some other time in order to make a delivery.

But the goal of this work differs from that of the TRAINS project which focused on developing an intelligent planning assistant that is conversationally proficient in natural language to predict future states of the world in the absence of complete knowledge (Allen and Schubert, 1991). Allen (1991), in his temporal planning work highlighted the preconditions for opening the door latch without considering the spatial qualification of the agent as one of the preconditions. This chapter is aimed at deciding the spatial qualification of an intelligent agent i.e. to determine if it is possible for the agent to be present at the desired location at a certain future time as a required precondition to carry out the action at that time. To assess the existing plan and re-plan to meet the domain's need at any time in case of uncertainty requires that we investigate the possibility of the vans to get to any of the hostel that has an urgent need to meet the deadline. This chapter is aimed at providing a planning reasoner that assists in the investigation using the spatial qualification model.

Planning the distribution of these products requires that the managing agent should take the following steps:

(i) Know the position of things according to previous plan.
(ii) Identify where there is need for re-planning
(iii) Draw up a new plan when necessary.

Steps (ii) and (iii) cannot be carried out except step (i) is done. Step (i) requires reasoning such as it applies to the logic of spatial qualification, which helps in the assessment of the existing plan. Therefore, the planner sends the travel plan to the SQM reasoner as input that is considered by SQM for validity check. If the plan is valid, then, it is slated for implementation, otherwise, it is slated for re-planning. These feedbacks are directed to the planner for information and further action. Figure 5.1 gives the framework of how SQM is applied in planning for interaction with the planning agent.

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This interaction of the SQM with the planning agent does not eschew the underlying concept of mixed-initiative planning used in the TRAINS project. Some of the lessons from the mixed-initiative planning used in the TRAINS project include the fundamental process of communication based on defeasible reasoning, where conclusions are subject to revision given new information or more time to reason. The defeasibility is as a result of the opacity and ambiguity of communication.

Although the communication aspect of the TRAINS project promotes collaborative reasoning, SQM seeks to address other sources of defeasibility such as incomplete knowledge and uncertain effects of actions other than its complexity, using the modalities in Modal Logics. The distribution process of Coca cola to hostels in University of Ibadan is a fixed multi-agent domain like the TRAINS domain.

5.3 Application of SQM in Coca-Cola Distribution from Mini Depot to hostels within University of Ibadan

Considering the distribution of Coca-Cola product from the Coca-Cola Mini Depot (CCMD) to all the hostels within University of Ibadan campus, the Mini Depot is known to have two pick-up vans meant for these distribution processes. This means that there are multi-agent actions taking place at varying locations and times in the problem domain. The domain agents are mainly the two pick-up vans and the supervisor agent that require intelligence to monitor and re-plan based on urgent request and need from any of the hostels. The distribution processes follow a star-like structure from CCMD to the various hostels, about 12 in number, and from one hostel to the other following the given designated routes.

Route Designation:

The route designation described here is applicable only to this research work. Due to the carrying capacity of the pick-up vans and the locations of the hostels, each van is assigned to a designated route which includes three hostels each based on their geographic locations. The designated routes for routines supply of product are:

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R1:	CCMD	→	MH	→	TrH	→	TdH	→	CCMD
R2:	CCMD	→	QED	→	KH	→	BH	→	CCMD
R3:	CCMD	→	TBH	→	ZH	→	IH	→	CCMD
R4:	CCMD	→	QIH	→	NH	→	AH	→	CCMD

Figure 5.2 gives the representation of these designated routes as they traverse from CCMD to the hostels for product distribution. Although this is based on the assumption that the driver of the van has to visit all the nodes in each of the routes, there are other possible ways. Some of the possible routes through which nodes in the designated routes can be visited are shown in figure 5.3.

The actual locations of CCMD and hostels within University of Ibadan are shown in the skeletal diagram in figure 5.4. The distances apart from one location to another are as obtained from Google Maps (Google-Maps, 2012). Figure 5.5 shows a section of the world map showing some of the nodes and the routes within University of Ibadan. Google maps uses the great circle display and distance calculation which returns the shortest distance between any two points on the surface of the sphere measured along a path on the surface of the sphere with centers that are coincident with the center of the sphere, different from Euclidean distance which measures the length of a straight line from one point to the other.

Spatially qualifying an agent involves the temporal knowledge of the world. This SQM application assumes the speed limit of 20km/sec for the pick-up vans to traverse on roads in the university campus. Hence, the time required to cover the obtained distances for each of the routes in equation (5.2) can be computed from equation (5.1)

$$\text{Speed} = \text{distance} / \text{time} \quad \ldots\ldots\ldots\ldots\ldots\ldots.. \quad (5.1)$$

Therefore,

$$\text{Time} = \text{distance} / \text{speed} \quad \ldots\ldots\ldots\ldots\ldots\ldots.. \quad (5.2)$$

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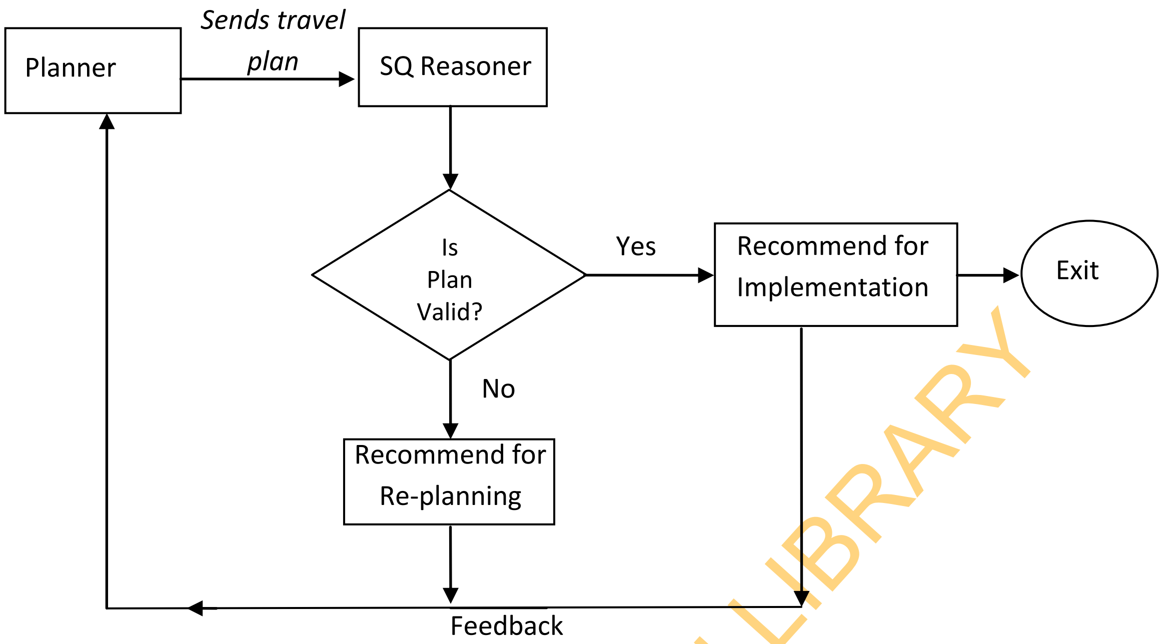

Figure 5.1: An Application Framework of SQM in Planning

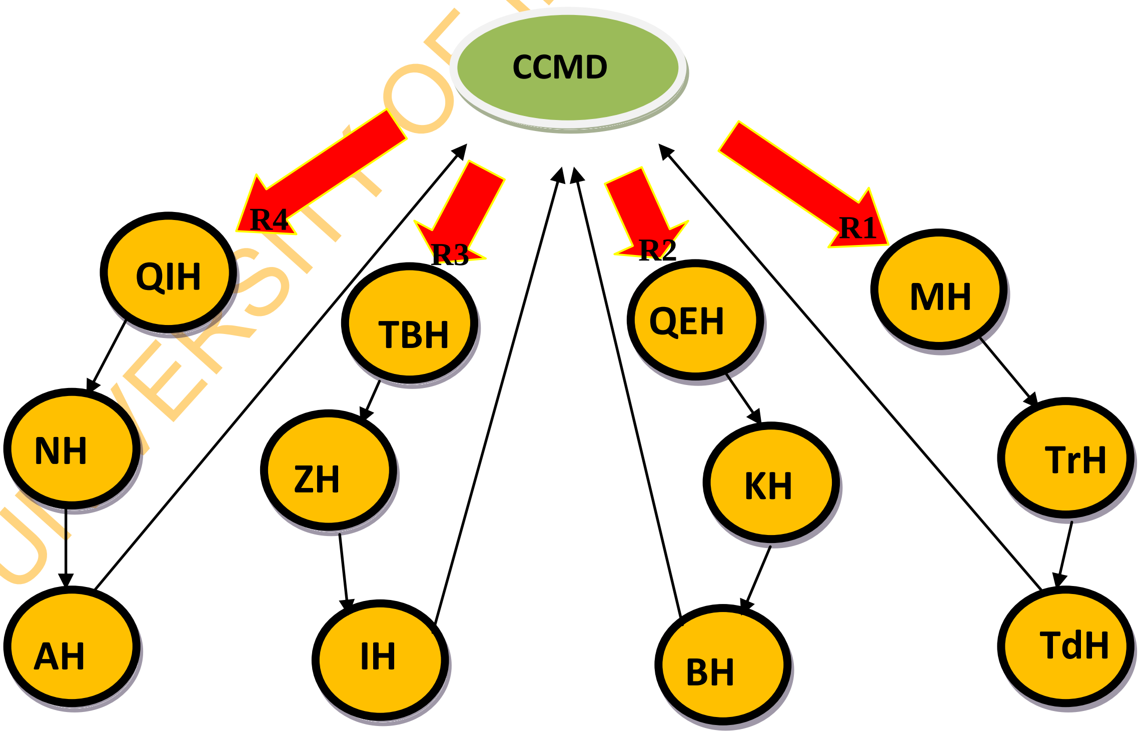

Figure 5.2: Designated routes for routine product distribution from CCMD

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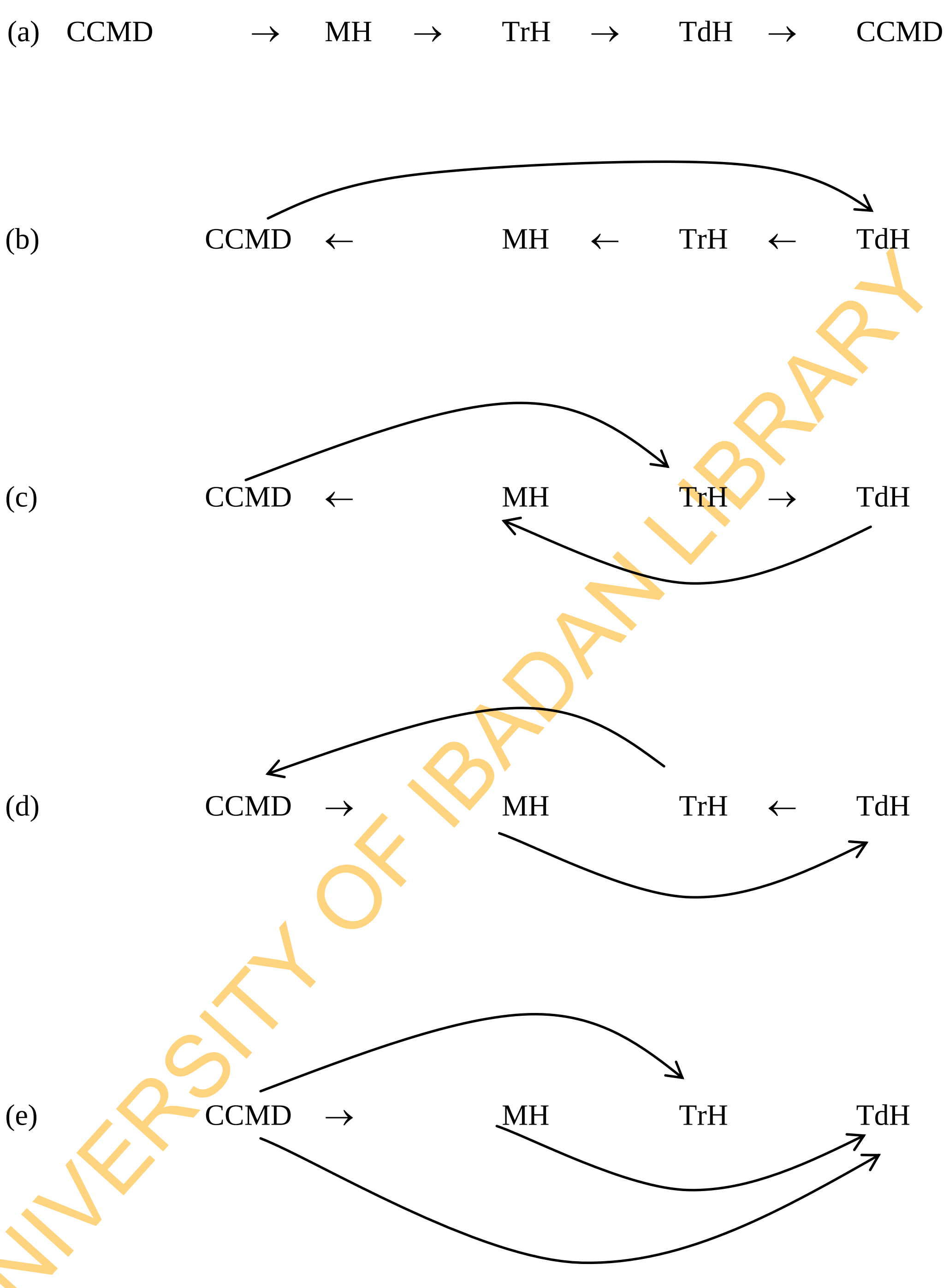

