ANALYSIS OF PRE-CAST WAFFLE SLABS USING YIELD LINE AND RANKINE GRASHOFF THEORIES

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A Thesis in the Department of Civil Engineering,
Submitted to the Faculty of Technology
in partial fulfillment of the requirement for the Award of

DEGREE OF DOCTOR OF PHILOSOPHY

of the

UNIVERSITY OF IBADAN, NIGERIA

September, 2012.

ABSTRACT

Failure in pre-cast waffle slabs can be attributed to factors like analytical error, poor handling during transportation and placement which factors often lead to partial/total failure of slabs. The conventional method of analysing waffle slabs focuses on the ribs, while the slab portions at the top are avoided. This has led to under reinforcement and subsequent failure of the slab portions that are usually in direct contact with loads. In this study, a method that incorporates both the slab and rib portions in the analysis of pre-cast waffle slabs was therefore developed.

Yield Line and Rankine Grashoff Theories (YLRGT) were combined for the analysis of pre-cast waffle slab. Six physical models of waffle slab were developed, each having five replicates, with the following dimensions: W1 (1353 x 430 x 58 mm), W2 (900 x 300 x 50 mm), W3 (1085 x 430 x 58 mm), W4 (407 x 364 x 50 mm), W5 (1312 x 300 x 58 mm) and W6 (860 x 360 x 50mm). Solid slabs of the same size and number designated S1, S2, S3, S4, S5 and S6 served as control. These models were validated using the slabs by testing for failure loads, deflections and crack width. Each slab was subjected to incremental load of 1.0 kN until failure occurred. Maximum bending moments were obtained for slab and rib portions using YLRGT, a finite element based method called ETABS was also used to analyse the slabs and results obtained were subjected to statistical analysis using ANOVA at p= 0.05.

The YLRGT analysis of the various physical models (slab portion, transverse and longitudinal ribs) yielded the following bending moments: W1 (5526.0, 34.5, 918.3) Nm, W2 (1122.0, 279.2, 36.5) Nm, W3 (2880.0, 27.2, 619.9) Nm, W4 (590.0, 171.9, 160.9) Nm, W5 (947.0, 37.0, 4.4) Nm and W6 (1276.0, 90.4, 36.2) Nm respectively. The ETABS combined both slab and ribs giving W1 (4729.0) Nm, W2 (581.0) Nm, W3 (3338.0) Nm, W4 (733.0) Nm, W5 (851.0) Nm and W6 (686.0) Nm. Deflections at failure for waffle slab were smaller compared to solid slabs: (W1 = 1.19 and S1 =3.56) mm, (W2 = 3.64 and S2 =9.28) mm, (W3 = 3.90 and S3 =7.44) mm, (W5 = 8.17 and S5 =12.18) mm, (W6 = 3.29 and S6 =3.89) mm with the exception of W4 (6.60 mm) and S4 (6.44mm), where deflection of waffle slab was higher than that of solid slab. Mean deflection of S1 was significantly higher than W1, while S2 was significantly higher

than W2. Average crack width for waffle (0.48 mm) and solid slabs (0.99 mm) were significantly different. High crack width in solid slab indicated lower shear strength.

The Yield Line and Rankine Grashoff Theories have facilitated the accurate analysis of pre-cast waffle slabs by separating the slab and rib portions.

Keywords: Yield line theory, Rankine Grashoff theory, Waffle slabs, Crack width.

Word Count: 464

DEDICATION

Unto God, who was, who is and who is to come. To Him is all the Glory For Ever.

ACKNOWLEDGEMENTS

blessed memory, Bukola Oke- Akinyele, Yinka and Seun Raji, Mr and Mrs Sola Jebutu and Family, Dr. S.O.S. Akinyemi and family, Dr. and Dr. (Mrs.) Aduramigba-Modupe, Dr. and Dr. (Mrs.) Sola Ojurongbe, Mr and Mrs. Sam. Olanipekun, Tunde and Debola Folorunso, and Tolu 'sholu', my uncles and aunties, I wish you all well.

To my Pastor and Spiritual guide, Rev. Abiodun Aladekomo and Mummy Aladekomo, all the members of the New Covenant Church Basorun, Ibadan, Rev. Odunjo & family, Rev. Adeniji and family, Pastor and Mrs Israel Adeyemi and family, Pastor Dele Adeyemi and family, Rev. Akin Feyisetan and family, and many others that deserve my appreciation.

To my beautiful daughters and little angels, Oreoluwa Oluwafeyijimi and Ooreoluwa Olaoluwasubomi, and my little boy Adeoluwa Olorunjuwon Akinwumi, I just love you all, you make my life worth it and thanks for all the disturbances, questions and the 'hiding' games, I really enjoy it all. To my Soul mate and confidant, Adejoke Olukemi Akinyele, you are always there for me; I really love you from the depth of my heart. Thank you for making my life great and better.

CERTIFICATION

I certify that this work was carried out by JOSEPH OLAWALE AKINYELE in the Department of Civil Engineering, University of Ibadan, Nigeria.

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LIST OF SYMBOLS

The most commonly used symbols are listed below. Exceptions from the list may appear, but this will then be noted in the text in connection with the actual symbol.

h = Height of a cross-section
A = Area of a cross-section
Ac = Area of a concrete cross-section
As = Area of reinforcement close to the bottom face
As' = Area of reinforcement close to the top face
Asc = Area of reinforcement in compression
hc = Distance from the bottom face to the centre of the bottom reinforcement
hc' = Distance from the top face to the centre of the top reinforcement
y_o = Compression depth
L = Length of an element
Lx, Ly = Length of a slab in the x and y direction, respectively
e = Eccentricity
u = Deflection
um = Deflection in the mid section
x, y, z = Cartesian coordinates
fc = Compressive strength of concrete
fy = Yield strength of reinforcement
ρ = Reinforcement ratio
$\Phi 0$ = Degree of Reinforcement
$\Phi 0x$, $\Phi 0y$ = Degree of Reinforcement in the x and y direction, respectively
p = Line load, uniform load per unit length
q = Surface load, uniform load per area unit
mp = yield moment in pure bending
mpx, mpy = yield moment in pure bending in the x and y direction, respectively
mf = yield moment for a given axial load
mfx, mfy = yield moment for a given axial load in the x and y direction, respectively
n = Axial load per unit length
nx, ny = Axial load per unit length in the x and y direction, respectively
Wi, We = internal and external work, respectively
Wc, Ws = concrete and reinforcement contribution to the dissipation, respectively
M = mass
T = time
N = load acting within a particular region (kN)
σ = stress
δ = vertical displacement of the load N on each region expressed as a fraction of Unity (m)

δ = unit weight

m   = ultimate design moment of resistant for the slab (kNm)

θ = rotation of the region about its axis of rotation (m/m)

a_r = reduced short span dimension

b_r = reduced long span dimension

k = a constant = 1

v = a variable = $0 \le 0.5$

 i = the fixity ratio at supports, e.g i_1, i_2.

If : $i_1 = i_2 = 1$, that support is a continuous support

And $i_1 = i_2 = 0$ that support is a simple support.

a_1 and b_1 = are the spacing of the ribs on the short and long span respectively.

q = total load per unit area

q_1 and q_2 = the load shared on the short and long span respectively

a = Shorter dimension of grid

b = Longer dimension of grid

M_x and M_y = moments on the short and long span respectively

Q_x and Q_y = the shear forces on the short and long span respectively.

 L_p =   Diameter of bars in the prototype

 L_m =   Diameter of bars in the model

 δ_p =   Characteristic strength of steel used in the prototype

 δ_m =   Characteristic strength of steel used in the model

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CHAPTER ONE

INTRODUCTION

1.1 Background

Reinforced concrete slabs are one of the most common structural engineering elements. They are used as floors and roofs, to carry vertical loads in structures such as buildings and bridges. A slab is part of a reinforced concrete structure which is often subjected to bending (tensile or compressive) but in rare cases, subjected to shear, such as a bridge deck. In most cases, slabs are horizontal members but they can be used as vertical members, such as walls, to infill panels, side to drains and sewers appurtenances (Oyenuga, 2001).

The various types of slabs include:

(a)   Solid slab,

(b)   Ribbed slab,

(c)   Flat slab, and

(d)   Waffle slab.

 The type to be preferred may depend on:

(i)   span of the slab,

(ii)   use of the space which may determine the span,

(iii)   load to be carried, and

(iv)   architectural aesthetics that is required.

Most of the researches carried out on yield line theory have basically been on solid slabs with little or no research on waffle slabs. According to Mosalam and Naito (2002), a limited number of experimental studies exist in the literature concerning waffle slabs.

Waffle slab has its genesis in a rather thick solid-slab floor from which the bottom layer concrete in tension is partially replaced by their ribs along orthogonal directions.

The ribs are reinforced with steel to resist flexural tensile stresses. The dimensions and spacing of ribs are decided in a manner so as to achieve better load distribution without requiring the shear reinforcement (Prasad *et al*. 2005).

Waffle slabs are generally employed in large span slabs, as spans become larger still, the required slab thickness for the flat plate and flat slab increases to the point where the slab may be unable to carry its own weight. A solution to this is to provide thickness so that reinforcement can be placed in a member at greater depth, but remove concrete from regions of the slab not required for strength (Konda, 2003). It is an extension of the ribbed floor slab in which the slab is ribbed in two directions. Hence, an inverted pot-like hollow is formed which serves as the ceiling for the floor below, (Figure 1.1.). Waffle slabs are all concrete. The inverted pot-like shape is formed through the use of a special mould. When compared with the conventional solid flat slab construction, waffle slabs allow a considerable reduction in dead load; can support heavy loads over a long span (Daniel and Onur, 2005). It is commonly used in parking garages of tall buildings with ramps and also in industrial facilities and warehouses (Buildings, 2005) and meet fire proofing requirements (Sadusky, 2004; Kenichi and Ai, 2005). Also, it has the advantage of medium to long span, light weight, economical in material usage and profiles may be expressed architecturally or used for heat transfer (The concrete centre, 2006). This slab is usually employed for architectural and structural reasons for large rooms such as auditoria, vestibules, theatre halls, show rooms of shops where column-free-space is often the main requirement (Krishna Raju, 1988).
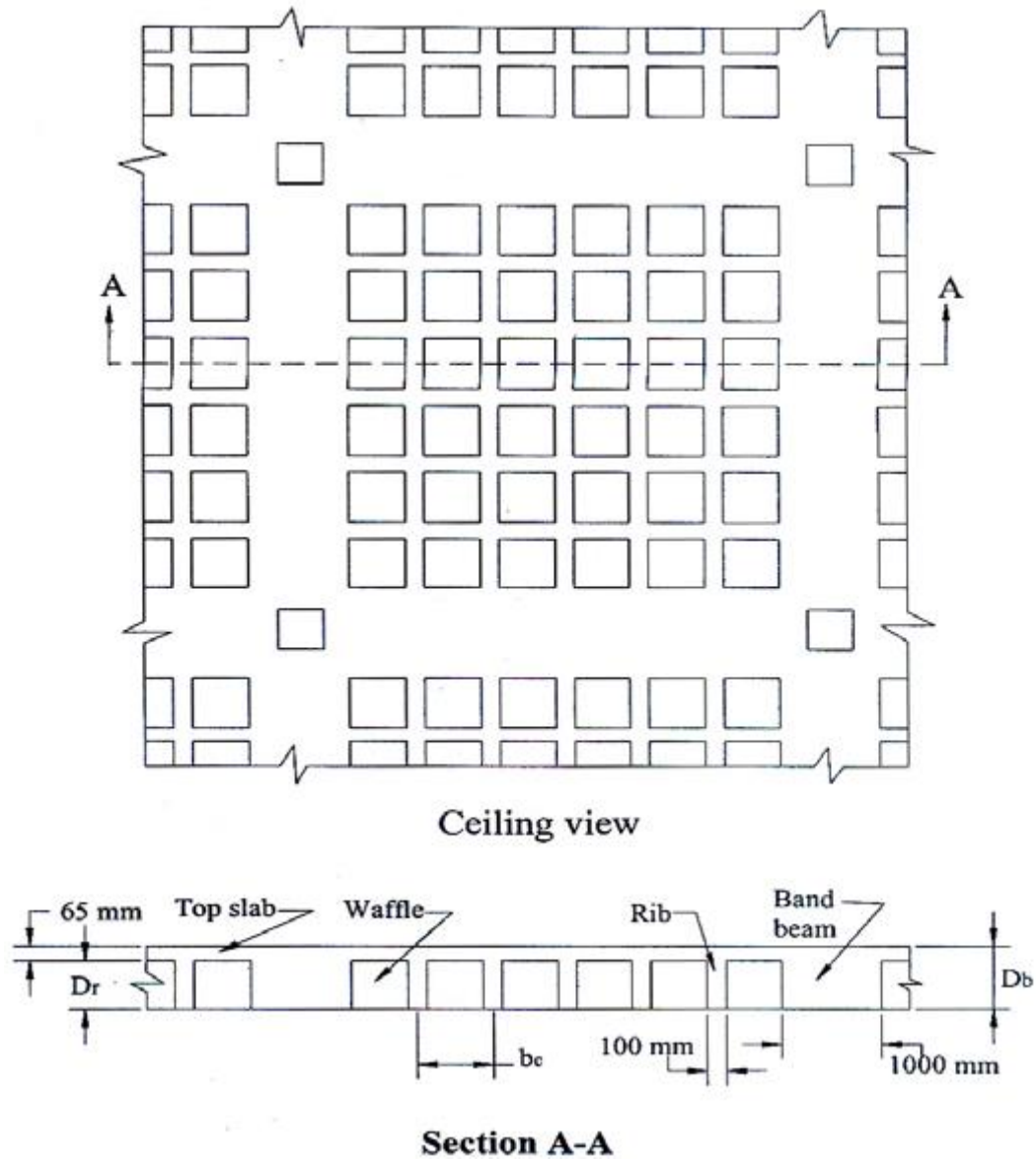
Figure 1.1: Detailed view of a waffle slab floor

According to Howard and Hansen (2002), waffle floors are used extensively in semiconductor factories as they provide high impedance mounts for manufacturing equipment that is extremely vibration sensitive. Also it has been used for cooling

towers, storage tanks, communication shelters, barriers and retaining walls, artificial reefs, building walls, hybrid columns and beams (Waffle-Crete international, 2009).

Deflection or failure in slab can occur due to some factors like overloading, under-reinforcement and poor span-depth ratio. Under overloading, failure of a slab will occur when the reinforcement yields first in a region of high moments. According to Wang *et al* (2003), when concrete is under triaxial compressive loading, both its strength and ductility will have a significant increase as a result of resistance to the compressive force by the concrete materials (molecules). Initially, at service load, the response of a slab is elastic with maximum steel stress and deflection occurring at the centre of the slab. At this stage, it is possible that some hairline cracking will occur on the suffix where the flexural tensile capacity of the concrete has been exceeded at mid span. Increasing the load hastens the formation of these hairline cracks. Further increment of the load will increase the size of the cracks and induce yield of the reinforcement, initiating the formation of large cracks emanating from the point of maximum deflection (Kennedy and Goodchild, 2004). This portion acts like a plastic hinge. On increasing the load further, the hinging region rotates plastically and the moments due to additional loads are redistributed to adjacent sections, the concrete section at the position of a yield line is incapable of carrying any further load, causing them to collapse (Thompson and Haywood, 1986; Macgregor, 1997).

Yield line theory investigates failure mechanism at the ultimate limit state. It does not deal with serviceability issues such as deflection *per se.* Nonetheless, deflection can be dealt with by simple formulae based on yield line moment (Kennedy and Goodchild, 2004). The basic assumption of the yield line theory is that a reinforced concrete slab, similar to a continuous beam or frame of a perfectly plastic material will

develop yield line hinges under overload, but will not collapse until a mechanism is formed (Dunham,1964). The theory also permits the prediction of the ultimate load of a slab system by postulating a collapse mechanism which is compatible with the boundary conditions (Buyukozturk 2004). Yield-line analysis is seen as a useful technique to determine the collapse load of slabs (Johansen, 1963). The band in which yielding has occurred are referred to as yield lines which divide the slab into a series of elastic plates.

The use of yield line analysis requires knowledge of the plastic flexural capacity of the slab, and the results serve as an upper bound on the ultimate load of the system. The results of yield line analysis for a given slab are either correct or too high and are highly dependent on proper selection of a failure mechanism (Park and Gamble, 2000).

The Rankine Grashoff theory of equating deflections at the junctions of ribs is used for the analysis of ribbed or grid floors. The method considers the load in a slab and distribute to all the ribs along both the longitudinal and transverse directions.

1.2   Aim and Objectives

The aim of this work was to develop a method of analysing precast waffle slabs of different sizes, under various support conditions.

The objectives of this study are to:

i. Apply Yield Line and Rankine Grashoff Theories (YLRGT) to waffle slabs subjected to axial loading.

ii. Determine the structural characteristics of waffle slabs.

iii. Develop a computer based method called YLRGT for the analysis and design of waffle slabs.

iv. Validation of the program developed.

1.3   Justification

Structural failure in building was attributed to both technical and human errors by Carper (1998). He listed some causes of failure in civil engineering structures to include: programming deficiency, site selection and site development errors, design errors, construction errors, material deficiencies, and operational errors. Of all the causes mentioned, the programming and design errors were the main problems that this study has tried to look at and proffer appropriate solutions.

Yield line design leads to slabs that are quick and easy to handle. The resulting slabs are light and have very low amounts of reinforcement in very regular arrangements. Above all, yield line design generates economic concrete slabs, because it considers failures at the ultimate limit state (Kennedy and Goodchild, 2004). Since waffle slab is considered to be economical due to its reduced concrete volume, if compared with solid slabs, Yield line theory will be applied in order to investigate its structural response to loading.

In most of the works that have been carried out on waffle slabs, it was only the ribs that were analysed with the provision of adequate reinforcements, but the slab portion has often being left unanalysed. Most designs simply provided wire meshes for the slab portion. Mosley *et al* (1999) simply adopted 12% of the cross section area of the slab portion as wire mesh reinforcements. However, this method has been found to be adequate for slabs that are cast in-situ only, since they are not subjected to the same eccentric forces generated as a result of lifting and transportation of precast waffle slab from the manufacturing plant to the site. In some factories where precast waffle slabs are manufactured using wire mesh in the slab portions, the precast slabs were subjected

to both hogging and sagging moments which resulted into cracks and some times outright failures during loading and placement.

Considering the problems of precast with wire mesh reinforcements, it became necessary to subject the loading conditions to further investigations. This study therefore, developed a method (YLRGT) which is based on yield line and Rankine Grashoff theories of analysis to provide adequate reinforcements for the precast slab.

1.4   Scope of the Study

Six waffle slabs were modelled as W1, W2, W3, W4, W5 and W6, in this research work. Six solid slabs of the same shape and size as the models labeled, S1, S2, S3, S4, S5 and S6, were constructed to serve as control to the models. Both one way and two-way slabs with simple supports on four, three and two sides respectively were investigated. The above specimens were used to determine and compare deflection, crack width and the load bearing capacities of both waffle and solid slabs. The computer program developed (YLRGT) was used to determine the bending moments of both types of slab.   The models were tested at the Department of Mechanical Engineering laboratory of The Polytechnic, Ibadan, Oyo state.

1.5   Problems Encountered

(i.) The inability of the universal tensile machine to stop at every incremental load of

1 kN made the reading of the crack width difficult at this interval, hence the crack widths were measured at failure loads only.

(ii.) The non-availability of crack meter or a crack detective microscope led to the use of a Vernier caliper in determining the crack widths of all the models.

CHAPTER TWO

LITERATURE REVIEW

2.1 History of Reinforced Concrete

Concrete is a material used in building construction, consisting of hard, chemically inert particulate substances, known as aggregates that is bonded by cement and water.

The Assyrians and Babylonians used clay as the bonding substance or cement. The Egyptians used lime and gypsum cement. In 1756, British engineer, John Smeaton made the first modern concrete (hydraulic cement) by adding pebbles as a coarse aggregate and mixing powdered brick into the cement. In 1824, English inventor, Joseph Aspdin invented Portland cement, which has remained the dominant cement used in concrete production. Joseph Aspdin created the first true artificial cement by burning ground limestone and clay together. The burning process changed the chemical properties of the materials and he created cement, stronger than what using plain crushed limestone would produce.

The other major part of concrete besides the cement is the aggregates which include sand, crushed stone, gravel, slag, ashes, burned shale, and burned clay. Fine aggregate (fine refers to the size of aggregate) is used in making concrete slabs and smooth surfaces. Coarse aggregate is used for massive structures or sections of cement.

Concrete that includes imbedded metal (usually steel) is called reinforced concrete or ferroconcrete. Reinforced concrete was invented in 1849 by Joseph Monier, who received a patent in 1867. He was a Parisian gardener who made garden pots and tubs of concrete reinforced with an iron mesh. Reinforced concrete combines the tensile or bendable strength of metal and the compressional strength of concrete to withstand heavy loads. Joseph Monier exhibited his invention at the Paris Exposition of 1867. Besides his pots and tubs, He promoted reinforced concrete for use in railway ties, pipes, floors, arches and bridges (Bellis, 2009).