Figure 5.3: Possible Routes for Nodes Visitation in R1

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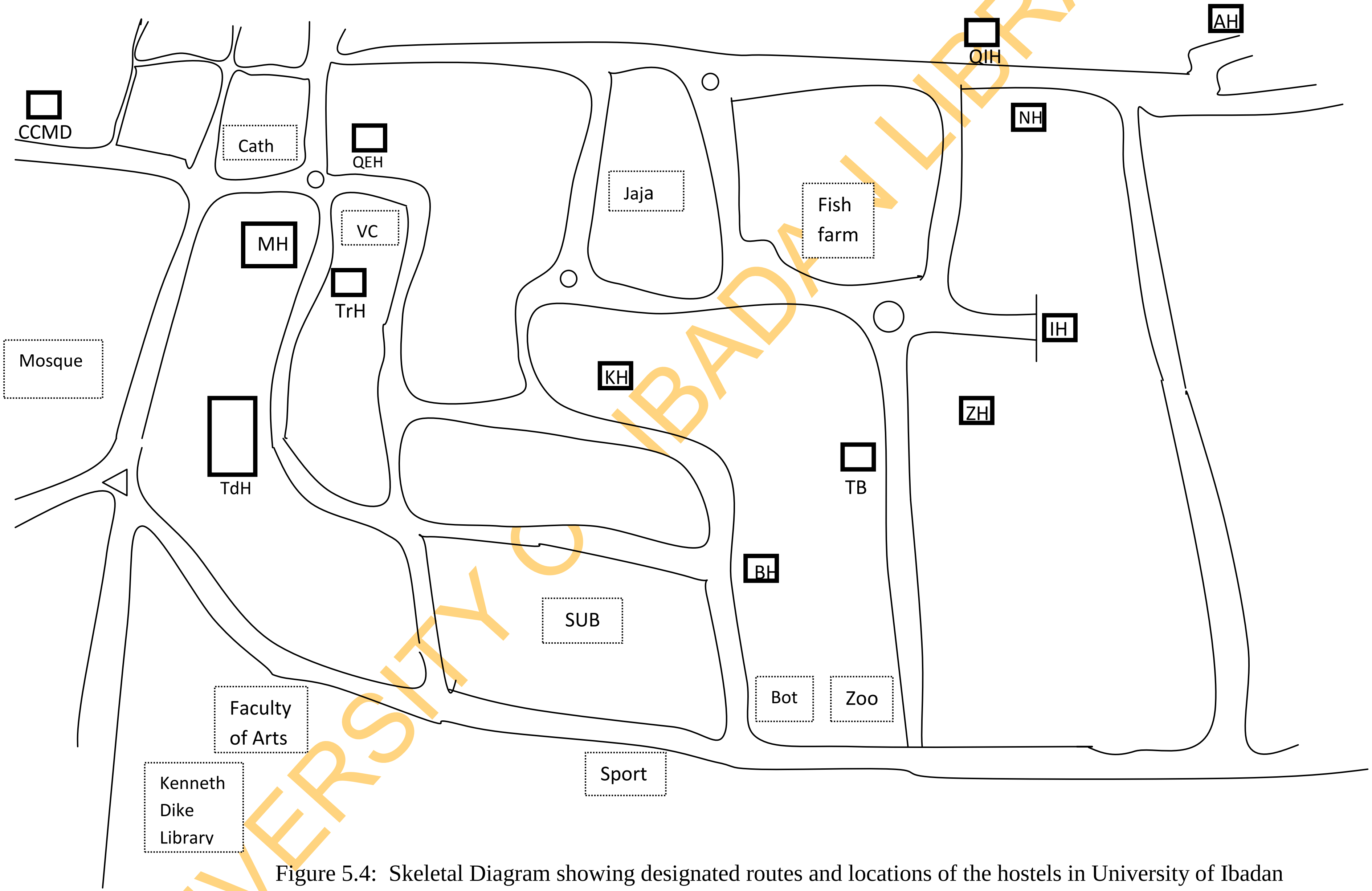

Figure 5.4: Skeletal Diagram showing designated routes and locations of the hostels in University of Ibadan

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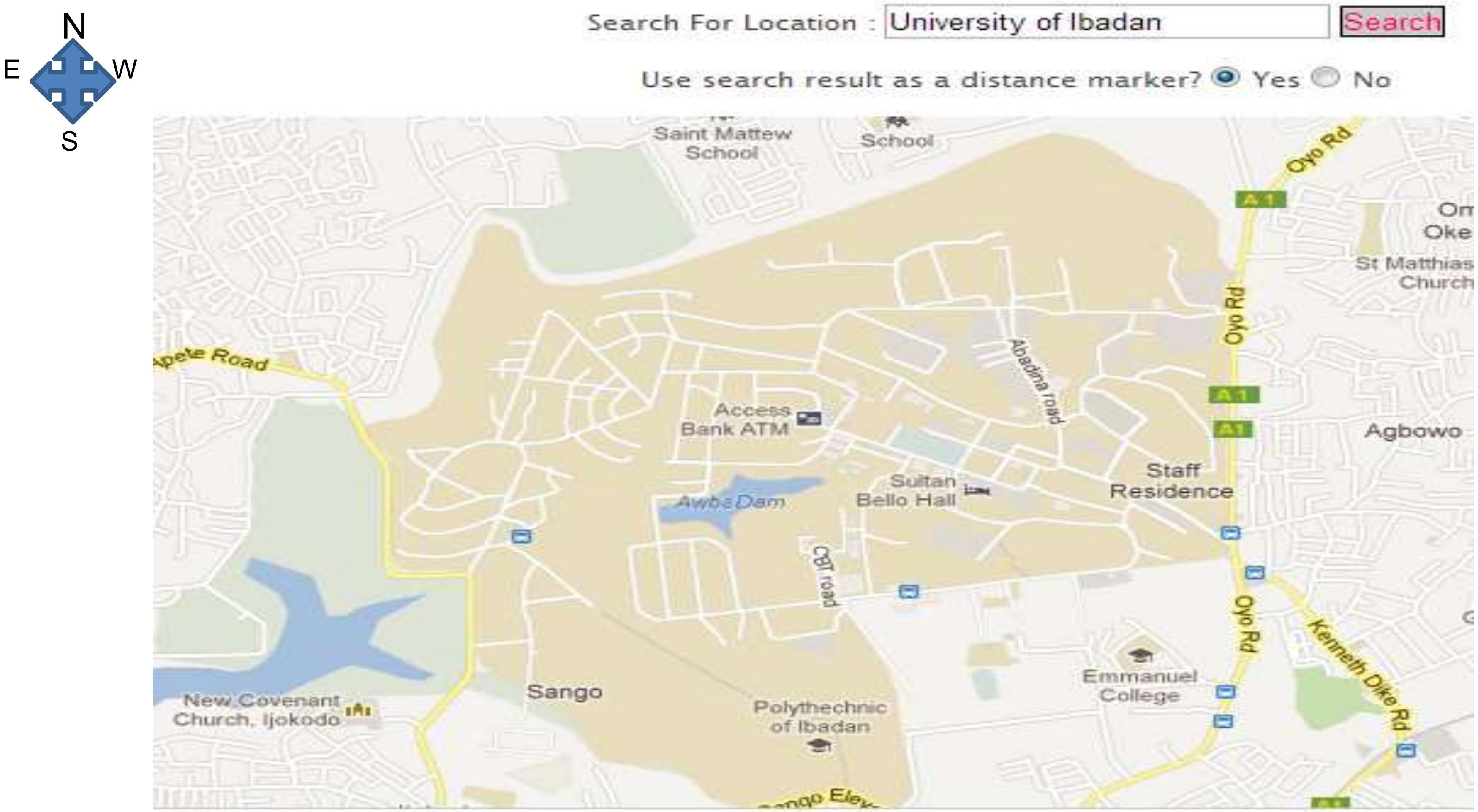

Figure 5.5: Screen shot of Google Maps Distance Calculator showing some routes in University of Ibadan

Thus, the needed components of the domain knowledge for the application of the spatial qualification logic are the distances of the routes and the time it takes to traverse on that route. This has been derived as given in table 5.1.

Measuring the distances between the designated routes and calculating the times required to cover the distances results in the summary presented in table 5.2.

For additional information on routes other than the designated routes and intra-routes for routine distribution, the matrix representation in table 5.3 is hereby given. Table 5.4 also provides the matrix with necessary information on inter-route distribution.

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Table 5.1: Hostels reachable from CCMD in University of Ibadan with their Codes/Abbreviations, distances and their corresponding times

Name of Hostel	Hostel Codes/ Abbreviations	Distances (Kilometers)	Time	
			Hours	Minutes
Queen Elizabeth II Hall	QEH	1.080	0.0540	3.240
Tedder Hall	TdH	0.802	0.0401	2.406
Mellanby Hall	MH	0.782	0.0391	2.346
Independence Hall	IH	1.541	0.0771	4.626
Sultan Bello Hall	BH	1.053	0.0527	3.162
Tafawa Balewa Hall	TBH	1.143	0.0572	3.432
Kuti Hall	KH	1.081	0.0541	3.246
Trenchard Hall	TrH	0.834	0.0417	2.502
New Hall	NH	1.617	0.0809	4.854
Queen Idia Hall	QIH	1.690	0.0845	5.070
Obafemi Awolowo Hall	AH	2.062	0.1031	6.186
Nnamdi Azikiwe Hall	ZH	1.323	0.0662	3.972

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Table 5.2: Route Descriptions for product distribution from CCMD through the routine designated reachable routes with their distances and corresponding time in University of Ibadan

Route Code	Reachable Paths	Distances (Kilometers)	Time Hours	Time Minutes
R1	CCMD – MH	0.782	0.039	2.34
	MH –TrH	0.062	0.003	0.18
	TrH – TdH	0.143	0.007	0.42
	TdH – CCMD	0.802	0.040	2.40
R2	CCMD – QEH	1.080	0.054	3.24
	QEH – KH	0.278	0.014	0.84
	KH – BH	0.181	0.009	0.54
	BH – CCMD	1.053	0.053	3.18
R3	CCMD – TBH	1.143	0.057	3.42
	TBH – ZH	0.184	0.009	0.54
	ZH – IH	0.227	0.011	0.66
	IH – CCMD	1.541	0.077	4.62
R4	CCMD – QIH	1.690	0.085	5.10
	QIH – NH	0.069	0.004	0.24
	NH – AH	0.531	0.027	1.62
	AH – CCMD	2.062	0.103	6.18

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Table 5.3: Route Descriptions for product distribution from CCMD through the Designated reachable intra-routes with their distances and corresponding time in University of Ibadan

ROUTE CODE	HOSTEL CODE	CCMD	MH	TrD	TdH
R1	CCMD	0/0			
	MH	0.782/2.34	0/0		
	TrD	0.844/2.52	0.662/0.18	0/0	
	TdH	0.987/2.94	0.205/0.60	0.143/0.42	0/0
R2		CCMD	QEH	KH	BH
	CCMD	0/0			
	QEH	1.080/3.24	0/0		
	KH	1.358/4.08	0.278/0.84	0/0	
	BH	1.439/4.62	0.459/1.38	0.181/0.54	0/0
R3		CCMD	TBH	ZH	IH
	CCMD	0/0			
	TBH	1.143/3.42	0/0		
	ZH	1.327/3.96	0.184/0.54	0/0	
	IH	1.554/4.62	0.411/1.20	0.227/0.66	0/0
R4		CCMD	QIH	NH	AH
	CCMD	0/0			
	QIH	1.690/5.10	0/0		
	NH	1.759/5.34	0.069/0.24	0/0	
	AH	2.290/6.96	0.600/1.86	0.531/1.62	0/0

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Table 5.4: Route Descriptions for product distribution from CCMD through the designated reachable inter-routes with their distances and corresponding time in University of Ibadan

HOSTEL CODE	CCMD	MH	TrH	TdH	QEH	KH	BH	TBH	ZH	IH	QIH	NH	AH
CCMD	0/0												
MH	0.782/2.34	0/0											
TrH	0.844/2.52	0.662/0.18	0/0										
TdH	0.987/2.94	0.205/0.60	0.143/0.42	0/0									
QEH	1.080/3.24	0.350/1.05	0.291/0.87	0.428/1.28	0/0								
KH	1.358/4.08	0.300/0.90	0.246/0.74	0.283/0.85	0.278/0.84	0/0							
BH	1.439/4.62	0.341/1.02	0.320/0.96	0.259/0.78	0.459/1.38	0.181/0.54	0/0						
TBH	1.143/3.42	0.416/1.25	0.560/1.68	0.348/1.04	0.470/1.41	0.185/0.56	0.092/0.27	0/0					
ZH	1.327/3.96	0.587/1.76	0.558/1.67	0.518/1.55	0.599/1.80	0.328/0.98	0.257/0.77	0.184/0.54	0/0				
IH	1.554/4.62	0.786/2.36	0.735/2.21	0.729/2.19	0.700/2.10	0.501/1.50	0.484/1.45	0.411/1.20	0.227/0.66	0/0			
QIH	1.690/5.10	0.961/2.88	0.925/2.78	0.880/2.64	0.892/2.68	0.681/2.04	0.636/1.91	0.547/1.64	0.378/1.13	0.195/0.59	0/0		
NH	1.759/5.34	0.906/2.72	0.874/2.62	0.829/2.49	0.863/2.59	0.633/1.90	0.578/1.73	0.497/1.49	0.320/0.96	0.185/0.56	0.069/0.24	0/0	
AH	2.290/6.96	1.404/4.21	1.373/4.12	1.328/3.98	0.392/1.18	1.156/3.47	1.077/3.23	0.998/2.99	0.827/2.48	0.719/2.16	0.600/1.86	0.531/1.62	0/0

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5.4 Spatial Qualification Logic Case Studies

In the case study under consideration in this research, the existing plan schedule and the incoming activity to be incorporated into the plan is assumed to be given as follows.

Plan Schedule:

The schedule by the managing agent is that pick-up van1 should service routes R1 and R3 from Tuesdays to Fridays while van2 should service routes R2 and R4 from Mondays to Thursdays respectively. The off duty days for the pick-up vans are meant for the vans to be serviced and kept clean. But when there is an urgent request, the manager can reason out the solution and can handle it otherwise.

Activity:

If an order is made that requires products to be delivered to BH in R2 not later than 10:00 a.m., is it possible for any of the two vans to make the delivery on or before the set time?

Plan Assessment using SQM:

The assessment of the existing plan can be done after randomly considering the following cases from the possible combinations of the vans' routes as (R1, R2), (R1, R3), (R1, R4), (R2, R3), (R3, R4) and (R4, R2) for van1 and van2 respectively as obtained in cases 1, 2 and 3 summarised in tables 5.5, 5.6, 5.7 and 5.8.

Case 1:

Given that van1 has departed to R1 at 7:30a.m and also that van2 has departed to R4 at 8:00a.m; and assuming that each van uses minimum of 30 minutes to offload the products at a particular hostel.

The logic of spatial qualification in chapter three can be applied as it will help the managing agent to assess existing plan and determine the possibility of delivering the product to BH by 10:00 a.m. With the logic, the managing agent can decide which of the vans will be available to make the delivery without disrupting the existing plan. In

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the absence of none, the managing agent will decide which of them to use with less disruption, to ease the re-planning process.

The application of axioms T_{A1} and T_{A2} of SQ logic gives further proofs of the possibility of the two vans to remain present at same location at another time greater than the former.

$$Present\_at(van1,CCMD,7{:}30) \Rightarrow \Box Present\_at(van1,CCMD,7{:}30)$$

$$Present\_at(van2,CCMD,8{:}00) \Rightarrow \Box Present\_at(van2,CCMD,8{:}00)$$

$$Present\_at(van1,CCMD,7{:}30) \quad \Rightarrow (\exists t'.\ 7{:}30 < t'$$
$$\Rightarrow \Diamond Present\_at(van1,CCMD,10{:}00))$$

$$Present\_at(van2,CCMD,8{:}00)$$
$$\Rightarrow (\exists t'.\ 8{:}00 < t'$$
$$\Rightarrow \Diamond Present\_at(van2,CCMD,10{:}00))$$

That is to say that knowing them to depart from CCMD also means it is possible for it to still be present at CCMD at a later time for some reasons. Hence, axiom T_{A1} and T_{A2} holds.