Since the strength of concrete in tension is very poor, this disadvantage of low tensile strength was overcome by the introduction of reinforcement. Hence; the name 'Reinforced Concrete' (Rajagopalan, 2005).

2.2 Analysis of Waffle Slabs

The exact analysis and design of waffle slab is complex, therefore designers adopted simple procedure based on the use of certain coefficients to distribute the load in both directions, with the assumption that the moment on parallel ribs in one direction are equal (Abdel-Karim and Mahmood, 2006). This type of assumption is not so reliable because so many parameters are left unconsidered during analysis. According to Abdel-Karim and Mahmood (2006), the stiffnesses of the beams on which the ribs are supported are not taken into consideration in distributing the moments along the different parallel ribs in each direction, and this is a disadvantage to the results of such analysis.

In the analysis of waffle slabs, Oyenuga (2001) used the coefficient for two-way spanning solid slabs in the BS 8110 part 1(1997) code. The analysis considered the pot/hollow dimensions of the waffle slabs in which provisions were made for top and bottom reinforcements of the ribs only. Mosley *et al* (1999) designed a waffle slab for a panel in which the ribs were designed as a "**T**" section. Adequate reinforcements were provided and the deflection of the ribs was also determined. However, there was provision for minimum reinforcements in the slab portion.

Rahman *et al* (2010), used a 3-D strut-and tie model to analyse waffle slab, the thickness of the bottom tie, vertical ties, inclined strut and nodal zones at bottom were taken as the thickness of the waffle ribs. Thickness of the top strut and nodal zones at

top was taken as the effective width of top slab and was less than or equal to the rib spacing. It was observed from the work that the ribs were well analysed using the strut-and tie model method, while the slab portion was not analysed.

2.3 Analytical Procedures

Different analytical methods had been developed in the past in order to solve the problem of analysis of waffle slabs. According to Prasad *et al* (2005), waffle slabs were generally analysed using the finite element method, grid or grillage analysis and orthotropic plate theory.

2.3.1 Finite element method

The Finite Element Method (FEM) is based on the division of the structures into small pieces (elements) whose behaviours are formulated to capture the local behaviour of the structure. Each element's definition is based on its material properties, geometry, location in the structure, and relationship with surrounding elements. These elements can be in the form of line elements, two dimensional elements and three-dimensional elements to represent the structure. The intersection between the elements are called nodal points in one dimensional problems, while in two and three dimensional problems, they are called nodal line and nodal planes respectively (Maher, 2007).   At the nodes, degrees of freedom (which are usually in the form of the nodal displacements and/ or their derivatives, stresses, or combinations of these) are assigned.

Models which use displacements are called displacements models and some models use stresses defined at the nodal points as unknown. Models based on stresses are called force or equilibrium models, while those based on combination of both displacements and stresses are termed mixed models or hybrid models (Beckett, 1973).

The mathematical assemblage of these elements into the complete structure allows for automated computation of the response of the entire structure. With finite element method, the entire floor can be analyzed at once.

Aalami and Kelly (2001), used FEM to analyze waffle slab, and two options were adopted. The first option was the modelling of the floor system with each waffle represented by its true geometry. Alternatively, the waffle stems were lumped together and positioned along the lines of support without changing the area, moment of inertial and section moduli of the structure. The result of the analysis was compared with two other methods, namely the Simple Frame Method (SFM), and Equivalent Frame Method (EFM). It was concluded that EFM was a refinement of the SFM, and that both methods were approximate. The degree of approximation depends on the extent to which a floor system deviates from a uniform, orthogonal support layout and constant slab thickness.

Although both the EFM and SFM gave safe solutions, they are not as reliable as the FEM. The study concluded that since the selection of load path is a prerequisite for the design of a concrete floor, using the FEM was able to prove that the selection of load path and analysis of the slab can be automated and give satisfactory results. This method needs more time and efforts in modelling than the grillage. The results obtained from the FEM depend on the mesh size, but by optimizing the mesh, the results of this method are considered more accurate than grillage. The FEM is a well-known tool for the solution of complicated structural engineering problems, as it is capable of accommodating many complexities in the solution. (Maher, 2007).

Tiedman *et al* (1993) showed that FEM is a numerical method with powerful technique for solution of complicated structural engineering problems. It most predicted

accurately the bridge behaviour under truck axle load. Quaqish (2005) presented the effect of skew angle on distribution of bending moments in bridge slabs using FEM, and the results were very reliable.

According to Bakht and Jaeger (1985), the finite element method has a number of advantages. These include the ability to model irregular shaped bodies composed of different materials, handle general loading and different kinds of boundary conditions.

2.3.2 Grillage analysis

This method is commonly used in the analysis of bridge decks. In this method, the deck is represented by an equivalent grillage of beams. The orientation of the longitudinal members should always be parallel to the free edges while the orientation of transverse members can be either parallel to the supports or orthogonal to the longitudinal beams (Maher, 2007). The method can be used to consider boundary conditions in waffle slab analysis, in which the shear force and bending moments are determined (Lee *et al*, 2006). The grillage numerical method has gained increased popularity in the static and dynamic analysis of plate structures (Zeng *et al*, 2007).

West (1973) reported a study, where 53 models and full-sized bridges were compared. The work recommended the use of grillage analysis for slabs and pseudo-slabs bridge decks. Tan *et al*. (1998) reported that the accuracy, simplicity and speed of grillage analysis make it the most suitable model for bridge analysis. It was found that the results obtained from grillage analysis compared with experiments and more rigorous method is accurate enough for design purposes (Maher, 2007).

Zeng *et al* (2007) in their work on grillage analysis of skewed bridges were able to develop some governing equations of motion for transverse and torsional vibration of the bridge deck as follows:

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial x^2} = f(x,t) \qquad \ldots\ldots\ldots \qquad \ldots\ldots\ldots\ldots.2.1$$

$$GJ\frac{\partial^2 \theta(x,t)}{\partial x^2} + m(x,t) = I_0\frac{\partial^2 \theta(x,t)}{\partial t^2} \qquad \ldots\ldots\ldots. \quad \ldots\ldots\ldots \qquad .2.2$$

Where:

$w(x,t)$ and $\theta(x,t)$ are the transverse and torsional displacements

EI and GJ are the flexural and torsional rigidity

ρ and A are the mass density and cross sectional area, I_0 is the mass moment of inertial per unit length.

$f(x,t)$ and $m(x,t)$ are the external transverse force (including bending moment) and torsional moment per unit length, applied at the nodes.

The effective flexural or torsional rigidity of a grillage element is equivalent to the corresponding rigidity of the strip of the plate,

$$EI = bD,$$
$$D = Eh^3/12(1-\upsilon^2),$$
$$GJ = bD,$$

Where:

h is the thickness of the plate

υ is the poison ratio, and

b is the width of a strip of the plate.

The deck was idealized as a grillage, with the girders and diaphragms coinciding with certain torsion beam members. Close spacing between grillage members provides accurate results, but increase computational efforts. The optimum assembly results form a compromise between accuracy, simplicity and efficiency.

According to Jaeger and Bakht (1982), the grillage analysis has become popular because of the following reasons:

a.   It can be used in cases where the bridges exhibit complicating features such as a heavy skew, edge stiffening and deep hunches over supports.

b.   The representation of a bridge as a grillage is ideally suited to carrying out the necessary calculations associated with analysis and design on digital computer.

c.   The grillage representation is conducive to giving the designer an idea of both the structural behaviour of the bridge and the manner in which bridge load is distributed and eventually taken to the supports.

In grillage design, the central intersection point load may be used as a worst-case loading condition, especially with a larger number of beams in each direction as long as local collapse is prevented. But a point load may often move around on the grid system. In such a case, the worst load point would not necessarily be at the central point. In this case, the worst load point is located between intersections (Ki-Sung *et al,* 2001).

Recently, Gordon and May (2004), showed that under certain conditions, the grillage analysis of slabs can give incorrect results, so that the use of FEM was preferred in their study. Grillage analysis was applied to 3 slabs: a square plate simply supported on four sides; a rectangular plate simply supported on two sides; and a skew plate simply supported on two sides. The results were compared with those obtained using a FEM software package (LUSAS), and where available, with theoretical solutions. In certain cases the grillage results exhibit significant errors, and the situation was not improved by a local refinements. It was concluded that the use of finite element method (FEM) is to be preferred, and that the use of the grillage method should be avoided.

2.3.3 The Plate theory

The plate theory is another method of analysing complex engineering problems especially thin walled plate structures. This method involves the use of solution in the form of simultaneous differential equations of the fourth order. Some assumptions are made in the use of the plate theory in analysing thin walled structures.

From Fig.2.1, the assumptions are:

a.   linear element of the plate extending through the plate thickness, normal to the mid surface, x-y plane, in the unstressed state, upon the application of load:

i. Undergoes at most a translation and a rotation with respect to the original coordinate system.

ii. Remains normal to the deformed middle surface.

b.   A plate resists lateral and in-plane loads by bending, transverse shear stresses, and in-plane action, not through block like compression or tension in the thickness direction. This assumption results from the fact that $h/a \ll 1$ and $h/b \ll 1$.