Considering Van1:

Using axiom T_{A3}, it is possible for van1 known to be present at CCMD at 7:30 to be present at another location, say MH, at a later time.

$$Reachable(van1,\ CCMD,\ MH,\ (7{:}30,\ (7{:}30+0{:}03)))$$
$$\Leftrightarrow 7{:}30 < 7{:}33 \ \wedge (Present\_at(van1,\ CCMD,\ 7{:}30)$$
$$\Rightarrow \Diamond Present\_at(van1,\ MH,\ 7{:}33))$$

If this holds for axiom T_{A3}, it holds for axiom T_{A5} as well, that means the reverse (commutatively) of the reachability is possible.

$$Reachable(van1,CCMD,MH,(7{:}30,7{:}33))$$
$$\Leftrightarrow Reachable(van1,\ MH,\ CCMD,\ (7{:}30,7{:}33))$$

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If van1 can reach MH from CCMD at an interval (7:30, 7:33), then it means that van1 can still be spatially qualified at different time interval as long as the interval is the same as the former, following axiom T_{A6}.

$$Reachable(van1, CCMD, MH, (7{:}30, 7{:}33))$$
$$\wedge\ (\forall t_3, t_4.\ t_3 < t_4 \wedge ((t_4 - t_3) \geq (7{:}33 - 7{:}30)))$$
$$\Rightarrow Reachable(van1, CCMD, MH, (t_3, t_4)))$$

Since van1 can reach MH from CCMD at time interval of (7:30, 7:33), again it can reach TrH from MH at interval (8:03, 8:04) with the additional time of 30 minutes for off-loading, then it means that van1 can reach TrH from CCMD at interval (7:30, 8:04). Then T_{A10} holds following the transitive axiom for reachability as follows

$$Reachable(van1, CCMD, MH, (7{:}30, 7{:}33)) \wedge$$
$$Reachable(van1, MH, TrH, (8{:}04, 8{:}05))$$
$$\Rightarrow Reachable(van1, CCMD, TrH, (7{:}30, 8{:}05)).$$

$$Reachable(van1, MH, TrH, (8{:}04, 8{:}05)) \wedge$$
$$Reachable(van1, TrH, TdH, (8{:}35, 8{:}36)) \wedge$$
$$\Rightarrow Reachable(van1, MH, TdH, (8{:}04, 8{:}36)).$$

And from T_{A3} we have that:

$$Reachable(van1, TdH, CCMD, (9.06, 9{:}09))$$
$$\Leftrightarrow 9.09 < 10{:}00 \wedge (Present\_at(van1, CCMD, 9{:}09)$$
$$\Rightarrow \Diamond Present\_at(van1, BH, 10{:}00)).$$

Thus, concluding that it is possible for van1 to reach BH at 10:00a.m without actually knowing or having all feedback and/or reports on the current presence of van1.

Considering van2:

The axioms equally hold for this case depending on their departure time.

Following from axiom T_{A3}, it is possible for van2 known to be present at CCMD at 8:00 to be present at another location, QIH, at a later time.

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$$Reachable(van2, CCMD, QIH, (8{:}00, (8{:}00+0{:}06)))$$
$$\Leftrightarrow 8{:}00 < 8{:}06 \ \wedge (Present\_at(van2, CCMD, 8{:}00)$$
$$\Rightarrow \Diamond Present\_at(van2, QIH, 8{:}06))$$

If this holds for axiom T_{A3}, it holds for axiom T_{A5} as well, that means the reverse (commutatively) of the reachability is possible.

$$Reachable(van2,CCMD,QIH,(8{:}00,8{:}06))$$
$$\Leftrightarrow Reachable(van2, QIH, CCMD, (8{:}00,8{:}06))$$

If van2 can reach QIH from CCMD at an interval (8:00, 8:06), then it means that van2 can still be spatially qualified at different time interval so long as the interval is the same as the former, following axiom T_{A6}.

$$Reachable(van2, CCMD, QIH, (8{:}00,8{:}06))$$
$$\wedge (\forall t_3, t_4.\ t_3 < t_4 \wedge ((t_4 - t_3) \geq (8{:}06 - 8{:}00)))$$
$$\Rightarrow Reachable(van2, CCMD, QIH, (t_3, t_4)))$$

Since van2 can reach QIH from CCMD at time interval of (8:00, 8:06), again it can reach NH from QIH at interval (8:36, 8:37) with the additional time of 30 minutes for off-loading, then it means that van2 can reach NH from CCMD at interval (8:00, 8:37). Then T_{A10} holds following the transitive axiom for reachability as follows:

$$Reachable(van2, CCMD, QIH, (8{:}00,8{:}06)) \wedge$$
$$Reachable(van2, QIH, NH, (8{:}36,8{:}37)) \wedge$$
$$\Rightarrow Reachable(van2, CCMD, NH, (8{:}00, 8{:}37)).$$

$$Reachable(van2, QIH, NH, (8{:}36,8{:}37)) \wedge$$
$$Reachable(van2, NH, AH, (9{:}07, 9{:}09)) \wedge$$
$$\Rightarrow Reachable(van2, QIH, AH, (8{:}36, 9{:}09)).$$

And from $T_{A3,}$ we have that:

$$Reachable(van2, AH, CCMD, (9.39,9{:}46))$$
$$\Leftrightarrow 9.46 < 10{:}00 \wedge (Present\_at(van2, CCMD, 9{:}46)$$
$$\Rightarrow \Diamond Present\_at(van2, BH, 10{:}00)).$$

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Thus, concluding that it is possible for van2 to reach BH at 10:00a.m without actually knowing or having all feedback and/or reports on the current presence of van2.

Table 5.5 summarizes the results for the transitions of the vans in case 1.

From table 5.5, axiom T_{A3} will return false which depicts impossibility concluding that it is not possible for van2 to reach BH without actually knowing or having all feedback and reports of the current presence of van2.

Case 2:

Given that van1 has departed to R3 at 8:30a.m and also that van2 has departed to R4 at 9:00a.m; and assuming that each van uses minimum of 30 minutes to offload the products at a particular hostel. Note that there is an order for products to be delivered at BH in R2 on or before 10:00a.m.

Considering van1

Using axiom T_{A3}, it is possible for the van known to be present at CCMD at 8:30 to be present at another location, TBH, at a later time.

$$Reachable(van1, CCMD, TBH, (8{:}30, (8{:}30+0{:}04)))$$
$$\Leftrightarrow 8{:}30 < 8{:}34 \wedge (Present\_at(van1, CCMD, 8{:}30)$$
$$\Rightarrow \Diamond Present\_at(van1, TBH, 8{:}34))$$

If this holds for axiom T_{A3}, it holds for axiom T_{A5} as well, that means the reverse (commutatively) of the reachability is possible.

$$Reachable(van1,CCMD,TBH,(8{:}30,8{:}34))$$
$$\Leftrightarrow Reachable(van1, TBH, CCMD, (8{:}30,8{:}34))$$

If van1 can reach TBH from CCMD at an interval (8:30, 8:34), then it means that van1 can still be spatially qualified at different time interval as long as the interval is the same as the former, following axiom T_{A6}.

$$Reachable(van1, CCMD, TBH, (8{:}30, 8{:}34))$$
$$\wedge (\forall t_3, t_4.\ t_3 < t_4 \wedge ((t_4 - t_3) \geq (8{:}34 - 8{:}30)))$$
$$\Rightarrow Reachable(van1, CCMD, TBH, (t_3, t_4)))$$

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Since van1 can reach TBH from CCMD at time interval of (8:30, 8:34), again it can reach ZH from TBH at interval (9:04, 9:05) with the additional time of 30 minutes for off-loading, then it means that van1 can reach ZH from CCMD at interval (8:30, 9:05). Then T_{A10} holds following the transitive axiom for reachability as follows

Reachable(van1, CCMD, TBH, (8:30,8:34)) ∧
Reachable(van1, TBH, ZH, (9:04,9:05))
⇒ *Reachable(van1, CCMD, ZH, (8:30, 9:05)).*

Reachable(van1, TBH, ZH, (9:04,9:05)) ∧
Reachable(van1, ZH, IH, (9:35, 9:36))
⇒ *Reachable(van1, TBH, IH, (9:04, 9:36)).*

Reachable(van1, IH, CCMD, (10.06,10:11))
⇔ *10:11 < 10:00* ∧ *(Present_at(van1, CCMD, 10:11))*
⇒ *◊Present_at(van1, BH, 10:00)).*

Thus, concluding that it is not possible for van1 to reach BH at 10:00a.m as it returns false since 10:11 > 10:00 a.m.

Considering van2:

The axioms equally hold for this case.

Using axiom T_{A3}, it is possible for the van known to be present at CCMD at 9:00 to be present at another location, QIH, at a later time.

Reachable(van2, CCMD, QIH, (9:00, (9:00+0:06)))
⇔ *9:00 < 9:06* ∧ *(Present_at(van2, CCMD, 9:00)*
⇒ *◊Present_at(van2, QIH, 9:06))*

If this holds for axiom T_{A3}, it holds for axiom T_{A5} as well, that means the reverse (commutatively) of the reachability is possible.

Reachable(van2,CCMD,QIH,(9:00,9:06))
⇔ *Reachable(van2, QIH, CCMD, (9:00,9:06))*

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If van2 can reach QIH from CCMD at an interval (9:00, 9:06), then it means that van2 can still be spatially qualified at different time interval as long as the interval is the same as the former, following axiom T_{A6}.

$$\begin{aligned} & Reachable(van2, CCMD, QIH, (9{:}00, 9{:}06)) \\ & \quad \wedge (\forall t_3, t_4.\ t_3 < t_4 \wedge ((t_4 - t_3) \geq (9{:}06 - 9{:}00))) \\ & \quad \Rightarrow Reachable(van2, CCMD, QIH, (t_3, t_4))) \end{aligned}$$

Since van2 can reach QIH from CCMD at time interval of (9:00, 9:06), again it can reach NH from QIH at interval (9:36, 9:37) with the additional time of 30 minutes for off-loading, then it means that van2 can reach NH from CCMD at interval (9:00, 9:37). Then T_{A10} holds following the transitive axiom for reachability as follows

$$\begin{aligned} & Reachable(van2, CCMD, QIH, (9{:}00, 9{:}06)) \wedge \\ & \quad Reachable(van2, QIH, NH, (9{:}36, 9{:}37)) \\ & \quad \Rightarrow Reachable(van2, CCMD, NH, (9{:}00, 9{:}37)). \end{aligned}$$

$$\begin{aligned} & Reachable(van2, QIH, NH, (9{:}36, 9{:}37)) \wedge \\ & \quad Reachable(van2, NH, AH, (10{:}07, 10{:}09)) \\ & \quad \Rightarrow Reachable(van2, QIH, AH, (9{:}36, 10{:}09)). \end{aligned}$$

From T_{A3}, we have

$$\begin{aligned} & Reachable(van2, AH, CCMD, (10.39, 10{:}46)) \\ & \quad \Leftrightarrow 10.46 < 10{:}00 \wedge (Present\_at(van2, CCMD, 10{:}46) \\ & \quad \Rightarrow \Diamond Present\_at(van2, BH, 10{:}00)). \end{aligned}$$

Thus, concluding that it is not possible for van2 to reach BH at 10:00a.m.

Also applying axiom T_{A1}, models of the truth of the presence of the vans at CCMD at certain times can be stated as shown in table 5.6.

From table 5.6, axiom T_{A3} will return false for both van1 and van2 since neither 10:11<10:00 nor 10:46<10:00 holds. This concludes that it is not possible for both van1 and van2 to reach BH at 10:00a.m. Therefore, this does not imply the possibility of presence.

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Table 5.5: Availability time for Van1 and Van2 on routes R1 and R4 respectively

Van	Route	Departure	Arrival	Departure	Arrival	Departure	Arrival	Departure	Arrival
Van1	R1	CCMD	MH	MH	TrH	TrH	TdH	TdH	CCMD
		7:30	7:33	8:03	8:04	8:34	8:35	9:05	9:08
Van2	R4	CCMD	QIH	QIH	NH	NH	AH	AH	CCMD
		8:00	8:06	8:36	8:37	9:09	9:09	9:39	9:46

Table 5.6: Availability time for Van1 and Van2 on routes R3 and R4 respectively

Van	Route	Departure	Arrival	Departure	Arrival	Departure	Arrival	Departure	Arrival
Van1	R3	CCMD	TBH	TBH	ZH	ZH	IH	IH	CCMD
		8:30	8:34	9:04	9:05	9:35	9:36	10:06	10:11
Van2	R4	CCMD	QIH	QIH	NH	NH	AH	AH	CCMD
		9:00	9:06	9:36	9:37	10:07	10:09	10:39	10:46

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Case 3:

Given that van1 departed from CCMD to R4 at 8:30 a.m and also that van2 has departed to R2 at 8:00a.m; and again assuming the maximum offloading time of 30 minutes for the products at any of the hostels. Note that an order for products to be delivered to BH in R2 is actually on van2's assigned route. The notification for the order came in by 8:00a.m.

Considering van1

Using axiom T_{A3}, it is possible for van1 known to be present at CCMD at 8:30 to be present at another location, QIH, at a later time.

$$Reachable(van1, CCMD, QIH, (8{:}30, (8{:}30+0{:}06)))$$
$$\Leftrightarrow 8{:}30 < 8{:}36 \wedge (Present\_at(van1, CCMD, 8{:}30)$$
$$\Rightarrow \Diamond Present\_at(van1, QIH, 8{:}36))$$

If this holds for axiom T_{A3}, it holds for axiom T_{A5} as well, that means the reverse (commutatively) of the reachability is possible.

$$Reachable(van1,CCMD,QIH,(8{:}30,8{:}36))$$
$$\Leftrightarrow Reachable(van1, QIH, CCMD, (8{:}30,8{:}36))$$

If van1 can reach QIH from CCMD at an interval (8:30, 8:36), then it means that van1 can still be spatially qualified at different time interval so long as the interval is the same as the former, following axiom T_{A6}.

$$Reachable(van1, CCMD, QIH, (8{:}30, 8{:}36))$$
$$\wedge(\forall t_3, t_4.\ t_3 < t_4 \wedge ((t_4 - t_3) \geq (8{:}36 - 8{:}30)))$$
$$\Rightarrow Reachable(van1, CCMD, QIH, (t_3, t_4)))$$

Since van1 can reach QIH from CCMD at time interval of (8:30, 8:36), again it can reach NH from QIH at interval (9:06, 9:07) with the additional time of 30 minutes for off-loading, then it means that van1 can reach NH from CCMD at interval (8:30, 9:07). Then T_{A10} holds following the transitive axiom for reachability as follows

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$$Reachable(van1, CCMD, QIH, (8{:}30,8{:}36)) \wedge$$
$$Reachable(van1, QIH, NH, (9{:}06,9{:}07))$$
$$\Rightarrow Reachable(van1, CCMD, NH, (8{:}30, 9{:}07)).$$

$$Reachable(van1, QIH, NH, (9{:}06,9{:}07)) \wedge$$
$$Reachable(van1, NH, AH, (9{:}37, 9{:}39))$$
$$\Rightarrow Reachable(van1, QIH, AH, (9{:}06, 9{:}39)).$$

Again, from T_{A3}*, we have*

$$Reachable(van1, AH, CCMD, (10.09,10{:}16))$$
$$\Leftrightarrow 10.16 < 10{:}00 \wedge (Present\_at(van1, CCMD, 10{:}16)$$
$$\Rightarrow \Diamond Present\_at(van1, BH, 10{:}00)).$$

Thus, concluding that it is not possible for van1 to reach BH at 10:00a.m.

Considering van2

Using axiom T_{A3}, it is possible for the van known to be present at CCMD at 8:00 to be present at another location, QEH, at a later time.

$$Reachable(van2, CCMD, QEH, (8{:}00, (8{:}00+0{:}04)))$$
$$\Leftrightarrow 8{:}00 < 8{:}04 \wedge (Present\_at(van2, CCMD, 8{:}00)$$
$$\Rightarrow \Diamond Present\_at(van2, QEH, 8{:}04))$$

If this holds for axiom T_{A3}, it holds for axiom T_{A5} as well, that means the reverse (commutatively) of the reachability is possible.

$$Reachable(van2,CCMD,QEH,(8{:}00,8{:}04))$$
$$\Leftrightarrow Reachable(van2, QEH, CCMD, (8{:}00,8{:}04))$$

If van2 can reach QEH from CCMD at an interval (8:00, 8:04), then it means that van2 can still be spatially qualified at different time interval as long as the interval is the same as the former, following axiom T_{A6}.

$$Reachable(van2, CCMD, QEH, (8{:}00, 8{:}04))$$
$$\wedge\ (\forall t_3, t_4.\ t_3 < t_4 \wedge ((t_4 - t_3) \geq (8{:}04 - 8{:}00)))$$
$$\Rightarrow Reachable(van2, CCMD, QEH, (t_3, t_4)))$$

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Since van2 can reach QEH from CCMD at time interval of (8:00, 8:04), again it can reach KH from QEH at interval (8:34, 8:35) with the additional time of 30 minutes for off-loading, then it means that van2 can reach KH from CCMD at interval (8:00, 8:35). Then T_{A10} holds following the transitive axiom for reachability as follows

$$\begin{aligned}&\mathit{Reachable(van2,\ CCMD,\ QEH,\ (8{:}00,8{:}04))} \wedge \\ &\quad \mathit{Reachable(van2,\ QEH,\ KH,\ (8{:}34,8{:}35))} \\ &\quad \Rightarrow \mathit{Reachable(van2,\ CCMD,\ KH,\ (8{:}00,\ 8{:}35))}.\end{aligned}$$

$$\begin{aligned}&\mathit{Reachable(van2,\ QEH,\ KH,\ (8{:}34,8{:}35))} \wedge \\ &\quad \mathit{Reachable(van2,\ KH,\ BH,\ (9{:}05,\ 9{:}06))} \\ &\quad \Rightarrow \mathit{Reachable(van2,\ QEH,\ BH,\ (8{:}34,\ 9{:}06))}.\end{aligned}$$

Also, from T_{A3}, we have

$$\begin{aligned}&\mathit{Reachable(van2,\ BH,\ CCMD,\ (9.36,9{:}40))} \\ &\quad \Leftrightarrow 9.40 < 10{:}00 \wedge \mathit{(Present\_at(van2,\ CCMD,\ 9{:}40)} \\ &\quad \Rightarrow \Diamond \mathit{Present\_at(van2,\ BH,\ 10{:}00))}.\end{aligned}$$

Thus, concluding that it is possible for van2 to reach BH at 10:00a.m.

Also, applying axiom T_{A1}, models of the truth of the presence of the vans at CCMD at certain times can be stated as shown in table 5.7.

From table 5.7, the axioms will lead to a false conclusion for van1 since 10:16<10:00 is not true but returns true for van2 since 9:40<10:00, thereby concluding that it is possible for van 2 to deliver the products at BH on or before 10:00 a.m.

To show that the model is scalable, cases where the company decides to increase the numbers of vans to three are also considered. The axioms in the model were also used with the given distances and times of the known locations. The possible combinations of the vans' routing were streamlined to four: (R1, R2, R3), (R1, R2, R4), (R1, R3, R4) and (R2, R3, R4) for van1, van2, and van3. Two of these combinations, (R1, R2, R3) and (R2, R3, R4), were randomly selected as case studies 4 and 5 respectively.

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Case 4:

Given that van1 has departed to R1 at 7:30 a.m, van2 to R2 at 8:30 a.m and van3 to R3 at 8:00a.m; still assuming the maximum offloading time of 30 minutes for the products at the hostels. Note that an order for products to be delivered to BH in R2 is actually on van2's assigned route. The notification for the order came in by 8:00a.m.

Considering van1

Using axiom T_{A3}, it is possible for the van known to be present at CCMD at 7:30 to be present at another location, MH, at a later time.

$$Reachable(van1, CCMD, MH, (7{:}30, (7{:}30+0{:}03)))$$
$$\Leftrightarrow 7{:}30 < 7{:}33 \wedge (Present\_at(van1, CCMD, 7{:}30)$$
$$\Rightarrow \Diamond Present\_at(van1, MH, 7{:}33))$$

If this holds for axiom T_{A3}, it holds for axiom T_{A5} as well, that means the reverse (commutatively) of the reachability is possible.

$$Reachable(van1,CCMD,MH,(7{:}30,7{:}33))$$
$$\Leftrightarrow Reachable(van1, MH, CCMD, (7{:}30,7{:}33))$$

If van1 can reach MH from CCMD at an interval (7:30, 7:33), then it means that van1 can still be spatially qualified at different time interval as long as the interval is the same as the former, following axiom T_{A6}.

$$Reachable(van1, CCMD, MH, (7{:}30, 7{:}33))$$
$$\wedge (\forall t_3, t_4.\ t_3 < t_4 \wedge ((t_4 - t_3) \geq (7{:}33 - 7{:}30)))$$
$$\Rightarrow Reachable(van1, CCMD, MH, (t_3, t_4)))$$

Since van1 can reach MH from CCMD at time interval of (7:30, 7:33), again it can reach TrH from MH at interval (8:03, 8:04) with the additional time of 30 minutes for off-loading, then it means that van1 can reach TrH from CCMD at interval (7:30, 8:04). Then T_{A10} holds following the transitive axiom for reachability as follows.

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$$Reachable(van1, CCMD, MH, (7{:}30,7{:}33)) \wedge$$
$$Reachable(van1, MH, TrH, (8{:}03,8{:}04))$$
$$\Rightarrow Reachable(van1, CCMD, KH, (7{:}30, 8{:}04)).$$

$$Reachable(van1, MH, TrH, (8{:}03,8{:}04)) \wedge$$
$$Reachable(van1, TrH, TdH, (8{:}34, 8{:}35))$$
$$\Rightarrow Reachable(van1, MH, TdH, (8{:}03, 8{:}35)).$$

Also, from T_{A3}, we have

$$Reachable(van1, TdH, CCMD, (9.05,9{:}08))$$
$$\Leftrightarrow 9.08 < 10{:}00 \wedge (Present\_at(van1, CCMD, 9{:}08))$$
$$\Rightarrow \Diamond Present\_at(van1, BH, 10{:}00)).$$

Thus, concluding that it is possible for van1 to reach BH at 10:00a.m.

Considering van2

Using axiom T_{A3}, it is possible for the van2 known to be present at CCMD at 8:30 to be present at another location, QEH, at a later time.

$$Reachable(van2, CCMD, QEH, (8{:}30, (8{:}30+0{:}04)))$$
$$\Leftrightarrow 8{:}30 < 8{:}34 \wedge (Present\_at(van2, CCMD, 8{:}30)$$
$$\Rightarrow \Diamond Present\_at(van2, QEH, 8{:}34))$$

If this holds for axiom T_{A3}, it holds for axiom T_{A5} as well, that means the reverse (commutatively) of the reachability is possible.

$$Reachable(van2,CCMD,QEH,(8{:}30,8{:}34))$$
$$\Leftrightarrow Reachable(van2, QEH, CCMD, (8{:}30,8{:}34))$$

If van2 can reach QEH from CCMD at an interval (8:30, 8:34), then it means that van2 can still be spatially qualified at different time interval as long as the interval is the same as the former, following axiom T_{A6}.

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$Reachable(van2, CCMD, QEH, (8{:}30, 8{:}34))$

$\wedge\ (\forall t_3, t_4.\ t_3 < t_4 \wedge ((t_4 - t_3) \geq (8{:}34 - 8{:}30)))$

$\Rightarrow Reachable(van2, CCMD, QEH, (t_3, t_4)))$

Since van2 can reach QEH from CCMD at time interval of (8:30, 8:34), again it can reach KH from QEH at interval (9:04, 9:05) with the additional time of 30 minutes for off-loading, then it means that van2 can reach KH from CCMD at interval (8:30, 9:05). Then T_{A10} holds following the transitive axiom for reachability as follows

$Reachable(van2, CCMD, QEH, (8{:}30,8{:}34)) \wedge$

$Reachable(van2, QEH, KH, (9{:}04,9{:}05))$

$\Rightarrow Reachable(van2, CCMD, KH, (8{:}30, 9{:}05)).$

$Reachable(van2, QEH, KH, (9{:}04,9{:}05)) \wedge$

$Reachable(van2, KH, BH, (9{:}35, 9{:}36))$

$\Rightarrow Reachable(van2, QEH, BH, (8{:}34, 9{:}36)).$

Again, from T_{A3}, we have

$Reachable(van2, BH, CCMD, (10.06,10{:}10))$

$\Leftrightarrow 10.10 < 10{:}00 \wedge (Present\_at(van2, CCMD, 10{:}10)$

$\Rightarrow \Diamond Present\_at(van2, BH, 10{:}00)).$

Thus, concluding that it is possible for van2 to reach BH at 9:36 since BH is in its route. This does not require waiting for van2 to return to CCMD at 10:10.

Considering van3

Using axiom T_{A3}, it is possible for vans known to be present at CCMD at 8:00 to be present at another location, TBH, at a later time.

$Reachable(van3, CCMD, TBH, (8{:}00, (8{:}00+0{:}04)))$

$\Leftrightarrow 8{:}00 < 8{:}04\ \wedge (Present\_at(van3, CCMD, 8{:}00)$

$\Rightarrow \Diamond Present\_at(van3, TBH, 8{:}04))$

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If this holds for axiom T_{A3}, it holds for axiom T_{A5} as well, that means the reverse (commutatively) of the reachability is possible.

Reachable(van3,CCMD,TBH,(8:00,8:04))

$\Leftrightarrow$ *Reachable(van3, TBH, CCMD, (8:00,8:04))*

If van3 can reach TBH from CCMD at an interval (8:00, 8:04), then it means that van3 can still be spatially qualified at different time interval as long as the interval is the same as the former, following axiom T_{A6}.

Reachable(van3, CCMD, TBH, (8:00, 8:04))

$\wedge$ ($\forall t_3, t_4.\ t_3 < t_4 \wedge ((t_4 - t_3) \geq (8{:}04 - 8{:}00))$)

$\Rightarrow$ *Reachable(van3, CCMD, TBH,* (t_3, t_4)*))*

Since van3 can reach TBH from CCMD at time interval of (8:00, 8:04), again it can reach ZH from TBH at interval (8:34, 8:35) with the additional time of 30 minutes for off-loading, then it means that van3 can reach ZH from CCMD at interval (8:00, 8:35). Then T_{A10} holds following the transitive axiom for reachability as follows

Reachable(van3, CCMD, TBH, (8:00,8:04)) $\wedge$

Reachable(van3, TBH, ZH, (8:34,8:35))

$\Rightarrow$ *Reachable(van3, CCMD, ZH, (8:00, 8:35)).*

Reachable(van3, TBH, ZH, (8:34,8:35)) $\wedge$

Reachable(van3, ZH, IH, (9:05, 9:06))

$\Rightarrow$ *Reachable(van3, TBH, IH, (8:34, 9:06)).*

Also, from T_{A3}, we have

Reachable(van3, IH, CCMD, (9.36,9:41))

$\Leftrightarrow$ *9:41 < 10:00* $\wedge$ *(Present_at(van3, CCMD, 9:41))*

$\Rightarrow$ $\Diamond$*Present_at(van3, BH, 10:00)).*

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Table 5.7: Availability time for Van1 and Van2 on routes R4 and R2 respectively

Van	Route	Departure	Arrival	Departure	Arrival	Departure	Arrival	Departure	Arrival
Van1	R4	CCMD	QIH	QIH	NH	NH	AH	AH	CCMD
		8:30	8:36	9:06	9:07	9:37	9:39	10:09	10:16
Van2	R2	CCMD	QEH	QEH	KH	KH	BH	BH	CCMD
		8:00	8:04	8:34	8:35	9:05	9:06	9:36	9:40

Table 5.8: Availability time for Van1, Van2 and Van3 on routes R1, R2 and R3 respectively

Van	Route	Departure	Arrival	Departure	Arrival	Departure	Arrival	Departure	Arrival
Van1	R1	CCMD	MH	MH	TrH	TrH	TdH	TdH	CCMD
		7:30	7:33	8:03	8:04	8:34	8:35	9:05	9:08
Van2	R2	CCMD	QEH	QEH	KH	KH	BH	BH	CCMD
		8:30	8:34	9:04	9:05	9:35	9:36	10:06	10:10
Van3	R3	CCMD	TBH	TBH	ZH	ZH	IH	IH	CCMD
		8:00	8:04	8:34	8:35	9:05	9:06	9:36	9:41

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Thus, concluding that it is possible for van3 to reach BH at 10:00a.m. since it will reach CCMD at 9:41a.m.

Also, applying axiom T_{A1}, models of the truth of the presence of the vans at CCMD at certain times can be stated as shown in table 5.8.

From table 5.8, the axioms conclude with truth possibilities for van1 since 9:08<10:00 is true; van2 is actually assigned to the needed route and 9:36<10:00 which is the arrival time at BH; and 9:41<10:00 is true for van3. The general conclusion is that it is possible for any of van1, van2 and van3 to deliver the products at BH on or before 10:00 a.m.

Case 5:

Given that van1 departed to R2 at 8:30a.m, van2 to R3 at 9:00a.m and van3 to R4 at 8:30 a.m; still assuming the maximum offloading time of 30 minutes for the products at the hostels. Note that an order for products to be delivered to BH in R2 is actually on van1's assigned route. The notification for the order came in by 8:00a.m.