From (ai), the following implied:

c.   A linear element through the thickness does not elongate or contract.

d.   The linear element remains straight upon load application.

e.   A normal stress in the direction normal to the mid surface is negligibly small and such can be discarded. (i.e. $\partial_z = 0$). (Vinson, 1989).
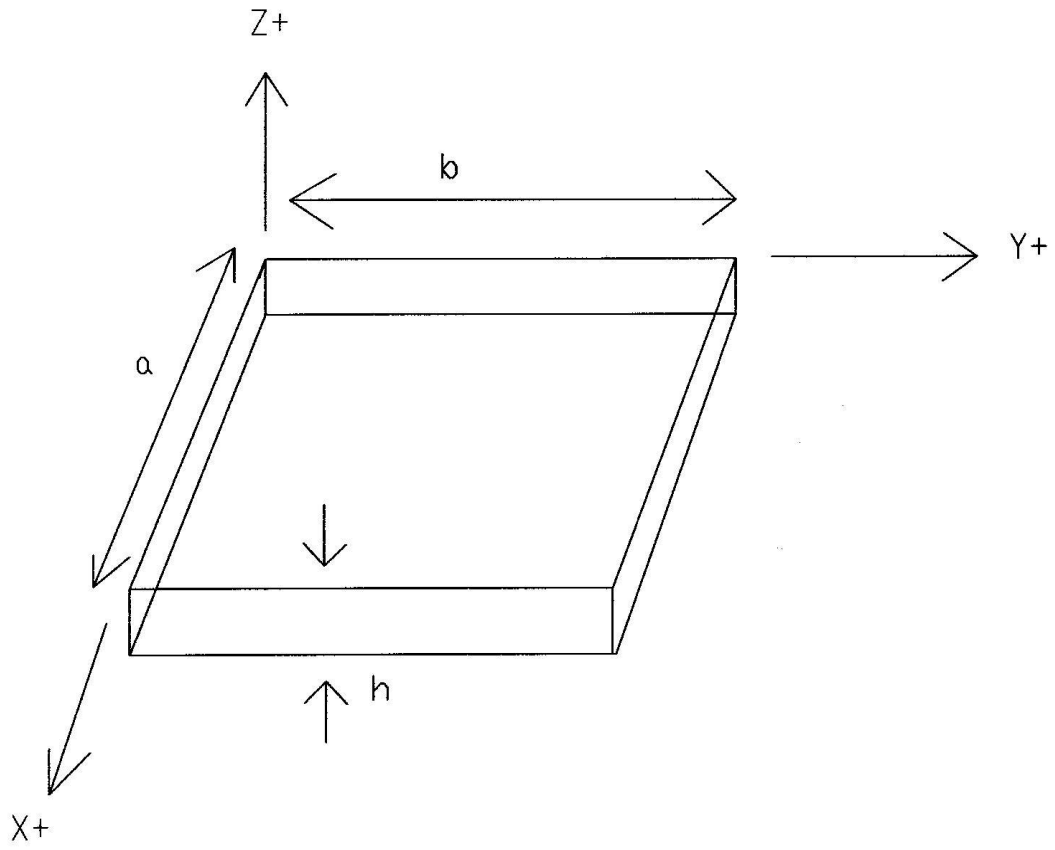
Figure 2.1: Plate Structure

According to Murray (1986), flat plates which are stiffened to different degrees in orthogonal directions (waffle slabs) behave like orthotropic plates, the theory of which was developed by Gening in 1860 and Boussinesq in 1879.

For transverse load Y, the governing equation is:

$$D_x \frac{\partial^4 \omega}{\partial x^4} + 2H \frac{\partial^4 \omega}{\partial x^2 \partial z^2} + D_z \frac{\partial^4 \omega}{\partial z^4} = Y \qquad \ldots\ldots\ldots\ldots.2.3$$

It was developed by Huber in 1914, and known as the "Huber's equation"

Where:

$D_X = (EI)_X / (1 - \upsilon_X \upsilon_Z) =$ average flexural rigidity of the stiffened plate under bending moment, Mx.

$D_Z = (EI)_Z / (1 - \upsilon_X \upsilon_Z) =$ average flexural rigidity of the stiffened plate under bending moment, Mz.

$$H = \tfrac{1}{2}(\upsilon_x D_Z + \upsilon_z D_X) + 2(GI)_{XZ}$$

$$2(GI)_{XZ} = \frac{\partial^2 \omega}{\partial x \partial z} / M_{XZ} = \text{average torsional rigidity} = G_{xz}t^3/12$$

$\upsilon_X \upsilon_Z$ = Poison's ratio in the X- and Z- directions.

$E = \sqrt{(E_x E_z)} =$ modified Young's modulus

$G_{xz} \approx E / 2\left(1 + \sqrt{\upsilon_x \upsilon_z}\right)$ = modified shear modulus.

Huber applied these equations to the analysis of a reinforced concrete slab stiffened by orthogonal ribs (waffle slabs).

Jamal (1998), when studying the effects of shear reinforcements in rib stiffened (waffle) concrete slab, used the orthotropic plate theory to analyse the structures.

This theory assumes that the orthotropy of the structures may be replaced by the orthotropy of the constituent material. Although the actual structural behaviour of a stiffened slab cannot be entirely replaced by that of an equivalent orthotropic slab, previous theoretical and experimental investigations indicated good agreement (El-Sebakhly, 1979).

Abdul-Wahab and Khalil (2000) tested eight large-scale (1/4 scale) models of reinforced concrete waffle slabs with varied rib spacing and rib depth to failure. The results for the rigidities in the elastic-uncracked and elastic-cracked ranges were compared with predicted values obtained from three different methods which included the orthotropic plate theory. The theoretical analysis based on the conventional orthotropic plate theory gave satisfactory predictions but involved an elaborate procedure for determining the torsional rigidities.

2.4 Computer Programs

There have been different computer programs that were developed by different researchers for the analysis of slabs of different shapes and configurations. It has been discovered that most of these programs, except very few, adopted the FEM of analysis of structures. Few of these programs are discussed here especially those that are applicable to the analysis of waffle slabs.

2.4.1 Fortran 77

In their study of the determination of the optimum dimensions of waffle slabs for medium size floors, Prasad *et al* (2005) adopted the computer program for grid analysis, written in Fortran 77 (Formula Translation). Before adopting the said program, waffle slabs were considered as made of grid or grillage beams. The loads

were distributed between longitudinal beams by bending and twisting of transverse beams. The stiffness matrix is developed on the basis of writing joint equilibrium in terms of stiffness co-efficients and unknown joint displacements. Straight members of constant cross-section were considered. The deformations considered were two orthogonal rotations in the horizontal plane and a vertical deflection at each node. Nodal displacements in the horizontal plane and rotations along the vertical axis were not considered keeping in view that they did not significantly contribute to the structural behaviour and hence were ignored.

The computer analysis resulted in determining the moment, shear force and torsion for each of the elements and deflection and rotation about the two orthogonal axes at each of the nodes.

2.4.2 SAP 2000

Structural Analysis Program (SAP) is another computer based program that was used to analyze both steel and concrete structures. Abdel-Karim and Mahmood (2006) used SAP 2000 program to analyze two-way ribbed simply supported rectangular waffle slab models, supported on beams of different stiffnesses. The analysis was done to determine the moment and shear distribution in the ribs and beams in each direction, to study the effect of the panel aspect ratio and the beam stiffnesses on the distribution of moment and shear in each reaction.

The model was a grid system and the ribs were 0.5 m apart in each direction. The slab rested on four pin supports at the corners. The used load was 2 kN/m^2. The results of the analysis gave the shear force and moment values at each joint from which the load factors could be calculated using empirical equations such as:

$$M_a = C_a\,wl_a{}^2\,/8 \qquad\qquad\qquad \ldots\ldots\ldots\ldots 2.\,4$$

Where:

$C_a=$ *moment load factor in certain direction.*

$W =$ *uniform load per metre square.*

$l_a =$ *span length.*

From the results of analysis, it was discovered that the distribution of moments and shear in each direction depended on the panel aspect ratio and the perimeter beam stiffnesses. In actual fact, these factors depended on the relative stiffnesses of the perimeter beams to the slab or ribs stiffnesses and not the absolute stiffnesses of the beams.

The beam shear load factors in each direction were calculated by the shear values in the beams from the results of the analysis using SAP 2000. Part of the conclusion reached was that, the results of the research could be used to calculate the moments and shears in the perimeter beams and in the ribs in each direction for the given aspect ratio; panel dimension; slab thickness, and beams dimensions for a given slab load.

2.4.3 Adapt Floor Program

This program also adopts FEM for the analysis of waffle slabs. It was found to be good for the analysis and design of post-tensioned waffle slabs, in which iron rod is replaced by cables that are under tension as in pre-stressed concrete design. The cables were placed between the waffle ribs and held under tension, while wire mesh was used in the slab area (Adapt Technical Note, 2006).

2.4.4 SAFIR Program

In his work on membrane action in fire exposed concrete floor system, Lim (2003) used this program to analyse slabs that are subjected to fire.

SAFIR is a non-linear finite element program which was developed at the University of Liege, Belgium, and is based on an earlier program, CEFICOSS. It consists of thermal and structural analysis components integrated into a single program. The thermal analysis component was used to determine the temperature distributions of the structural members which were used in the structural analysis. SAFIR's structural analysis capabilities include 2D and 3D analysis of steel, concrete and composite members and can account for geometrical and material non-linearity.

2.4.5 RCC Program

The Reinforced Concrete Council (RCC) also developed a computer program that adopted the Microsoft excel package to analyse and design different civil engineering structures based on BS 8110, and the results of this program have been found to be appropriate.

2.4.6 ETABS

ETABS is a program that can greatly enhance an engineer's analysis and design capabilities for structures. Part of that power lies in an array of options and features. The other part lies on the simplicity of its use.

The basic approach for using the program is very straightforward. The user establishes grid lines, places structural objects relative to the grid lines using points, lines and areas, and assigns loads and structural properties to those structural objects. (For example, a line object can be assigned section properties; a point object can be assigned

spring properties; an area object can be assigned slab or deck properties). Analysis and design are then performed based on the structural objects and their assignments. Results are generated in graphical or tabular form that can be forwarded to a printer or to a file for use in other programs. (ETABS User guide, 2005).

2.5 The Yield Line Theory

Yield line theory is an ultimate load analysis. It establishes either the moments in an element (e.g. a load) at the point of failure or at which an element will fail. Yield lines not only signify the location of maximum principal moments (where yielding occurs), but also the location of zero shears (Gohnert, 2006). The shapes and locations of Yield zone are affected by the support structures and their rigidity (Geng *et al*, 2006). The theory gives a conservative estimate of strength (Ferguson, 1965). It may be applied to many slabs both with and without beams. It can deal with openings, holes, irregular shapes and with any support condition (Punmia *et al*, 2006; Chee *et al* 2008). Yield line design is a plastic method: it is different from 'normal' elastic method (Kennedy and Goodchild, 2004).The technique requires the postulation of a kinematically admissible yield-line or fracture pattern from which the corresponding collapse load is determined through the principle of virtual displacements (Ramsay and Johnson, 1997).

2.5.1 Energy dissipation

In order to calculate the load-carrying capacity from an upper bound solution, the energy dissipation has to be known. In yield line theory, it is the energy dissipated that is used in the analysis of the slabs, because both the external energy and internal energy dissipated must be considered.

Gudmand-Høyer (2003) calculated dissipation in a yield line on the basis of Coulomb yield condition for concrete in order to verify K.W. Johansen's method. An effort was made to evaluate the error made using Johansen's proposal for orthotropic rectangular slabs and it was found that the method is sufficiently correct for practical purposes. Also, for deflected slabs that are believed to have a high load carrying capacity, it was assumed that the axis of rotation corresponds to the neutral axis of a slab part and dissipation was found from the moment capacities about these axes. The Johansen's proposal was also used to find the load carrying capacities in these cases. He compared his results with that of numerical calculations of the dissipation and generated some numerical equations for energy dissipation in slabs, based on contribution from concrete and the reinforcements. Some of the equations are shown below in conjunction with Figures 2.2, 2.3 and 2.4.

If the axes of rotation for two slab parts are not at the same depth measured from the slab surface, the relative displacement discontinuity is no longer perpendicular to the yield line. The angle between the displacement discontinuity and the yield line changes with the depth from the slab surface and this must be taken into account when calculating the dissipation (Figure 2.2).
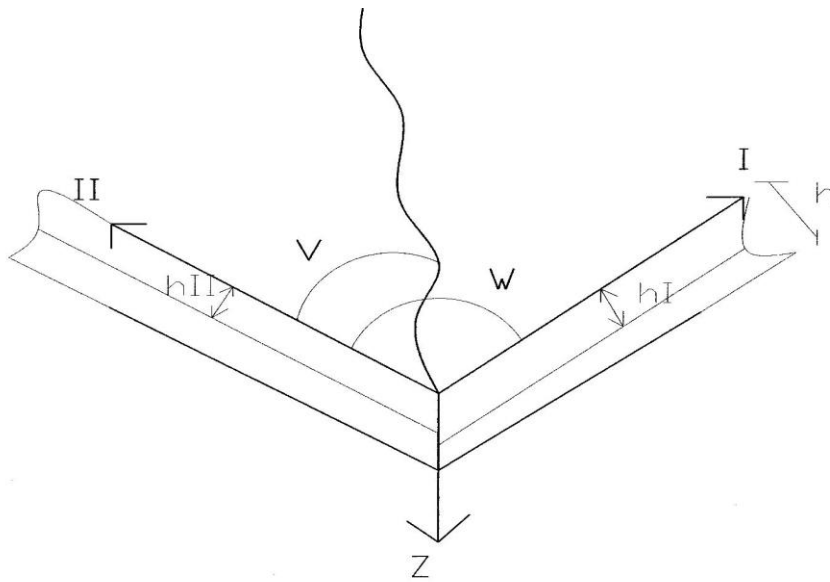
Figure 2.2: Yield line from the angle of a slab (Source: Gudmand-Hoyer 2003)
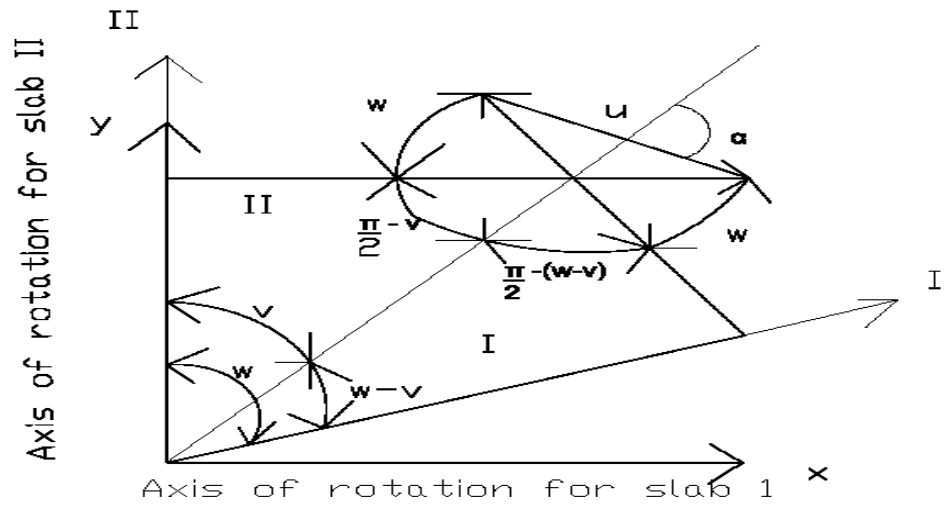
Figure 2.3: Displacement for slab parts (Source: Gudmand-Hoyer 2003)
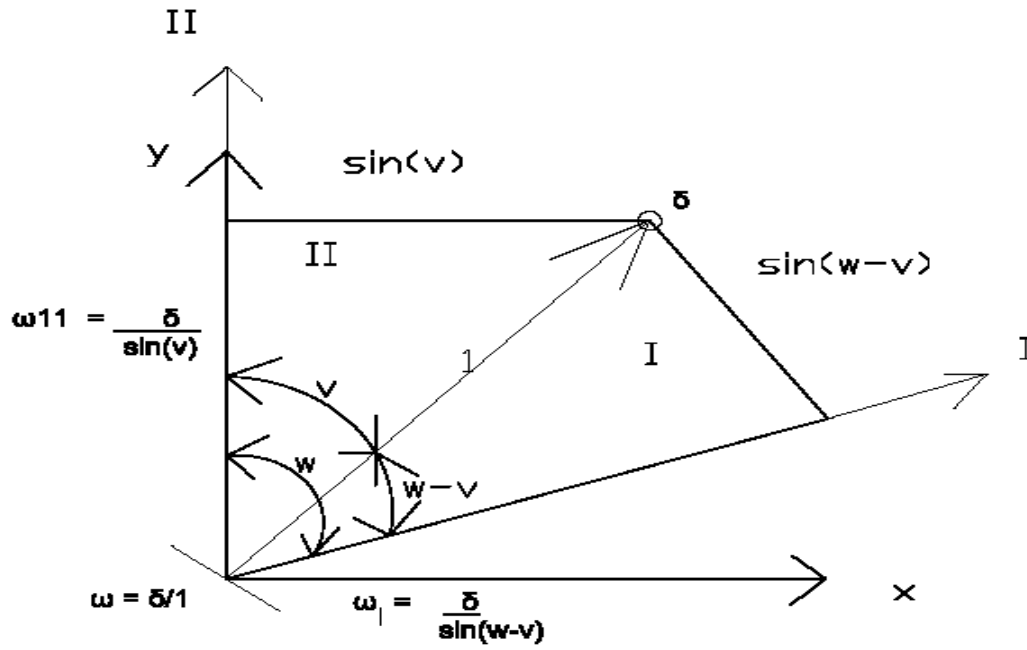
Figure 2.4: Geometry relation between rotations   (Source: Gudmand-Hoyer 2003)

2.5.2 The contribution from the concrete:

The concrete contribution from the yield line was calculated from the dissipation formulas for plane stress assuming a modified Coulomb material. Setting the tensile strength of concrete to zero, the contribution to the dissipation (per unit length) from the concrete was calculated as:

$$\mathbf{W_c} = \int_0^h \frac{1}{2} f_c u(1 - \sin(\alpha)) dx \qquad \ldots\ldots\ldots\ldots \ 2.5$$

u being the relative displacement and α the angle between the displacement and the yield line, Figure 2.3.

But u and α depend on z, which is the depth from the top surface to the point considered. u_1 and u_{II} are the displacements of slab part I and II, respectively, u was calculated as:

$$u = \sqrt{u_I^2 + u_{II}^2 + 2u_I u_{II} \cos(w)} \qquad \ldots\ldots\ldots\ldots \ 2.6$$

The angle between the displacement and the yield line α varies with respect to u_I and u_{II} depending on whether they are positive or negative.

The relationship between the two rotations about *I* and *II* were found from the geometrical conditions demanding the same displacement at a point of yield line.

From Figure 2.4, the rotations may be calculated as:

$$\omega_I = \frac{\omega}{\sin(w-v)}$$

<div align="right">…………. 2.7</div>

$$\omega_{II} = \frac{\omega}{\sin(v)}$$

Here ω is the rotation of slab part line about an axis along the yield line.

In the calculation of the displacement, it was assumed that the rotation is small and the displacement may therefore be calculated as the product of rotation and the height. The displacements u_I, u_{II} and u was calculated as:

$$u_I = h\frac{\omega}{\sin(w-v)}\left(\frac{h_I}{h} - \frac{z}{h}\right) \Leftrightarrow$$

<div align="right">…………… 2.8</div>

$$\frac{u_I}{h\omega} = \frac{1}{\sin(w-v)}\left(\frac{h_I}{h} - \frac{z}{h}\right)$$

$$u_{II} = h\frac{\omega}{\sin(v)}\left(\frac{h_{II}}{h} - \frac{z}{h}\right) \Leftrightarrow$$

<div align="right">………….. 2.9</div>

$$\frac{u_{II}}{h\omega} = \frac{1}{\sin(v)}\left(\frac{h_{II}}{h} - \frac{z}{h}\right)$$

.

Inserting (2.8) and (2.9) into (2.6) leads to:

$$u = \sqrt{\left(h\frac{\omega}{\sin(w-v)}\left(\frac{h_I}{h}-\frac{z}{h}\right)\right)^2 + \left(h\frac{\omega}{\sin(v)}\left(\frac{h_{II}}{h}-\frac{z}{h}\right)\right)^2 \atop + 2\left(h\frac{\omega}{\sin(w-v)}\left(\frac{h_I}{h}-\frac{z}{h}\right)\right)\left(h\frac{\omega}{\sin(v)}\left(\frac{h_{II}}{h}-\frac{z}{h}\right)\right)\cos(w)} \Leftrightarrow$$

.........2.10

$$\frac{u}{h\omega} = \sqrt{\left(\frac{\left(\frac{h_I}{h}-\frac{z}{h}\right)}{\sin(w-v)}\right)^2 + \left(\frac{\left(\frac{h_{II}}{h}-\frac{z}{h}\right)}{\sin(v)}\right)^2 + \frac{2\left(\frac{h_I}{h}-\frac{z}{h}\right)\left(\frac{h_{II}}{h}-\frac{z}{h}\right)\cos(w)}{\sin(w-v)\sin(v)}}$$

The angle α varies and if the situation in which:

($u_I > 0$ & $u_{II} > 0$): is considered, then α can be calculated as:

$$\alpha = -\left(\frac{\pi}{2}+v-Arc\cos\left(\frac{u^2+u_{II}^2-u_I^2}{2u_{II}u}\right)\right)$$

.........2.11

It is seen that the contribution to the dissipation from the concrete is a function of both the position of the axis of rotation h_I, h_{II}, and h, the rotation ω and the compressive strength, f_c. The dissipation may be calculated in a dimensionless form as:

$$\frac{W_c}{h^2\omega f_c} = \int_0^1 \frac{1}{2}\frac{u}{h}(1-\sin(\alpha))d\frac{z}{h}$$

.... 2.12

2.5.3   Contribution from reinforcement

Gudmand-Høyer went further to determine the contribution from the reinforcement, based on Figure 2.5. If the reinforcement is placed in a direction perpendicular to the axis of rotation at a distance from the slab surface as shown in Figure 2.5, the contribution from the reinforcement to the dissipation per unit length becomes:

$$W_S = \omega_I \cos(w-v)\left(A_{s,I}\left|h - h_{cI} - h_I\right| + A_{s,I}\left|h_I - h_{cI}\right|\right) + \\ \omega_2 \cos(v)\left(A_{s,II}\left|h - h_{cII} - h_{II}\right| + A_{s,II}\left|h_{II} - h_{cII}\right|\right)$$