Considering van1

Using axiom T_{A3}, it is possible for the van known to be present at CCMD at 8:30 to be present at another location, QEH, at a later time.

$$
\begin{aligned}
&\mathit{Reachable}(van1, CCMD, QEH, (8{:}30, (8{:}30+0{:}04))) \\
&\qquad \Leftrightarrow 8{:}30 < 8{:}34 \ \wedge (\mathit{Present\_at}(van1, CCMD, 8{:}30) \\
&\qquad \Rightarrow \Diamond \mathit{Present\_at}(van1, QEH, 8{:}34))
\end{aligned}
$$

If this holds for axiom T_{A3}, it holds for axiom T_{A5} as well, that means the reverse (commutatively) of the reachability is possible.

$$
\begin{aligned}
&\mathit{Reachable}(van1,CCMD,QEH,(8{:}30,8{:}34)) \\
&\qquad \Leftrightarrow \mathit{Reachable}(van1, QEH, CCMD, (8{:}30,8{:}34))
\end{aligned}
$$

If van1 can reach QEH from CCMD at an interval (8:30, 8:34), then it means that van1 can still be spatially qualified at different time interval as long as the interval is the same as the former, following axiom T_{A6}.

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$$Reachable(van1, CCMD, QEH, (8{:}30, 8{:}34))$$
$$\wedge (\forall t_3, t_4.\ t_3 < t_4 \wedge ((t_4 - t_3) \geq (8{:}34 - 8{:}30)))$$
$$\Rightarrow Reachable(van1, CCMD, QEH, (t_3, t_4)))$$

Since van1 can reach QEH from CCMD at time interval of (8:30, 8:34), again it can reach KH from QEH at interval (9:04, 9:05) with the additional time of 30 minutes for off-loading, then it means that van1 can reach KH from CCMD at interval (8:30, 9:05). Then T_{A10} holds following the transitive axiom for reachability as follows

$$Reachable(van1, CCMD, QEH, (8{:}30,8{:}34)) \wedge$$
$$Reachable(van1, QEH, KH, (9{:}04,9{:}05))$$
$$\Rightarrow Reachable(van1, CCMD, KH, (8{:}30, 9{:}05)).$$

$$Reachable(van1, QEH, KH, (9{:}04,9{:}05)) \wedge$$
$$Reachable(van1, KH, BH, (9{:}35, 9{:}36))$$
$$\Rightarrow Reachable(van1, QEH, BH, (9{:}04, 9{:}36)).$$

Also, from T_{A3}, we have

$$Reachable(van1, BH, CCMD, (10.06,10{:}10))$$
$$\Leftrightarrow 10.10 < 10{:}00 \wedge (Present\_at(van1, CCMD, 10{:}10))$$
$$\Rightarrow \Diamond Present\_at(van1, BH, 10{:}00)).$$

Thus, concluding that it is possible for van1 to reach BH at 10:00a.m since it was assigned to this route.

Considering van2

Using axiom T_{A3}, it is possible for van2 known to be present at CCMD at 9:00 to be present at another location, TBH, at a later time.

$$Reachable(van2, CCMD, TBH, (9{:}00, (9{:}00+0{:}04)))$$
$$\Leftrightarrow 9{:}00 < 9{:}04 \wedge (Present\_at(van2, CCMD, 9{:}00)$$
$$\Rightarrow \Diamond Present\_at(van2, TBH, 9{:}04))$$

If this holds for axiom T_{A3}, it holds for axiom T_{A5} as well, that means the reverse (commutatively) of the reachability is possible.

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$Reachable(van2,CCMD,TBH,(9{:}00,9{:}04))$

$\Leftrightarrow Reachable(van2, TBH, CCMD, (9{:}00,9{:}04))$

If van2 can reach TBH from CCMD at an interval (9:00, 9:04), then it means that van2 can still be spatially qualified at different time interval as long as the interval is the same as the former, following axiom T_{A6}.

$Reachable(van2, CCMD, TBH, (9{:}00, 9{:}04))$

$\wedge (\forall t_3, t_4.\ t_3 < t_4 \wedge ((t_4 - t_3) \geq (9{:}04 - 9{:}00)))$

$\Rightarrow Reachable(van2, CCMD, TBH, (t_3, t_4)))$

Since van2 can reach TBH from CCMD at time interval of (9:00, 9:04), again it can reach ZH from TBH at interval (9:34, 9:35) with the additional time of 30 minutes for off-loading, then it means that van2 can reach ZH from CCMD at interval (9:00, 9:35). Then T_{A10} holds following the transitive axiom for reachability as follows

$Reachable(van2, CCMD, TBH, (9{:}00,9{:}04)) \wedge$

$Reachable(van2, TBH, ZH, (9{:}34,9{:}35))$

$\Rightarrow Reachable(van2, CCMD, ZH, (9{:}00, 9{:}35)).$

$Reachable(van2, TBH, ZH, (9{:}34,9{:}35)) \wedge$

$Reachable(van2, ZH, IH, (10{:}05, 10{:}06))$

$\Rightarrow Reachable(van2, TBH, IH, (9{:}34, 10{:}06)).$

Again, T_{A3} gives

$Reachable(van2, IH, CCMD, (10.36,10{:}41))$

$\Leftrightarrow 10.41 < 10{:}00 \wedge (Present\_at(van2, CCMD, 10{:}41)$

$\Rightarrow \Diamond Present\_at(van2, BH, 10{:}00)).$

Thus, concluding that it is not possible for van2 to reach BH at 10:00 since 10:41 < 10:00 returns false.

Considering van3

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Using axiom T_{A3}, it is possible for the van known to be present at CCMD at 8:30 to be present at another location, QIH, at a later time.

$$Reachable(van3, CCMD, QIH, (8{:}30, (8{:}30+0{:}06)))$$
$$\Leftrightarrow 8{:}30 < 8{:}36 \wedge (Present\_at(van3, CCMD, 8{:}30)$$
$$\Rightarrow \Diamond Present\_at(van3, QIH, 8{:}36))$$

If this holds for axiom T_{A3}, it holds for axiom T_{A5} as well, that means the reverse (commutatively) of the reachability is possible.

$$Reachable(van3,CCMD,QIH,(8{:}30,8{:}36))$$
$$\Leftrightarrow Reachable(van3, QIH, CCMD, (8{:}30,8{:}36))$$

If van3 can reach QIH from CCMD at an interval (8:30, 8:36), then it means that van3 can still be spatially qualified at different time interval as long as the interval is the same as the former, following axiom T_{A6}.

$$Reachable(van3, CCMD, QIH, (8{:}30, 8{:}36))$$
$$\wedge (\forall t_3, t_4.\ t_3 < t_4 \wedge ((t_4 - t_3) \geq (8{:}36 - 8{:}30)))$$
$$\Rightarrow Reachable(van3, CCMD, QIH, (t_3, t_4)))$$

Since van3 can reach QIH from CCMD at time interval of (8:30, 8:36), again it can reach NH from QIH at interval (9:06, 9:07) with the additional time of 30 minutes for off-loading, then it means that van3 can reach NH from CCMD at interval (8:30, 9:07). Then T_{A10} holds following the transitive axiom for reachability as follows

$$Reachable(van3, CCMD, QIH, (8{:}30,8{:}36)) \wedge$$
$$Reachable(van3, QIH, NH, (9{:}06,9{:}07))$$
$$\Rightarrow Reachable(van3, CCMD, NH, (8{:}30, 9{:}07)).$$

$$Reachable(van3, QIH, NH, (9{:}06,9{:}07)) \wedge$$
$$Reachable(van3, NH, AH, (9{:}37, 9{:}39))$$
$$\Rightarrow Reachable(van3, QIH, AH, (9{:}06, 9{:}39)).$$

Again, from T_{A3}, we have

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$$Reachable(van3, AH, CCMD, (10.09,10{:}16))$$

$$\Leftrightarrow 10{:}16 < 10{:}00 \wedge (Present\_at(van3, CCMD, 10{:}16))$$

$$\Rightarrow \Diamond Present\_at(van3, BH, 10{:}00)).$$

Thus, concluding that it is not possible for van3 to reach BH at 10:00a.m since 10:16 < 10:00 returns false.

Also, applying axiom T_{A1}, models of the truth of the presence of the vans at CCMD at certain times can be summarised as shown in table 5.9.

From table 5.9, the axioms conclude with truth possibilities for van1 since 10:10<10:00 is true since the van is actually on its route and 9:36 the arrival time at BH; but false for van2 and van3 since 10:41<10:00 and 10:16<10:00 respectively does not hold. The general conclusion is that it is possible for van1 to deliver the products at BH on or before 10:00 a.m. but impossible for van2 and van3.

5.5 Results from the spatial reasoning process

The resulting availability times of the two vans in each of the three cases are as shown in table 5.10.

The chart in figure 5.6 gives the resulting availability time of the vans for cases 1, 2 and 3 showing their possibility or otherwise of meeting the deadline at 10:00a.m. As shown in figure 5.6, the thick black line depicts the deadline of 10:00a.m. It is possible for van1 at 9:11a.m in case study 1 to meet the deadline at 10:00a.m while impossible for van2 at 10:23a.m. In case study 3, it is not possible for van1 to deliver on or before 10:00a.m while van2 can reach BH at 9:39a.m. Both vans cannot meet the deadline of 10:00a.m for case study 2, therefore, impossible to deliver the products.

Figure 5.7 shows the resulting availability time of the vans in cases 4 and 5 and their possibility or otherwise of meeting the deadline of 10:00 a.m.

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Table 5.9: Availability time for Van1, Van2 and Van3 on routes R2, R3 and R4 respectively

Van	Route	Departure	Arrival	Departure	Arrival	Departure	Arrival	Departure	Arrival
Van1	R2	CCMD	QEH	QEH	KH	KH	BH	BH	CCMD
		8:30	8:34	9:04	9:05	9:35	9:36	10:06	10:10
Van2	R3	CCMD	TBH	TBH	ZH	ZH	IH	IH	CCMD
		9:00	9:04	9:34	9:35	10:05	10:06	10:36	10:41
Van3	R4	CCMD	QIH	QIH	NH	NH	AH	AH	CCMD
		8:30	8:36	9:06	9:07	9:37	9:39	10:09	10:16

Table 5.10: Availability Time at CCMD for three combinations of two vans on their designated routes

	Availability Time		
Vans\Cases	Case 1	Case 2	Case 3
Van1	9:11	10:16	10:24
Van2	10:23	10:23	9:39

Table 5.11: Availability Time at CCMD for three combinations of three vans on their designated routes

	Availability Time	
Vans\Cases	Case 4	Case 5
Van1	9:08	10:10
Van2	10:10	10:41
Van3	9:45	10:16

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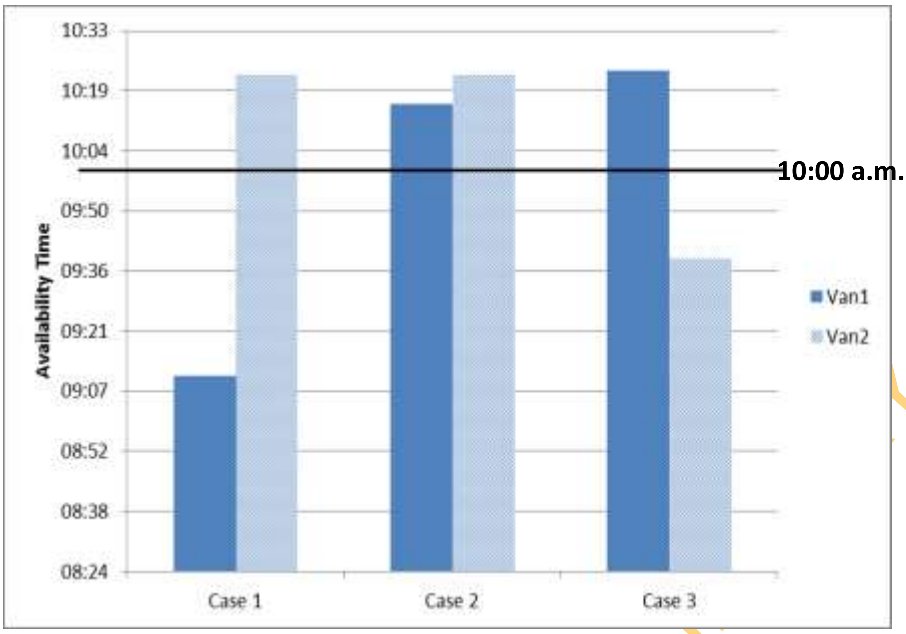

Figure 5.6: Chart showing possibility levels in case studies with deadline of 10:00 a.m. using 2 vans

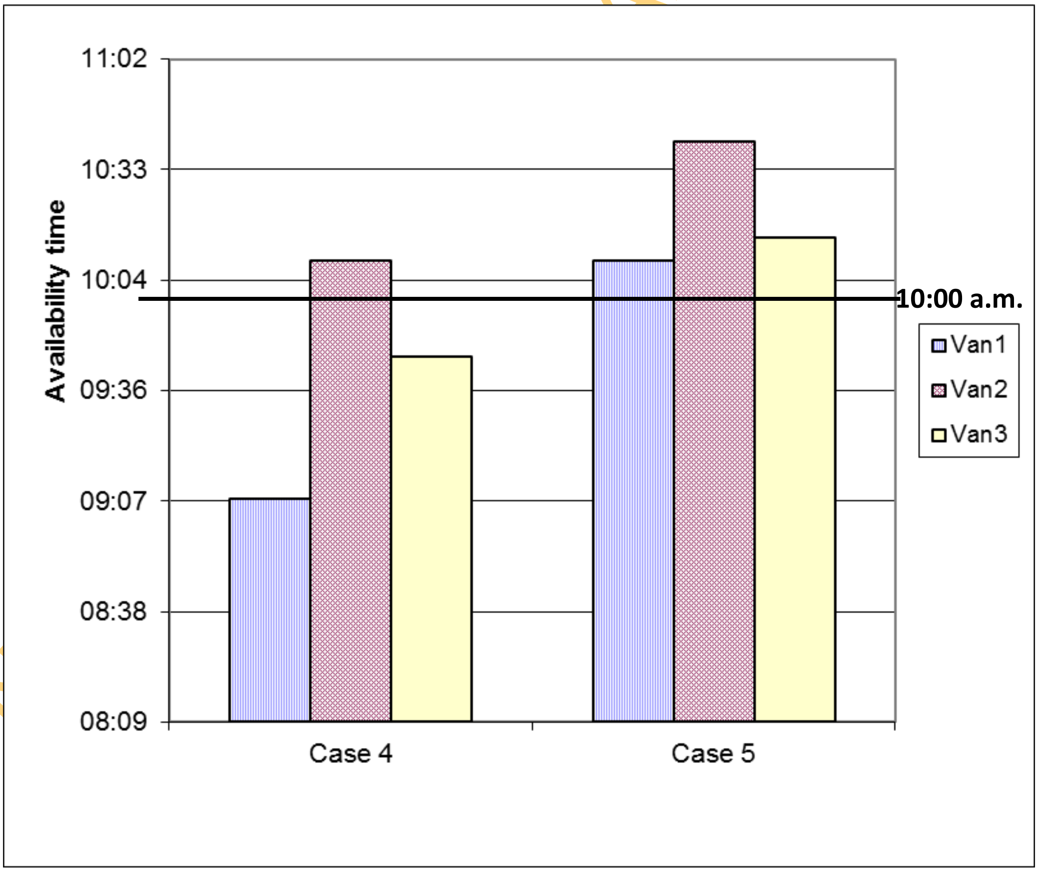

Figure 5.7: Chart showing possibility levels in case studies with deadline of 10:00 a.m. using 3 vans

As shown in figure 5.7, the thick black line depicts the deadline of 10:00a.m. It is possible for van1 and van3 at 9:08a.m and 9:45a.m respectively in case study 4 to meet the deadline at 10:00a.m. It is also possible for van2 at 9:36a.m to reach BH since BH is its routes before crossing the thick black line. In case study 5, it is not possible for van1, van2 and van3 by 10:10a.m, 10:41a.m and 10:16a.m respectively to deliver on or before 10:00a.m.