…………… 2.13

Where ω_I and ω_{II} are the rotations about axis I and II, respectively. These are determined in Equation 2.6 and the expression may be written as:

$$\frac{W_s}{\omega h^2 f_c} = \frac{1}{\tan(w-v)}\left(\Phi_{0,I}\left|1 - \frac{h_{cI}}{h} - \frac{h_I}{h}\right| + \Phi_{0,I}\left|\frac{h_I}{h} - \frac{h_{cI}}{h}\right|\right) + \\ \frac{1}{\tan(v)}\left(\Phi_{0,II}\left|1 - \frac{h_{cII}}{h} - \frac{h_{II}}{h}\right| + \Phi_{0,II}\left|\frac{h_{II}}{h} - \frac{h_{cII}}{h}\right|\right)$$

…………… 2.14

If the corner is right-angled the dissipation becomes:

$$\frac{W_s}{\omega h^2 f_c} = \tan(v)\left(\Phi_{0,I}\left|1 - \frac{h_{cI}}{h} - \frac{h_I}{h}\right| + \Phi_{0,I}{}'\left|\frac{h_I}{h} - \frac{h_{cI}}{h}\right|\right) + \\ \frac{1}{\tan(v)}\left(\Phi_{0,II}\left|1 - \frac{h_{cI}}{h} - \frac{h_{II}}{h}\right| + \Phi_{0,II}{}'\left|\frac{h_{II}}{h} - \frac{h_{cII}}{h}\right|\right)$$

………….2.15

Finally, the results from the theoretical equations derived were compared with those obtained from laboratory test specimens, where some slabs were loaded to failure.

It was discovered that the theoretical load – carrying capacity was too high. This phenomenon was ascribed to the fact that concrete does not behave entirely according to plastic theory and an effectiveness factor was proposed to make the theory reliable.

Beside the load-carrying capacity, it was discovered that the deflection was wrong. Not only wrong when it came to numerical value, but also when it came to the relationship between axial force and deflection. However, the comparison with test results showed that the theory developed may be used if the deflection at failure is known and a proper effectiveness factor is introduced. But if deflection at failure is not known, a conservative simplified method, which will lead to a large underestimation for low axial forces was proposed, the method was therefore recommended as rough estimate.
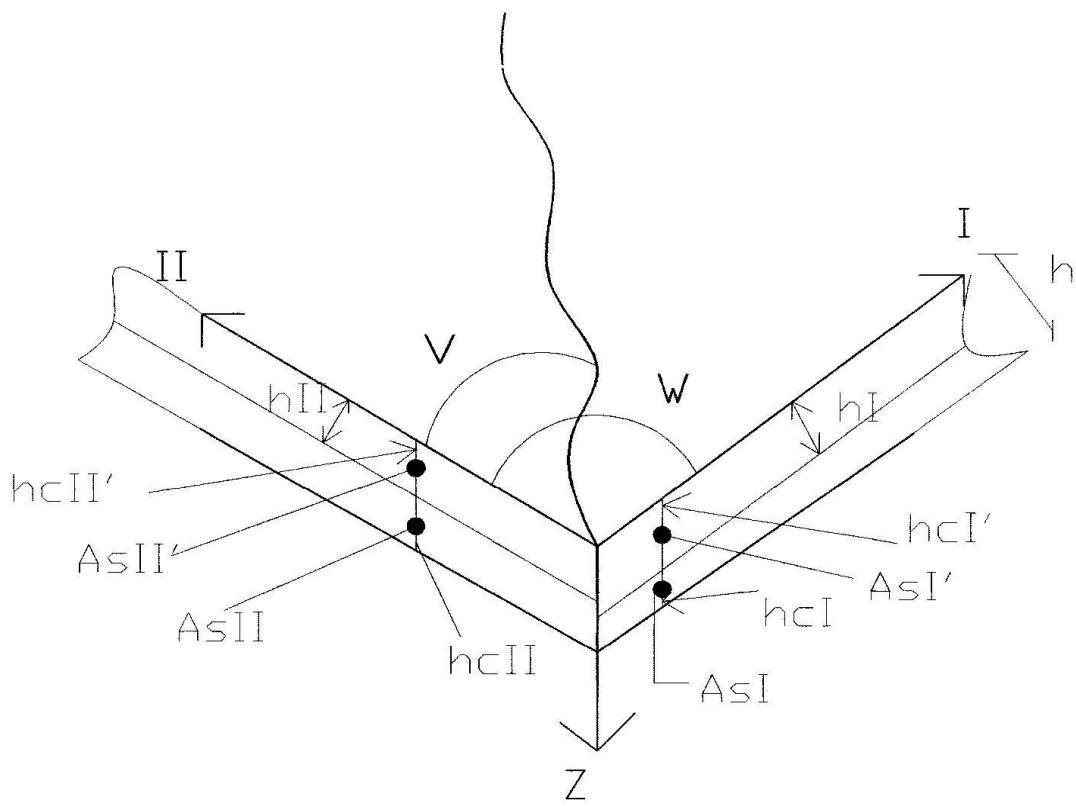
Figure 2.5: Reinforcement arrangement

2.6 Isotropic and Orthotropic Slabs

The arrangement of reinforcements in slabs has given rise to both isotropic and orthotropic slabs. An isotropic slab is one with the same amount of bottom reinforcement both ways. This is because the yield moments on both ways are equal, and by assuming equal effective depth, this type of slab is easy to deal with when using the yield line analysis and design method. Orthotropic slabs have different amounts of reinforcements in the two directions; in this case, the yield moments in both axes are mutually perpendicular to each other (Nilson, 1997: Buyukozturk, 2004).

According to Kennedy and Goodchild (2004), the analysis of such orthotropic slabs can be done using the affine transformation. In these, the stronger direction is assumed to have the moment capacity, M, and in the weaker direction the capacity of the slab is assumed to be μM. The value of μ is usually based on the relative amounts of reinforcement the designer wishes to use in the two directions. The use of μ will help to transform the orthotropic slabs to isotropic slabs and can be treated as such, applying all the usual formulae and methods.

2.7 Definition of Upper Bound and Lower Bound Theorem

i.   The lower bound of the true collapse load is that external load for which a distribution of moments can be found satisfying the requirements of equilibrium and boundary conditions so that the moments at any location do not exceed the yield moment.

ii.   The upper bound of the true collapse load is that external load for which the internal work done by the slab for a small increment of displacement is equal to the

external work done by that external load for the same amount of small increment of displacement.

Thus, the collapse load satisfying the lower bound theorem is always lower than or equal to the true collapse load. On the other hand, the collapse load satisfying the upper bound theorem is always higher than or equal to the true collapse load. The yield line analysis is an upper bound method in which the predicted failure load of a slab for given moment of resistance (capacity) may be higher than the true value. Thus, the solution of the upper bound method (yield line analysis) may result into unsafe design if the lowest mechanism could not be chosen. However, it has been observed that the prediction of the most probable true mechanism in slab is not difficult. Thus, the solution is safe and adequate in most of the cases (Kharagpur, 2009).

2.8 Difference in Load Distribution in Waffle Slabs and Solid Slabs

Load distribution in waffle slabs and solid slabs are different due to the orientation of these slabs. Waffle slabs have hollows in between the slabs, while solid slabs have no hollow in between. The arrangement of reinforcements in both slabs is different as a result of the hollow in the waffle slabs. Due to the arrangements of reinforcement and the span ratio of solid slab, there is possibility for one-way slab, however, waffle slabs do not have a one-way slab because of the reinforcement orientation and the presence of ribs within the slab. In a typical one-way slab, the floor comprises beams spanning in one direction between columns and a slab spanning between them. The system is designated as one-way slab because all loads in the slab are transferred primarily in one direction (to the beams), since the transverse slab span is infinite.

For a typical two-way slab, the floor comprises beams spanning both directions between columns, and a slab framing between the beams. The system is designated two-way because, in a situation where the aspect ratio of the slab has a value near unity, the slab transfers load to the beams in two directions. This type of floor system is generally quite efficient.

2.8.1 Load path designation in waffle slabs

In waffle slabs, loads are transferred from the slabs to the ribs and to the perimeter beams. Abdel-Karim and Mahmood (2006), when investigating the effect of beam stiffnesses on the load distribution in waffle slabs, was able to discover that the distribution of moments along the parallel ribs in the short and long directions depends, on the panel aspect ratio, the relative stiffness of the slab ribs and on the beams on which these ribs are supported.

From the model analysis of slabs, using SAP2000, the following observations were made. It was noticed that, as the beam stiffness was increased, the shear load factor in the short direction increased for a specified panel aspect ratio. Also, as the panel aspect ratio was increased, the beam shear load factor increased in the short direction and decreased in the long direction. It was also observed that moments varied in each direction along the panel width. It was seen that edge strip ribs had larger moments than the middle strip ribs for beams with small cross section (moment of inertia: stiffness) and the middle strip ribs had larger moments than the edge strip ribs for larger beams cross section(moment of inertia: stiffness). Hence, the ribs gave different bending moments and shear forces in each direction as a result of varying stiffnesses. The orientation and the amount of the reinforcement provided depended on

the bending moments (the function of the reinforcements is to resist the bending moment).

2.8.2 Load path designation in solid slabs

Reinforced concrete is very unique in it behaviour, and this has made it popular as construction material. In solid slabs; at flexural failure, concrete slabs develop hinge lines. A hinge line causes much of the reinforcement passing through it to resist the moment along it length, contributing to the safety of the slab.

Once a slab has cracked, the reinforcement determines the manner in which the applied load is resisted; it is the orientation and the amount of reinforcement that govern the path that the load takes to the supports (Aalami, 2005).

Prior to the calculation of the design moments and shears, the first thing that must be considered is to anticipate the load path, which set the orientation and position of the reinforcement. Sivagamasundari and Kumara, 2008, opined that the major work of the longitudinal reinforcement is to provide flexural strength for the concrete slab. For example, in a solid two-way slab, the function of the distribution bar is to distribute the load from the slab to the bottom or main bar, while the bottom bar will distribute the load to the supports at the edges of the slab, both the distribution and main bars are designed for in this type of slab. The amount of bending moment in each direction will depend on the ratio of the two spans and the condition of restraint at each support (Mosley and Bungey, 1990), while in one-way slab it is only the main bars that is designed, although appropriate provision is made for distribution bar in this type of slab. Top (torsion) reinforcement is provided at the supports or edges of slabs to prevent cracks as concrete is known to be weak in tension (BS 8110, 1997).