The results presented in figures 5.6 and 5.7 show the cases against the availability time of the vans, where the availability time is the point in time when the vans will be available for any other distribution process. Following the initial plan, any planner viewing the resulting charts can easily tell when it is possible for any of the vans to make the next delivery from those with availability time below the thick horizontal line marking the deadline of 10:00 a.m. Where there is no van available, it may call for re-planning to meet the deadline. In addition to this, figure 5.5 also show that as the number of vans increases, the likelihood of the possibility of the van reaching the desired location also increases since a van is likely to be assigned to that route.

From the results, the possibility and/or impossibility of a certain van at CCMD at a certain time to make delivery ay BH by 10:00 a.m can be inferred. For example,

Case 1: It is possible for van1 at CCMD by 9:11 a.m. to make delivery at BH by 10:00 a.m.

It is not possible for van2 at CCMD by 10:23 a.m. to make delivery at BH by 10:00 a.m

Case 2: It is not possible for both van1 and van2 to CCMD by 10:16 a.m. and 10:23 a.m. respectively to make delivery at BH of R2 by 10:00 a.m.

Case 3: It is possible for van2 at CCMD by 9:39 a.m. to make delivery at BH by 10:00 a.m. More so, van2 is assigned to R2 where we have BH.

It is not possible for van1 at CCMD by 10:24 a.m. to make delivery at BH by 10:00 a.m.

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Case 4: It is possible for van1 at CCMD by 9:08 a.m. to make delivery at BH by 10:00 a.m.

It is also possible for van2 at CCMD by 10:10 a.m. to make delivery at BH by 10:00 a.m. since van2 is assigned to R2 where BH situates.

It is possible for van3 at CCMD by 9:45 a.m. to make delivery at BH by 10:00 a.m.

Case 5: It is possible for van1 at CCMD by 10:10 a.m. to make delivery at BH by 10:00 a.m. since BH is along its designated route.

It is not possible for both van2 and van3 at CCMD at 10:41 a.m. and 10:16 a.m. respectively to make delivery at BH at 10:00 a.m.

Based on the prior spatial and temporal knowledge of the vans on each of the routes, the vans spatial presence at a location at a future time can be deductively inferred from the following constraints depending on its assigned route.

1. $\forall x, t_1, t_2.$ Reachable (x, CCMD, MH, (t_1, t_2)) $\Leftrightarrow t_2 - t_1 >= 2{:}34$
2. $\forall x, t_1, t_2.$ Reachable (x, MH, TrH, (t_1, t_2)) $\Leftrightarrow t_2 - t_1 >= 0{:}18$
3. $\forall x, t_1, t_2.$ Reachable (x, TrH, TdH, (t_1, t_2)) $\Leftrightarrow t_2 - t_1 >= 0{:}42$
4. $\forall x, t_1, t_2.$ Reachable (x, TdH, CCMD, (t_1, t_2)) $\Leftrightarrow t_2 - t_1 >= 2.40$
5. $\forall x, t_1, t_2.$ Reachable (x, CCMD, QEH, (t_1, t_2)) $\Leftrightarrow t_2 - t_1 >= 3{:}24$
6. $\forall x, t_1, t_2.$ Reachable (x, QEH, KH, (t_1, t_2)) $\Leftrightarrow t_2 - t_1 >= 0{:}84$
7. $\forall x, t_1, t_2.$ Reachable (x, KH, BH, (t_1, t_2)) $\Leftrightarrow t_2 - t_1 >= 0{:}54$
8. $\forall x, t_1, t_2.$ Reachable (x, CCMD, TBH, (t_1, t_2)) $\Leftrightarrow t_2 - t_1 >= 3{:}42$
9. $\forall x, t_1, t_2.$ Reachable (x, TBH, ZH, (t_1, t_2)) $\Leftrightarrow t_2 - t_1 >= 0{:}54$
10. $\forall x, t_1, t_2.$ Reachable (x, ZH, IH, (t_1, t_2)) $\Leftrightarrow t_2 - t_1 >= 0{:}66$
11. $\forall x, t_1, t_2.$ Reachable (x, IH, CCMD, (t_1, t_2)) $\Leftrightarrow t_2 - t_1 >= 4{:}62$

Examples of the spatiotemporal knowledge required for inferring the possibility of agent's spatial presence at a future location (BH) and time (10:00) are presented using the two model predicates: *Reachable* and *Present_at* as follows:

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1a. Reachable (van1, CCMD, QEH, (9:11, 9:15))

Reachable (van1, QEH, KH, (9:15, 9:16))

Reachable (van1, KH, BH, (9:16, 9:17))

Present_at (van1, CCMD, 9:11)

Present_at (van1, QEH, 9:15)

Present_at (van1, KH, 9:16)

Present_at (van1, BH, 9:17)

Then, it is possible.

3a. Reachable (van2, CCMD, QEH, (9:39, 9:43))

Reachable (van2, QEH, KH, (9:43, 9:44))

Reachable (van2, KH, BH, (9:44, 9:45))

Present_at (van2, CCMD, 9:39)

Present_at (van2, QEH, 9:43)

Present_at (van2, KH, 9:44)

Present_at (van2, BH, 9:45)

Then, it is possible.

4a. Reachable (van1, CCMD, QEH, (9:08, 9:11))

Reachable (van1, QEH, KH, (9:11, 9:12))

Reachable (van1, KH, BH, (9:12, 9:13))

Present_at (van1, CCMD, 9:08)

Present_at (van1, QEH, 9:11)

Present_at (van1, KH, 9:12)

Present_at (van1, BH, 9:13)

Then, it is possible.

4c. Reachable (van3, CCMD, QEH, (9:45, 9:49))

Reachable (van3, QEH, KH, (9:49, 9:50))

Reachable (van3, KH, BH, (9:50, 9:51))

Present_at (van3, CCMD, 9:45)

Present_at (van3, QEH, 9:49)

Present_at (van3, KH, 9:50)

Present_at (van3, BH, 9:51)

It is possible

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5. Reachable (van1, CCMD, QEH, (8:30, 8:34))
Reachable (van1, QEH, KH, (8:34, 9:05))
Reachable (van1, KH, BH, (9:05, 9:36))
Present_at (van1, CCMD, 8:30)
Present_at (van1, QEH, 8:34)
Present_at (van1, KH, 9:05)
Present_at (van1, BH, 9:36)
It is possible.

In the planning system considered in the case studies, the following facts are well established using the axioms forming the logical theory. The facts include:

a. Presence of van at location and at a certain time
b. Reachability of locations concerned within certain time durations
c. Possibility of the vans reaching the desired locations at said time stamps.

The axioms in the logic of spatial qualification are used to infer reachability of two locations. The application of the SQM to the planning of a distribution process has shown how it is useful in assessing any existing plan and re-plan when necessary to meet deadlines. These results also show that the effectiveness of any good plan also depends on the available resources.

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CHAPTER SIX

SUMMARY, CONCLUSION AND RECOMMENDATION

6.1 Summary

The research described in this thesis has introduced spatial qualification problem, an aspect of spatial reasoning, to be concerned with the impossibility of knowing an agent's presence at a specific location and time. The logic of spatial qualification has been formalised using the qualitative reasoning approach. The ability to reason with incomplete knowledge or reduced data set makes qualitative reasoning approach most suitable for spatial reasoning. Qualitative reasoning does not mean the absence of numbers, rather combining reduced sets of numbers with comparative approach to make new inferences. Therefore, qualitative reasoning works as a complement with quantitative reasoning making computation of commonsense properties a reality. Thus, qualitative reasoning with the formalised logical theory is based on prior knowledge.

The formal model resulting from the formalisation is known as the spatial qualification model (SQM). The model has established that we can still reason to determine the possibility of an agent's presence at a certain location and time with location antecedents. This is a missing precondition that has established an agent's spatial and temporal presence prior its participation in an action.

In a bid to formalise the logical model in this work, the quantified (first-order) modal logic commonly used to represent sentences in natural language processing has been found suitable for representing the knowledge of the domain. This representation, although it uses some quantities, is qualitative in our quest to avoid complex mathematical models, such as, those used in probabilistic models and fuzzy logic.

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The field of qualitative spatial reasoning has led to the emergence of a number of spatial calculi addressing the regional connections. The region connection calculi (RCC-8), being one of these calculi, have been found suitable in the definition of some of the spatial relations used in our formalism. The determination of an agent's spatial presence at a future time known to be uncertain is conveniently represented using the standard modal operators: *necessarily* $\Box$ and *possibly* $\Diamond$. This permitted us to define the *Reachable* relation which would be used in determining the possibility of an agent to be present at a known spatial location at a certain time. The definition of the *Reachable* relation is based on an uncertain presence of an agent at a certain time thus represented with the *Present_at* relation with a modality: $\Diamond$*Present_at(x,l,t)*.

Modal logic has proven to be the sure way to logically represent that an agent is possibly present at a location and time. To give our logical model the required expressiveness, the first-order was combined with modal logic to benefit from the quantifiers and its expressive nature. The hybridization of the two representational languages gave rise to the Quantified Modal Logic which was adopted in the formalism.

The semantics of the SQM was described using the Kripke's Possible World semantics due to accessibility relation which helps us to determine the reachability of two locations from each other. A comparative study of the properties of the formalised model with that of the standard modal system such as S4, S5 systems and Barcan's axiom was carried out. A further proof of the SQM was done using the analytic tableau proof method, where the possibility of an agent's spatial presence at a certain time is said to be semi-decidable.

The notions of persistence, discretisation and commutative distance coverage were used as parameters in formalising the concept of spatial qualification. Resulting from the formalisation process is the body of axioms called spatial qualification model with the presence log and reachability of locations as determinants for an agent's spatial presence. Properties of SQM as shown in axioms KP1, KP2, 4P and TP were equivalent to axioms K, P, 4 and T that make up an S4 system of axiom. Further comparison with the S5 system which has axioms K, P, 4, T and B shows a variance as

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SQM failed to satisfy the property of axiom B: $\Diamond\Box\phi \Rightarrow \phi$. Since domains remains constant across possible worlds in SQM, Barcan's axiom holds.

A proof system for reasoning with the formalised theory was developed using analytical tableau method. The tableau proofs for axioms that make up the SQM demonstrates the semi-decidability as they only led to closure when it is in the affirmative but their negation do not necessarily lead to closure.

The theory was applied to an agent's local distribution planning task with set deadline. Cases with known departure time and routes, which may result in the possibility or impossibility of an agent's spatial presence, were considered. Cases showing the use of SQM to assessed and reason about plans in planning domain resulted in our ability to make inferences such as "it is possible" or "it is not possible" as the case may be for a certain van: van1 or van2 or van3 to make the products delivery at a certain location at a certain time with prior knowledge of the van's past location and time.

Depending on the route, the application of SQM to the product distribution planning domain resulted in agent's feasible availability times, within or outside the set deadline to assess the agent's spatial qualification in agreement with possible cases in the planning task.

6.2 Conclusion

The spatial qualification model has qualitatively formalised the logic (reasoner) to assess and reason with the spatial qualification problem without resorting to any complex mathematical model. The formalised model would be useful in reasoning with agent's spatial presence in planning domains or other domains where alibi reasoning may be required. A typical instance is seen as the planning of processes with deadline was actually assessed by considering the presence of the distribution van at a location that is reachable from the location where supply is to be made.

The formalism gives rise to a body of axioms named Spatial Qualification Model (SQM) with the presence log and reachability of locations as determinants for agent's spatial presence. The SQM therefore demonstrated the characteristics of an S4 system of axioms but fell short of being an S5 system. Barcan's axiom held, confirming

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constant domain across possible worlds in the formalised model. Explicating the axioms in the SQM using PWS enabled the understanding of tableau proof rules.

Through closed tableaux, the SQM was demonstrably semi-decidable in the sense that the possibility of an agent's presence at a certain location and time was only provable in the affirmative, while its negation was not.

Depending on the route, the application of SQM to the product distribution planning domain resulted in agent's feasible availability times, within or outside the set deadline to assess the agent's spatial qualification in agreement with possible cases in the planning task.

The model successfully determines the possibility or impossibility of an agent's spatial and temporal presence, making it suitable to assess plans of product distribution task from one location to the other for vans' availability or its spatial qualification.

6.3 Contribution of the study to Knowledge

This work introduces spatial qualification as an important precondition to reasoning about spatial actions. This qualification will help the manager to assess existing plan and re-plan when it is necessary in order to attain the project's goal.

Through this research, the spatial qualification model which demonstrates the ability of using qualitative reasoning in constructing useful spatial calculi or logic of presence required to solve any spatial qualification problem was established. This advances the works done in the qualitative spatial reasoning field.

This introduces the modalities of modal logics into spatial reasoning field through its combination with first-order logics to have quantified modal logic used in solving the spatial qualification problem. This demonstrates the ability to represent and reason with uncertain and incomplete spatial knowledge with the introduction of the modalities of modal logic to first-order language. Its significance is seen in the application domain employed in the main thesis where planned actions are qualified with the *necessarily* or *possibly* modalities.

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6.4 Recommendations for further studies

One possible extension of the current work is towards collaborative spatial qualification reasoning, where the reasoner depends on other agents to determine its conclusion.

We also look forward to the full automation and applicability of spatial qualification model in varying domains other than planning such as alibi reasoning and homeland security.

Another aspect of furtherance of this work is on the use of evolutionary computing approaches to solve the spatial qualification problem and comparing the results with that of SQM.

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REFERENCES

Aiello, M. and Ottens, B. 2007. The Mathematical Morpho-Logical View on Reasoning about space. *Proceedings of the 20th International Joint Conference on Artificial Intelligence (IJCAI'07)*. 205-211.

Akinkunmi, B. O. 2000. On the Expressive Limits of Reified Theories. Journal of *Logic Computation* 10(2): 297-313, Oxford University Press.

Akinkunmi, B. O. and Osofisan, A. O. 2004. Temporal Properties of Repetitive Entities. *Artificial Intelligence Review* 22: 291-338, Kluwer Academic Publishers, Netherlands.

Alechina, N. 2003. Modal Logic. *Lecture Notes of Midlands Graduate School in Foundations of Computer Science, April 2003.* http://www.cs.nott.ac.uk/~nza/modal.html .

Allen, J. 1983. Maintaining knowledge about temporal intervals. *Communications of the ACM* 26 (11): 832-843.

Allen, J. 1984. Towards a General Theory of Action and Time. *Artificial Intelligence* 23: 123-154.

Allen, J. F. 1991. Planning as Temporal Reasoning. *Proceeding of the 2nd International Conference on Principles of Knowledge Representation and Reasoning*, Cambridge, MA. Retrieved. July 26, 2010, from http://www.cs.rochester.edu/~james/kr91.pdf..

Allen, J. F. and Ferguson, G. 1994. Actions and Events in Interval Temporal Logic. Journal of Logic and Computation, Special Issues on Actions and Processes. 4(5): 531-579. Retrieved October 15, 2010, from Doi:10.1093/logcom/4.5.531.

Allen, J. F. and Schubert, L. K. 1991. “The TRAINS Project”, *TRAINS Technical Note* 91-1, Computer Science Dept. University of Rochester.

Allen, J. F., Kautz, H., Pelavin, R. and Tenenberg, J. 1991. Reasoning About Plans. Morgan Kaufmann Publishers, San Mateo, CA.

Allen, J. P, Barry, K. P., McCormick, J. M. and Paul, R. A. 2004. Plan Execution Monitoring with distributed intelligent agents for battle command [5441-13]. *Progress in Biomedical Optics and Imaging (SPIE-Proceedings Series).* International society for optical engineering 5441: 69-77.

Asher, N. and Vieu, L. 1995. Towards a geometry of common Sense: A Semantics and a Complete Axiomatization of Mereotopology. *Proceedings of IJCAI’95* 846-852.

Ballarin, R. 2010. Modern Origins of Modal Logic. Stanford Encyclopedia of Philosophy. Retrieved February 16, 2012 from http://plato.stanford.edu/entries/logic-modal-origins/

Barwise, J. and Perry, J. 1983. Situations and Attitudes. Bradford Books–MIT Press, 1983.

Bennett, B. 1994. Spatial reasoning with propositional logic. J. Doyle, E. Sandewall, P. Torasso (Eds.), *Principles of Knowledge Representation and Reasoning: Proceedings 4th International Conference (KR94)*, Bonn, Germany, Morgan Kaufmann, San Mateo, CA 51-62.

Bennett, B. 1996. Modal logics for qualitative spatial reasoning. Bulletin of the Interest Group in Pure and Applied Logic (IGPL) 4(1): 23-45.

Bennett, B. 2001a. Space, Time, Matter and Things. *Proceedings of the Second International Conference on Formal Ontology in Information Systems (FOIS’01).* Edited by C. Welty and B. Smith, pp. 105-116, Ogunquit, Maine, U.S.A. Retrieved from http://www.comp.leeds.ac.uk/qsr/pub/bennett-fois01.pdf.

Bennett, B. 2001b. A categorical Axiomatization of Region-Based Geometry. *Fundamenta Informaticae* 46: 145-158. IOS Press.

UNIVERSITY OF IBADAN LIBRARY

Bennett, B. 2010. Foundations for an Ontology of Environment and Habitat. *Proceedings of the Sixth International Conference (FOIS-2010).* Edited by A. Galton and R. Mizoguchi, Frontiers in Artificial Intelligence and Applications 209:31-44, IOS Press. http://www.comp.leeds.ac.uk/qsr/pub/Bennett10fois.pdf

Bennett, B. and Agarwal, P. 2007. Semantic Categories Underlying the Meaning of Place. *Spatial Information Theory, proceedings of the 8th International Conference (COSIT-07)*, Melbourne, Australia and Lecture Notes in Computer Science 4736: 78-95, Springer. http://www.comp.leeds.ac.uk/qsr/pub/Bennett07place. pdf

Bennett, B., Cohn, A. G., Torrini, P. and Hazarika, S. M. 2000. Region-Based Qualitative Geometry. *Research Report Series.* School of Computer Studies LS2 9JT, University of Leeds, U.K. http://www.comp.leeds.ac.uk/brandon/brandon_bib.html

Benton, B., Coles, Amanda and Coles, Andrew 2012. Temporal planning with preferences and time-dependent continuous costs. In proceedings of ICAPS'12. AAAI Press 279-287.

Bogaert, P. 2008. A Qualitative Calculus for moving point objects constrained by networks. *A Ph.D. Thesis in Geography*. Department of Geography. University of Ghent.

Borgo, S., Guarino, N., and Masolo, C. 1996. A Pointless Theory of Space based on Strong Connection and Congruence. *Proceedings of Principles of Knowledge Representation and Reasoning (KR96)*. Boston, MA, Morgan Kaufmann 220-229.

Braysy, O. and Gendreau, M. 2005. Vehicle Routing Problem with Time Windows, Part I: Route Construction and Local Search Algorithms. In Transportation Science 39(1):104-118.

Casati, R. and Varzi, A. C. 1997. Spatial Entities. Oliviero Stock (ed.), *Spatial and Temporal Reasoning*, Dordrecht: Kluwer 73-96.

UNIVERSITY OF IBADAN LIBRARY

Clementini, E., Di Felice, P., and Hernandez, D. 1997. Qualitative Representation of Positional information. *Artificial Intelligence* 95:317-356.

Cohn, A. G. 1997. Qualitative Spatial Representation and Reasoning technique. G. Brewka, C. Habel, and B. Nebel (eds.). *KI-97: Advances in Artificial Intelligence. Lecture Notes in Computer Science* 1303:1-30 Springer.

Cohn, A. G. 1999. Qualitative Spatial Representations. *Proceeding of the International Joint Conference on Artificial Intelligence IJCAI-99 Workshop on Adaptive Spatial Representations of Dynamic Environment* 33-52.

Cohn, A. G. and Hazarika, S. M. 2001. Qualitative Spatial Representation and Reasoning: An Overview. *Fundamenta Informaticae* 43: 2-32. IOS Press.

Cohn, A. G. and Renz, J. 2008. Qualitative Spatial Representation and Reasoning. *Handbook of Knowledge Representation.* F. van Harmelen, V. Lifschitz and B. Porter (eds.) 551-596.

Cohn, A. G., and Renz, J. 2007. Qualitative Spatial Reasoning. *Handbook of Knowledge Representation*, F. van Harmelen, V. Lifschitz, and B. Porter (eds.), Elsevier, Oxford.

Cohn, A. G., Bennett, B., Gooday, J. and Gotts, N. M. 1997. Qualitative Spatial Representation and Reasoning with the Region Connection Calculus. *Geoinformatica*, 1: 1-44. Kluwer Academic Publishers, Boston.

Cole, S., and Hornsby, K. 2005. Modeling noteworthy events in a geospatial domain. *Lecture Notes in Computer Science* 3799: 77-89.

Coppin, B. 2004. Artificial Intelligence Illuminated. Jones and Bartlett Publishers, Inc. United States of America.

Davis, E. 2006. The expressivity of quantifying over regions. *Journal of Logic and Computation* 16: 891-916.

Davis, E. 2011. Elementarily Equivalent Structures for Topological Languages over Regions in Euclidean Space

UNIVERSITY OF IBADAN LIBRARY

Davlin, K. 2006. Situation Theory and Situation Semantics. Stanford University. *Handbook of the History of Logic*. Dov M. Gabbay and John Woods (Eds.). Elsevier 7:601-664 Retrieved January 20, 2014 from www.stanford.edu/~kdevlin/papers/HHL _SituationTheory .pdf.

Dean, T., Kaelbling, L. P., Kirman, J. and Nicholson A. 1993. Planning with Deadlines in Stochastic domains. *Proceedings of AAAI93*. http://lis.csail.mit.edu/pubs/pk/deanAAAI93.pdf.

Devlin, K. 2004. Jon Barwise's Papers on Natural Language Semantics, Bulletin of Symbolic Logic. 10:54-84.

Dosen, K. 1992. The first axiomatization of relevant logic. *Journal of Philosophical Logic*. 21:339-356. Kluwer Academic Publishers, Netherlands.

Dylla, F.; Frommberger, L.; Wallgrun, J. O.; Wolter, D.; Nebel, B.; and W¨olfl, S. 2007. SailAway: Formalizing navigation rules. *Proceedings of the Artificial and Ambient Intelligence Symposium on Spatial Reasoning and Communication (AISB'07)*.

Egenhofer, M. J. 2010. Qualitative Spatial-Relation reasoning for Design. *Workshop on Studying Visual and Spatial Reasoning for Design Creativity*, Aix-en-Provence, France.

Egenhofer, M. J. and Franzosa, R. D. 1991. Point-Set Topological Spatial Relations. *International Journal of Geographical Information Systems* 5(2): 161-174.

Erwig, M., Güting, R. H., Schneider, M. and Vazirgiannis, M. 1999. Spatio-Temporal Data Types: An Approach to Modeling and Querying Moving Objects in Databases. *GeoInformatica 3(*3).

Eschenbach, C., Habel, C. and Kulik, L. 1999. Representing Simple Trajectories as Oriented Curves. A. N. Kumar & I. Russell (eds.), *Proceedings of 12th International FLAIRS Conference*. pp. 431-436. Menlo Park, California. AAAI Press.

UNIVERSITY OF IBADAN LIBRARY

Escrig, M. T. 2005. Integrating qualitative and quantitative reasoning spatio-temporal models and their application to robotics. TIC2003-07182. *Jornada de Seguimiento de Proyectos, Programa Nacional de Tecnologías Informáticas*.

Ferguson, G. M. 1995. Knowledge Representation and Reasoning for Mixed-Initiative Planning. *A Ph.D Thesis.* Department of Computer Science, The College of Arts and Sciences, University of Rochester, Rochester, New York.

Fitting, M. 1998. On Quantified Modal Logic. *Fundamenta Informaticae* 39(1-2):105-121.

Fitting. M. 1997. Barcan Both Ways. Department of Mathematics and Computer Science, Lehman College (CUNY) Bronx, New York.

Fitting, M. 2008. Possible World Semantics for First Order LP. http://comet.lehman.cuny.edu/fitting/bookspapers/pdf/papers/FOLP.pdf

Forbus, K. 1981. A Study of Qualitative and Geometric Knowledge in Reasoning about Motion, TR-615, MIT AI Lab, Cambridge, MA.

Forbus, K. 1996. Qualitative Reasoning. *Draft Chapter for the CRC Handbook of Computer Science.* Retrieved June 2, 2012 from http://www.qrg.northwestern.edu/papers/files/crc7.pdf

Forbus, K. D. 1995. Qualitative Spatial Reasoning: framework and frontiers. In: Chandrasekaran, D, Glasgow J, Narayanan, N. H. (eds.) *Diagramatic reasoning: cognitive and computational perspectives. AAAI/MIT*, Menlo Park 183-202.

Forbus, K. D. 2008. Qualitative Modeling. *Handbook of Knowledge Representation.* F. van Harmelen, V. Lifschitz and B. Porter (eds.) 361-393.

Forbus, K. D., Nielsen, P. and Faltings, B. 1987. Qualitative Kinematics: a framework. McDermott, J. (ed.) *Proceedings of the 10th international joint conference on artificial intelligence (IJCAI'87)*. Morgan Kaufmann, Los Altos, Milan, Italy 430-435.

Forbus, K. D., Usher, J. and Chapman, V. 2004. Qualitative Spatial Reasoning about Sketch Maps. *AI Magazine* 25(3).

UNIVERSITY OF IBADAN LIBRARY

Forbus, K., Nielsen, P. and Faltings, B. 1991 "Qualitative Spatial Reasoning: The CLOCK Project", *Artificial Intelligence* 51(1-3).

Frank, A. 1991. Qualitative Spatial Reasoning with cardinal directions. *Proceedings of 7th Osterreichische AI – Tagung* 57-167, Berlin, Springer.

Freksa C. 1991. Qualitative Spatial Reasoning. D. M. Mark and A. U. Frank (ed.), *Cognitive and Linguistics Aspects of Geographic Space* 361-372.

Freksa, C. 1992. Using Orientation Information for Qualitative Spatial Reasoning. A. U. Frank, I. Campari, U. Formentini (eds.), *Theories and Methods of Spatio-Temporal Reasoning In Geographic Space, LNCS.* 639: 162-178. Berlin Springer-Verlag.

Frommberger, L. 2008a. Learning to behave in space: a qualitative spatial Representation for robot navigation with reinforcement learning. *International Journal on Artificial Intelligence Tools* 17(3):465–482. © World Scientific Publishing Company.

Frommberger, L. 2008b. Representing and Selecting Landmarks in Autonomous Learning of Robot Navigation. C. Xiong et al. (Eds.): *ICIRA 2008, Part I, LNAI 5314:* 488–497 © Springer-Verlag Berlin Heidelberg.

Galton A. P. and Hood, J. M. 2005. Anchoring: A New Approach to handling indeterminate Locations in GIS. Mark, D. M., Cohn, A. G.(Eds.). *Proceedings of COSIT'05,* Springer, Berlin.

Galton, A. 1994. Instantaneous Events. *Temporal Logic: Proceedings of the ICTL Workshop*, ed. H. J. Ohlbach 4–11. Saarbrücken: Max-Planck-Institut für Informatik, Technical Report MPI-I-94-230.

Galton, A. 1995. A Qualitative Approach to Continuity. P. Amsili, M. Borillo, and L. Vieu, L. (eds.). *Proceedings of the 5th Workshop on Time, Space, and Movement: Meaning and Knowledge in the Sensible World. CNSR (TSM'95)* 17-30.

UNIVERSITY OF IBADAN LIBRARY

Galton, A. P. 1996. Taking dimension seriously in qualitative spatial reasoning. W. Wahlster (ed.), *Proceedings of the 12th European Conference on Artificial Intelligence* 501-505. John Wiley and Sons.

Galton, A. P. 2009. Spatial and Temporal Knowledge Representation. *Earth Sci Inform.* 2:169-187. Springer-Verlag.

Georgeff, M. P. and Lansky, A. L. 1987. Reactive Reasoning and Planning. *Proceedings of AAAI-87*. Copyright ©1987, AAAI (www.aaai.org).

Gerevini, A. and Nebel, B. 2002. Qualitative Spatio-Temporal Reasoning with RCC-8 and Allen's Interval Calculus: Computational Complexity. F. v. Harmelen, ed., *15th European Conference on Artificial Intelligence (ECAI'02)*, Lyon, France 312-316. http://www.frontiersinai.com/ecai/ecai2002/pdf/p0312.pdf.

Goldblatt, R. 2005. Mathematical Modal Logic: A view of its evolution. In Handbook of the History of Logic. Vol. 6. Dov. M. Gabbay and John Woods (Eds.).Elsevier BV.

Google-Maps, 2012. Google Maps Distance Calculator. http://www.daftlogic.com/projects-google-maps-distance-calculator.htm

Gotts, N. M. 1996. An Axiomatic approach to topology for spatial information systems. *Technical Report 96.25*, School of Computer Studies, University of Leeds.

Guarino, N. 1998. Formal Ontology and Information Systems. N. Guarini (ed.). *Proceedings of Formal Ontology and Information Systems (FOIS'98)*, Trento, Italy. Amsterdam, IOS Press 3-15.

Hazarika, S. M. 2005. Qualitative spatial change: space time histories and continuity. *A Ph.D. Thesis*, University of Leeds, Leeds, UK, 250 pp.