<center>**CHAPTER THREE**</center>

<center>**METHODOLOGY**</center>

3.1 Preamble

Mathematical equations were imputed into the developed computer program and used to analyse developed models, while basic experimentation was carried out to determine, the deflection, crack width and the ultimate failure of the various types of slab models.

3.2   Mathematical Formulation for the Experimental Studies

The bending moment formula based on the yield line theory was derived as given below. These formulae have been taken from the solutions given by Johansen in 1943 and 1968. The solution below is for a one-way simply supported slab on two sides, using the work method of analysis (Figure 3.1)

3.2.1 Analysis of Slabs adopting Yield Line Theory

Assuming:

Internal work done, E = External work done, D

$$\sum \left(N \times \delta \right) = \sum \left(m \times l \times \theta \right)$$

$$2 \times n \times \frac{L}{2} \times w \times \frac{\delta_{max}}{2} = 2 \times m \times l \times \theta$$

Here, the length of the projected yield line, *'l'*, onto the axis of rotation is *w*.

Also θ, equate to Tan $\theta = \delta_{max} / (L/2)$.

Therefore:

$$\frac{2nLw}{2} \times \frac{\delta_{\max}}{2} = 2mw \times \frac{\delta_{\max}}{L/2}$$
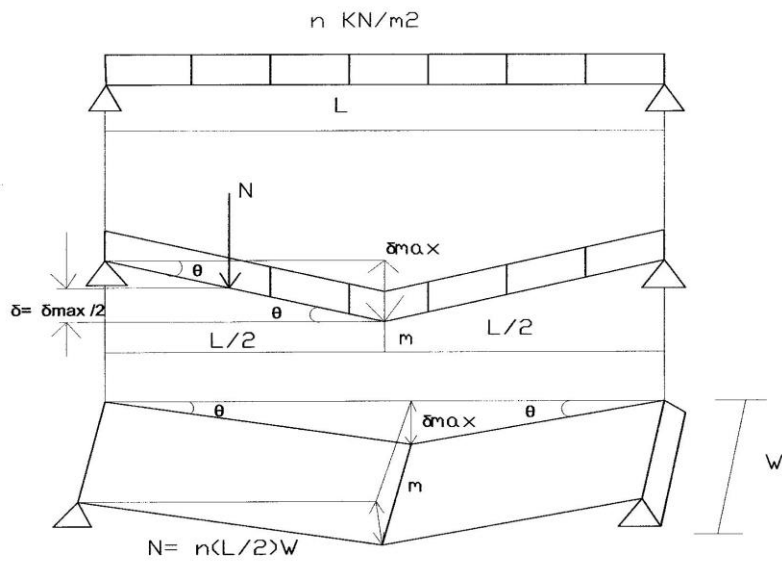
Figure 3.1: Simply supported one-way slab

$$\therefore \frac{2nL}{4} = \frac{4m}{L}$$

Rearranging

$$m = \frac{nL^2}{8} \qquad\qquad\qquad \text{……………. 3.1}$$

For two way slabs, the equation can be developed using Fig 3.2, as follows,

(Kong and Evans, 1987):

Assuming the slab measures b and L.

$$E = D$$
$$\sum(N \times \delta) = \sum(m \times L \times \theta)$$

E = external energy.

$$E = N\left[2 \times \frac{1}{3} b\alpha L + \frac{1}{2} b(1 - 2\alpha)L\right]$$

$$= \frac{NbL}{6}(3 - 2\alpha) \qquad \text{………………..3.2}$$

D= internal energy.

Energy dissipation for yield line AE

$$= m\left[\frac{\alpha L}{b/2} + \frac{b/2}{\alpha L}\right]$$

Energy dissipation for yield line EF

$$= m\left[2 \times \frac{(1-2\alpha)L}{b/2}\right]$$
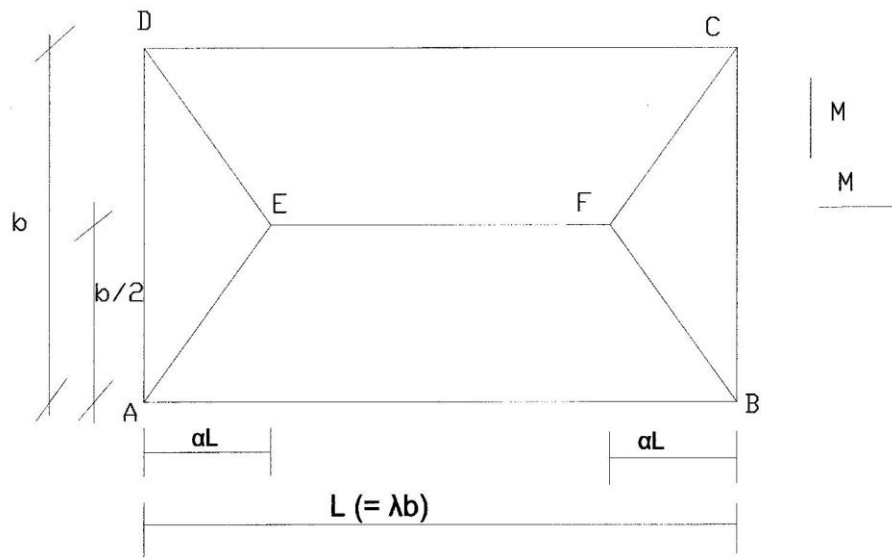
Figure 3.2: Yield lines on two-way simply supported slab

Therefore, total energy dissipation for yield lines AE, DE, BF, CF, and EF

$$= 4m\left[\frac{\alpha L}{b/2} + \frac{b/2}{\alpha L}\right] + 2m\frac{(1-2\alpha)L}{b/2}$$

$$= \frac{2m(1+2\lambda^2\alpha)}{\lambda\alpha} \qquad \text{(Where } \lambda = l/b) \ldots\ldots\ldots\ldots \quad 3.3$$

Equating   (3.2) and (3.3)

$$D = E$$

$$\frac{2m(1+2\lambda^2\alpha)}{\lambda\alpha} = \frac{NbL}{6}(3-2\alpha)$$

$$m = \frac{NL^2}{12}\cdot\frac{(3-2\alpha)\alpha}{1+2\lambda^2\alpha} \qquad\qquad \ldots\ldots\ldots\ldots.. \quad 3.4$$

Equations 3.1 and 3.4 have been simplified by Kennedy and Goodchild (2004), to accommodate different boundary conditions expected in a slab. Equation 3.1 was simplified to 3.5, while 3.4 became 3.6. Furthermore, equation 3.6 has been modified in this work to equation 3.7, in order to take care of the conservative nature of yield line theory in solid slabs, (Akinyele 2011). Equations 3.5 and 3.7 were therefore adopted in the computer program in Tables 3.1, 3.2 and 3.3.

$$m = \frac{nL^2}{2\left(\sqrt{1+i_1} + \sqrt{1+i_2}\right)^2} \qquad\qquad \ldots\ldots\ldots\ldots.3.5$$

$$m = \frac{n \times a_r \times b_r}{8\left(1 + \dfrac{b_r}{a_r} + \dfrac{a_r}{b_r}\right)} \qquad\qquad \dots\dots\dots\dots\dots 3.6$$

$$m = \frac{n \times a_r \times b_r}{8\left(1 + \dfrac{b_r}{a_r} + \dfrac{a_r}{b_r}\right)}(k + v) \qquad\qquad \dots\dots\dots\dots\dots\dots..3.7$$

Where:

N = load acting within a particular region (kN)

δ = vertical displacement of the load N on each region expressed as a fraction of

   Unity (m)

m = ultimate design moment of resistant for the slab (kNm)

L = length of slab (m)

θ = rotation of the region about its axis of rotation (m/m)

n = the ultimate uniformly distributed load (kN/m^2)

a_r = reduced short span dimension

b_r = reduced long span dimension

k = is a constant = 1

v = is a variable = $0 \leq 0.5$

i = the fixity ratio at supports, e.g i_1, i_2.

If : $i_1 = i_2 = 1$, that support is a continuous support

And $i_1 = i_2 = 0$ that support is a simple support.

Table 3.1: Template for the computer analysis and design of simply supported waffle slab adopting the yield line theory.

	References	Calculation/output
1	ANALYSIS AND DESIGN OF WAFFLE SLABS USING THE YIELD LINE THEORY	
2	CODE BS. 8110 PARTS 1 1997.	
3	Designed by: AKINYELE J.O	
4	Simply supported on all Four Sides	
5	Slab Long Span (ly) (m)	0.00
6	Slab Short Span (lx) (m)	0.00
7	Rib Width (mm)	0.00
8	Distance between ribs on Long Span (m)	0.00
9	Distance between ribs on Short Span (m)	0.00
10	Depth of Slab portion (mm)	0.00
11	Depth of ribs (mm)	0.00
12	Total depth of Slab (mm)	=SUM(H10,H11)
13	Effective depth of slab (mm)	=H10-H29
14	Slab dead load (kN/m^2)	=H10*24*10^-3*1.4
15	Imposed load on Slab (kN/m^2)	0.00
16	Finishes and partition loads	0.00
17	Total load on slab (kN/m^2)	= SUM(H14:H15:H16)
18	For discontinuous supports, fixity	0.00
19	For continuous supports on any side, fixity	1
20	Fixity ratio for short span left. I_1	0.00
21	Fixity ratio for long span bottom. I_2	0.00
22	Fixity ratio for short span right. I_3	0.00
23	Fixity ratio for long span top. I_4	0.00
24	Reduced side a_r	=2*H6/(SQRT(1+H21)+SQRT(1+H23))
25	Reduced side b_r	=2*H5/(SQRT(1+H20)+SQRT(1+H22))
26	Bending moment in slab (kNm)	=1.0*H17*H24*H25/(8*(1+(H25/H24)+(H24/H25)))
27		
28	DESIGN	
29	Cover to reinforcement (mm)	0.00
30	Concrete characteristic strength (N/mm^2)	0.00
31	Steel characteristic strength (N/mm^2)	0.00
32	Maximum lever arm factor	0.95