Hornsby, K., and Cole, S. 2007. Modeling Moving Geospatial Objects from an Eventbased Perspective. *Transactions in GIS*, 11(4):555–573.

UNIVERSITY OF IBADAN LIBRARY

Ibrahim, Z. M. and Tawfik, A. Y. 1998. A Qualitative Spatio-temporal Abstraction of a Disaster Space. *Applications of Uncertainty Formalisms* 379-396.

Isli, A. and Moratz, R. 1999. Qualitative spatial representation and reasoning: algebraic models for relative position, *Technical Report FBI-HH-M-284/99*, Universita¨ t Hamburg, FB Informatik, Hamburg.

Janowicz, K. 2009. The role of place for the spatial referencing of heritage data. *The cultural heritage of Historic European Cities and Public Participatory in GIS workshop*. University of New York, UK. 17-18.

Janson, T. M. V. 2012. Montage Semantics: The Standford Encyclopedia of Philosophy. Winter Edition 2012. Edward N. Zalta (Ed.).

Katoen, J. P. 2007. CTL, LTL and CTL*. Lecture #18: *Lecture Notes of Model Checking*. http://www-i2.informatik.rwth-aachen.de/i2/fileadmin/user_upload/documents/Introduction_to_Model_Checking/mc_lec18.pdf

Koomen, J. A. G. M. 1989. Reasoning about Recurrence. *A Ph.D. Thesis in Computer Science.* University of Rochester, Rochester, New York.

Kripke, S. 1963. Semantical Considerations on Modal Logic. Philosophica Fennica 16,83-94.

Kuipers, B. J. 2001. Qualitative Simulation. *Artificial Intelligence* 29:289-338. Retrieved April 20, 2013. http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.26.5127&rep=rep1&type=pdf

Kuipers, B. J. 1994. Qualitative Reasoning: Modeling and Simulation with Incomplete Knowledge. MIT Press, Cambridge, MA, 1994.

Ladkin, P. 1987. Models of axioms for time intervals. *Proceedings of the 6th National Conference on Artificial Intelligence*, Seattle, WA, July 13–17, 234–239. The AAAI Press, Menlo Park, CA.

Latombe, J. C. 1991. Robot Motion Planning, Kluwer Academic Publishers, Dordrecht.

UNIVERSITY OF IBADAN LIBRARY

Latombe, J. C. 1988. Motion Planning With Uncertainty: The Preimage Backchaining Approach. *A Research Report*. Department of Computer Science, Stanford University, Stanford, California. ftp://db.stanford.edu/pub/cstr/reports/cs/tr/88/1196/CS-TR-88-1196.pdf

Ma, J. and Knight, B. 1994 A general temporal theory. *Comput. J.* 37(2):114–123.

Ma, J. and Knight, B. 2003 Representing the dividing instant. *Comput. J.* 46(2):213-222.

MacFarlane, J. 2011. Propositional Modal Logic* *Philosophy 142 Lecture Notes.* http://johnmacfarlane.net/142/modal-propositional-logic.pdf

McCarthy, J. 1986. Applications of Circumscription to formalizing commonsense knowledge. *Artificial Intelligence* 28:89-116. htttp://www-formal.stanford.edu/jmc/applications.html. Re-printed in McCarthy, J. 1990. Formalization of Commonsense, papers by John. McCarthy. Edited by V.Lifchitz. Ablex, Norwood, N. J.

McCarthy, J. and Hayes, P. 1969. Some philosophical problems for the standpoint of artificial intelligence. *Machine Intelligence* 4:463-502.

McDermott, D. 1982. A temporal logic for reasoning about processes and plans. *Cognitive science* 6:101-155.

Mennis, J. L., Peuquet, D. J., and Qian, L. 2000. A conceptual framework for incorporating cognitive principles into geographical database representation. International Journal of geographical information science 14(6):501-520.

Menzel, C. 1990. Actualism, Ontological Commitment, and Possible World Semantics. *Synthese* 85: 355-389,35.

Muller, P. 1998a. A qualitative theory of motion based on spatiotemporal primitives. *Principles of Knowledge Representation and Reasoning*.

Muller, P. 1998b. Space-Time as a primitive for space and motion. *Proceedings of FOIS, Frontiers in Artificial Intelligence Applications*.

UNIVERSITY OF IBADAN LIBRARY

Nair, V. C. P. 2003. On extending BDI Logics. *A Ph.D. Thesis*. Knowledge Representation and Reasoning Unit (KRRU), School of Information Technology, Faculty of Engineering and Information Technology, Griffith University, Gold Coast Campus, Queensland, Australia.

Oberle, D., Volz, R., Staab, S. and Motik, B. 2003. An extensible ontology software environment. (Technical Report and Handbook). Institute AIFB, University of Karlsruhe (TH). http://kaon.semanticweb.org/

Pan, F. 2005. Temporal Aggregates for Web Services on the Semantic Web. *Proceedings of the IEEE International Conference on Web Services (ICWS),* Orlando, Florida 831-832, IEEE Computer Society.

Pan, F. 2007. Representing Complex Temporal Phenomena for the Semantic Web and Natural Language. *A Ph.D. Dissertation.* Computer Science Department, Faculty of the Graduate School, University of Southern Carlifornia.

Pan, F. and Hobbs, J. 2006. Temporal Arithmetics Mixing months and days. *Proceedings of the 13th International Symposium on Temporal Representation and Reasoning (TIME)* 212-217, Budapest, Hungary, IEEE Computer Society.

Pan, F., Mulkar, R. and Hobbs, J. 2006a. Extending TimeML with Typical Durations of Events. *Proceedings of Annotating and Reasoning about time and events Workshop at the 21st International Conference (COLING-ACL)* 38-45, Sydney, Australia.

Pan, F., Mulkar, R. and Hobbs, J. 2006b. Learning event durations from Event Descriptions. *Proceedings of the 21st International Conference on Computational Linguistics and the 44th Annual Meeting of the Association for Computational Linguistics (COLING-ACL)* 393-400, Sydney, Australia.

Pan, F., Mulkar, R. and Hobbs, J. 2007. Modeling and Learning Vague Event Durations for Temporal Reasoning. *Proceedings of the 22nd Conference on Artificial Intelligence (AAAI),* Nectar, Track 1659-1662, Vancouver, Canada.

Paulson, L. C. 2002. Logic and Proof. Computer Laboratory, University of Cambridge, U.K.

UNIVERSITY OF IBADAN LIBRARY

Perry, J. 1998. Semantics, possible worlds. E. Craig (ed.), *Routledge Encyclopedia of Philosophy.* Routledge, London. www-csli.stanford.edu/~jperry/PHILPAPERS/posswld.pdf. Perry

Price, C. J., Travé-Massuyès, L., Milne, R., Ironi, L., Bredeweg, B., Lee, M. H., Struss, P., Snooke, N., Lucas, P. and Cavazz, M. 2005. Qualitative Futures. B. Rinner et al. (eds.), *19th International Workshop on Qualitative Reasoning QR-05*, May 18th - 20th, 2005, Graz, Austria, ISBN 3-9502019-0-4, 29 – 37.

Ramsay, A. 1989. Formal methods in AI. School of Cognitive Science, University of Sussex, Cambridge University Press, Cambridge, New York.

Randell, D. A., Cui, Z. and Cohn, A. G. 1992. A Spatial logic based on region and connection. *Proceeding of 3rd International Conference on Knowledge Representation and Reasoning*, Morgan Kaufmann, Los Altos 55-66.

Renz, J. and Mitra, D. 2004. Qualitative direction calculi with arbitrary granularity. *PRICAI 2004: Trends in Artificial Intelligence, 8th Pacific Rim International Conference on Artificial Intelligence* 65–74.

Renz, J. and Nebel, B. 1999. On the complexity of qualitative spatial reasoning: A maximal tractable fragment of the Region Connection Calculus. *Artificial Intelligence* 108: 69-123. Elsevier

Renz, J. and Nebel, B. 2007. Qualitative Spatial Reasoning using constraint calculi, M. Aiello, I. Pratt Hartmann, J. van Benthem (eds.). *Handbook of Spatial Logics,* Springer.

Retz-Schmidt, G. 1988. Various views on spatial propositions. *AI. Magazine* 9 (2): 95-105.

Russell and Norvig 2003. Artificial Intelligence: A Modern Approach. 2nd Ed. Prentice Hall Series, New Jersey.

Sabri, K. E. 2009. Semantic Tableau Proof for First-Order Logic. McMaster University. imps.**mcmaster**.ca/courses/CAS-701-04/.../**Sabri**-sem-**tableau**.pdf

UNIVERSITY OF IBADAN LIBRARY

Schubert, L. K. 2000. The situations we talk about. In J. Minker (ed.). Logic-Based Artificial Intelligence. Kluvier Academic Publisher, Dortrecht.

Shakarian, P., Dickerson, J. P. and Subrahmanian, V. S. 2011. Adversarial Geospatial Abduction Problems. *ACM Tansactions on Intelligent Systems and Technology,* Vol. 20.

Sheremet, M, Wolter, F. and Zakharyaschev, M. 2010. A Modal logic framework for reasoning about comparative distances and topology. Annals of Pure and Applied Logic 16(4):534-559.

Starr, W. 2008. Montague and Possible World Semantics (Necessarily, Possibility and propositions)

Tarski, A. 1929. Foundations of the Geometry of Solids. *Logic, Semantics, Metamathematics (Papers from 1923 to 1938)*, 1956, Oxford, Clarendon Press.

Thielscher, M. 2001. The Qualification Problem: A Solution to the Problem of Anomalous Model. *Artificial Intelligence* 131(1-2):1-37.

Traum, D. R., Allen, J. F., Ferguson, G., Heeman, P. A., Hwang, C. H., Kato, T., Martin, N. Poesio, M. and Schubert, L. K. 1994. Integrating Natural Language Understanding and Plan Reasoning in the TRAINS-93 Conversation System. *A Technical Report.* citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.50.198...

Uribe, T. E., Chuadhri, V., Hayes, P. J. and Stickel, M. E. 2002. Qualitative Spatial Reasoning for Question-Answering: Axiom Reuse and Algebraic Methods. *Artificial Intelligence Center Compilation* copyright © 2002, AAAI (www.aaai.org). http://www.aaai.org/Papers/Symposia/Spring/2002/SS-02-06/SS02-06-019.pdf

Van De Wedge, N., Cohn, A. G., Tre, G. D. and De Maeyer, P. 2006. A Qualitative trajectory calculus as a basis for representing moving objects in GIS. *Control and Cybernetics.*

Van de Weghe, N., Cohn, A. G. and Maeyer, P. 2004. A qualitative representation of trajectory pairs. *Proceedings of European Conferences on Artificial Intelligence* (ECAI 2004).

UNIVERSITY OF IBADAN LIBRARY

Van de Weghe, N., Kuipers, B., Bogaert, P. and Maeyer, P. 2005. A qualitative trajectory calculus and the composition of its relations, *First International Conference on GeoSpatial Semantics (GeoS2005)* 3799: 181–211. Springer,

Villa, L. 1994. A Survey on Temporal Reasoning in Artificial Intelligence. *AI Communications* 7(1): 4-28.

Williams, B. C. and de Kleer, J. 1991. Qualitative reasoning about physical systems: a return to roots. *Artificial Intelligence* 51(1-9). Elsevier Science Publishers.

Winter, S. Bennett, R., Truelove, M., Rajabifad, A., Duckham, M., Kealy, A. and Leach, J. 2010. Spatially Enabling 'Place' information. Rajabifard, A. (Ed.), *Spatially Enabling Society: Research, Emerging Trends, and Critical Assessment, GSDI Association*, Singapore.

Wolter, F., and Zakharyaschev, M. 2000a. Spatial reasoning in RCC-8 with Boolean region terms. In Horn, W. (Ed.), *Proceedings of the 14th European Conference on Artificial Intelligence (ECAI 2000)* 244–248. IOS Press.

Wolter, F., and Zakharyaschev, M. 2000b. Spatio-temporal representation and reasoning based on RCC-8. Cohn, A., Giunchiglia, F., & Seltman, B. (Eds.), *Proceedings of the 7th Conference on Principles of Knowledge Representation and Reasoning (KR2000)* 3–14. Morgan Kaufmann.

Wolter, F., and Zakharyaschev, M. 2002. Qualitative spatio-temporal representation and reasoning: a computational perspective. Lakemeyer, G., & Nebel, B. (Eds.), *Exploring Artificial Intelligence in the New Millenium* 175–216. Morgan Kaufmann.

Zalta, E. N. 1995. Basic Concepts in Modal Logic. *Centre for the study of Language and Information*. Stanford University. http://mally.**stanford**.edu/notes.pdf

Zimmerman, K. and Freksa, C. 1996. Qualitative Spatial Reasoning using orientation, distance, path knowledge. *Applied Intelligence* 6(1):49-58.

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PAPERS PUBLISHED FROM THE WORK

JOURNALS:

1. Bassey, P. C. and Akinkunmi, B. O. (2013). Introducing the Spatial Qualification Problem and Its Qualitative Model. African Journal of Computing and ICTs. 6(1): 191-196.

2. Akinkunmi, B. O. and Bassey, P. C. (2013). A logical theory of spatial qualification using qualitative reasoning approach. International Journal of Artificial Intelligence and Applications (IJAIA). 4(2). 45-56. Indexed in: DOI:10.5121/ijaia.2013.4204

CONFERENCES:

1. Bassey, P. C. and Akinkunmi, B. O. (2012). Towards a logical theory of spatial qualification. Presented at the Research Consortium during the 24th National Conference of Nigeria Computer Society on "Towards a Cashless Nigeria" at Uyo, Nigeria. An Award winning paper and presentation.

2. Bassey, Patience C. and Akinkunmi, B. O. (2013). A Proof System for the Spatial Qualification Logic. Presented at the International Conference on Science and Sustainable Development in Nigeria. Ibadan May, 2013. Under review to appear in the special edition of Journal for Science Research.

GRANTS OBTAINED DURING THE PROGRAMME

1. Conference Grant – Postgraduate School, University of Ibadan - N20,000 (Taken)

2. All Africa Fellowship Grant – University of Cape Town, South Africa - Return Ticket, Accommodation and Subsistence Allowances of R260 for three months. (Taken)

3. Travel Grant for IJCAI 2013 Conference at Beijing, China - Association for Computing Machinery for Women (ACM-W) - $1,200 (Not Taken)

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AWARDS RECEIVED DURING THE PROGRAMME

1. An Award for 2nd Runner Up during Research Consortium on Information Technology Innovation (RECITI 2012) held during the 24th National Conference of Nigeria Computer Society on "Towards a Cashless Nigeria" at Uyo, Akwa Ibom State, Nigeria. Given by Nigeria Computer Society at the 11th International Conference held at Iloko-Ijesa, Osun State, Nigeria on 24th – 26th July, 2013.

   Paper Title: Towards a Logical Theory of Spatial Qualification

   Authors: Patience C. Bassey and B. O. Akinkunmi, Computer Science Department, University of Ibadan, Ibadan, Nigeria.

   Paper Presenter: Patience C. Bassey

   Price: N10,000.00

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