33	Maximum K-value	0.156
34	Moment M	=H26*1.0E06
35	K-value	=H34/(H30*1.0E03*H13^2)
36	Lever arm factor	=0.5+SQRT(0.25-H35/0.9)
37	Used Lever arm	=MIN(H32,H36)
38	Area of Steel (mm^2)	=H34/(0.95*H31*H37*H13)
39	PROVIDE=	
40	ANALYSIS & DESIGN OF RIBS	
41	Adopting Rankine Grashoff theory	
42	Number of ribs on long span	0.00
43	Number of ribs on short span	0.00
44	Weight of slab (kN/m^2)	=H17
45	Total load on slab (kN)	=H44*H5*H6
46	Weight of ribs kN/m	=(H7*H11)*1.0E-06*24
47	Total weight of ribs on long span (kN)	=H42*H46*H5
48	Total weight of ribs on short span (kN)	=H43*H46*H6
49	Total weight of Finishes (kN)	=H5*H6*1.0
50	Total Live load (kN)	=H5*H6*1.5
51	Total load on grid floor (kN)	=H45+H47+H48+H49+H50
52	Q= load per area (W2)	=H51/(H5*H6)
53	Denominator for Qx and Qy	=SUM(H5^4,H6^4)
54	Load sheared on short span Qx (kN/m^2)	=H52*(H5^4/H53)
55	Load sheared on long span Qy (kN/m^2)	=H52*(H6^4/H53)
56	Moment on short span Mx (kNm)	=(H54*H9*H6^2)/8
57	Moment on Long Span My (kNm)	=(H55*H8*H5^2)/8
58	Shear force on short span Vx (kN)	=(H54*H8*H6)/2
59	Shear force on long span Vy (kN)	=(H55*H9*H5)/2
60	DESIGN	
61	Steel characteristic strength (N/mm^2)	0.00
62	Short span design moment (kNm)	=H56*1.0E06
63	Cover to ribs	0.00
64	Effective depth of ribs	=H11-H63
65	K-value	=H62/(H30*H7*H64^2)
66	Lever arm factor	=0.5+SQRT(0.25-H65/0.9)
67	Used lever arm	=MIN(H66,H32)
68	Area of steel (mm^2)	=H62/(0.95*H61*H67*H64)
69	PROVIDE	

70	Long span design moment (kNm)	=H57*1.0E06
71	K-value	=H70/(H30*H7*H64^2)
72	Lever arm factors	=0.5+SQRT(0.25-H71/0.9)
73	Used laver arm	=MIN(H72,H32)
74	Area of steel (mm^2)	H70/(0.95*H61*H73*H64)
75	PROVIDE	
76	END DESIGN	

Table 3.2: Template for the computer analysis of simply supported waffle slab adopting the yield line theory (3 sides simply supported)

	References	Calculation / output
1	ANALYSIS AND DESIGN OF WAFFLE SLAB USING THE YIELD LINE THEORY	
2	CODE B.S. 8110 PART 1 1997.	
3	Designed by: Akinyele J.O	
4	Supported on 3-sides	
5	Slab long Span (ly) (m)	0.00
6	Slab short Span (lx) (m)	0.00
7	Depth of slab part (mm)	0.00
8	Distance between ribs on long span (m)	0.00
9	Distance between ribs on short span (m)	0.00
10	Rib width (mm)	0.00
11	Depth of ribs (mm)	0.00
12	Total depth of slab (mm)	=SUM(H7,H11)
13	Cover to reinforcements (mm)	0.00
14	Effective depth of slab (mm)	=H7-H13
15	Slab Dead load (kN/m^2)	=H7*24*10^-3*1.4
16	Slab Imposed load (kN/m^2)	0.00
17	Finishes and Partition loads	0.00
18	Total load on Slab (kN/m^2)	=SUM(H15:H16:H17)
19	CASE 1 $= h_1 + h_2 < b$	
20	Fixity for 1st short Span I_1	0.00
21	Fixity for supported long span I_2	0.00
22	Fixity for 2nd short span I_3	0.00
23	Reduced span b_r	=2*H5/(SQRT(1+H20)+SQRT(1+H22))
24	K'= redistribution factor	=2*H6/(3*H23)
25	.h = factor to determine yield length	=H6/H24+SQRT(H24^2+(H21+H23+H24/2*H6)+1)
26	Bending moments in slab (kN.m)	=1.0*(H18*H6*H23)/8*(((H21*H23)/4*H6)+(H6/H25))
27	CASE 2 $= h_1 + h_2 = b$	
28	.b' = dimensions	=2*H6/SQRT(1+H21)
29	Bending moments in slab (kN.m)	=1.0*(H18*H23*H28)/8*(1+(H28/H23)+(H23/H28))
30	Design	

31	Concrete characteristic strength	0.00
32	Steel characteristic strength	0.00
33	Maximum lever arm factor	0.95
34	Maximum K-value	0.156
35	Used bending moment	=MAX(H26,H29)
36	Moments M	H35*1.0E06
37	K-value	=H36/(H31*1.0E03*H14^2)
38	Lever arm factor	=0.5+SQRT(0.25-H37/0.9)
39	Used laver arm	=MIN(H33,H38)
40	Area of steel (mm^2)	=H36/(0.95*H32*H39*H14)
41	Provide	
42	Analysis and design of ribs	
43	ADOPTING RANKINE GRASHOFF THEORY	
44	No of ribs on long span (ly)	0.00
45	No of ribs on short span (lx)	0.00
46	Weight of slab (kN/m^2)	=H18
47	Total load on slab (kN)	=H46*H5*H6
48	Weight of ribs (kN/m)	=(H10*H11)*1.0E-06*24
49	Total weight of ribs on long span(kN)	=H44*H48*H5
50	Total weight of ribs on short span(kN)	=H45*H48*H6
51	Total weight of finishes (kN)	=H5*H6*1
52	Total Live Load (kN)	=H5*H6*1.5
53	Total load on grid floor (kN)	=H47+H49+H50+H51+H52
54	.q = load per area (m^2)	=H53/(H5*H6)
55	Denominator for qx and qy	=SUM(H5^4,H6^4)
56	Load sheared on short span qx(kN/m^2)	=H54*(H5^4/H55)
57	Load sheared on long span qy (kN/m^2)	=H54*(H6^4/H55)
58	Moments on short span Mx (kNm)	=(H56*H9*H6^2)/8
59	Moments on long span My (kNm)	=(H57*H8*H5^2)/8
60	Shear force on short span Vx (kN)	=(H56*H8*H6)/ 2
61	Shear force on long span Vy (kN)	=(H57*H9*H5)/ 2
62	Design	
63	Steel characteristic strength (N/mm^2)	0.00
64	Short span design moment (kNm)	=H58*1.0E06
65	Cover to ribs	0.00
66	Effective depth of ribs	=H11-H65

67	K-value	=H64/(H31*H10*H66^2)
68	Lever arm factor	=0.5+SQRT(0.25-H67/ 0.9)
69	Used lever arm	=MIN(H33,H68)
70	Area of steel (mm^2)	=H64/(0.95*H63*H66*H69)
71	Provide	
72	Long span design moments (kNm)	=H59*1.0E06
73	k-value	=H72/(H31*H10*H66^2)
74	Lever arm factor	=0.5+SQRT(0.25-H73/ 0..9)
75	Used lever arm	=MIN(H74,H33)
76	Area of steel (mm^2)	=H72/(0.95*H63*H75*H66)
77	Provide	
78	END DESIGN	

Table 3.3: Template for computer analysis of waffle slabs, supported on two short sides, adopting the yield line theory.

	References	Calculation / output
1	ANALYSIS AND DESIGN OF WAFFLE SLAB USING THE YIELD LINE THEORY	
2	CODE B.S. 8110 PART 1 1997.	
3	Designed by: Akinyele J.O	
4	Simply supported on two sides	
5	Slab long Span (ly) (m)	0.00
6	Slab short Span (lx) (m)	0.00
7	Depth of slab part (mm)	0.00
8	Distance between ribs on long span (m)	0.00
9	Distance between ribs on short span (m)	0.00
10	Rib width (mm)	0.00
11	Depth of ribs (mm)	0.00
12	Total depth of slab (mm)	=SUM(H7,H11)
13	Cover to reinforcements (mm)	0.00
14	Effective depth of slab (mm)	=H12-H13
15	Slab Dead load (kN/m^2)	=H7*24*10^-3*1.4
16	Slab Imposed load (kN/m^2)	0.00
17	Finishes and Partition loads	0.00
18	Total load on Slab (kN/m^2)	=SUM(H15:H16:H17)
19	Fixity for 1st support I$_1$	0.00
20	Fixity for 2nd support I$_2$	0.00
21	Bending moments in slab (kN.m)	=H17*H5^2/(2*(((SQRT(1+ H19)) + (SQRT(1+H20)))^2))
22	Design of slab	
23	Concrete characteristic strength	0.00
24	Steel characteristic strength	0.00
25	Maximum lever arm factor	0.95
26	Maximum K-value	0.156
27	Moments M	=H21*1.0E06
28	K-value	=H27/(H23*1.0E03*H14^2)
29	Lever arm factor	=0.5+SQRT(0.25-H28/ 0.9)
30	Used lever arm	=MIN(H25,H29)
31	Area of steel (mm^2)	=H27/(0.95*H24*H30*H14)
32	Provide	

33	ANALYSIS & DESIGN OF RIBS	
34	Adopting Rankine Grashoff theory	
35	Number of ribs on long span	0.00
36	Number of ribs on short span	0.00
37	Weight of slab (kN/m^2)	=H18
38	Total load on slab (kN)	=H37*H5*H6
39	Weight of ribs kN/m	=(H10*H11)*0.000001*24
40	Total weight of ribs on long span (kN)	=H35*H39*H5
41	Total weight of ribs on short span (kN)	=H36*H39*H6
42	Total weight of Finishes (kN)	=H5*H6*1
43	Total Live load (kN)	=H5*H6*1.6
44	Total load on grid floor (kN)	=H38+H40+H41+H43+H42
45	Q= load per area (m^2)	=H44/(H5*H6)
46	Denominator for Qx and Qy	=SUM(H5^4,H6^4)
47	Load sheared on short span Qx (kN/m^2)	=H45*(H5^4/H46)
48	Load sheared on long span Qy (kN/m^2)	=H45*(H6^4/H46)
49	Moment on short span Mx (kNm)	=(H47*H9*H6^2)/8
50	Moment on Long Span My (kNm)	=(H48*H8*H5^2)/8
51	Shear force on short span Vx (kN)	=(H47*H8*H6)/2
52	Shear force on long span Vy (kN)	=(H48*H9*H5)/2
53	DESIGN	
54	Steel characteristic strength (N/mm^2)	0.0
55	Short span design moment (kNm)	=H49*1.0E06
56	Cover to ribs	0.00
57	Effective depth of ribs	=H11-H56
58	K-value	=H55/(H23*H10*H57^2)
59	Lever arm factor	=0.5+SQRT(0.25-H58/0.9)
60	Used lever arm	=MIN(H59,H25)
61	Area of steel (mm^2)	=H55/(0.95*H54*H60*57)
62	PROVIDE	
63	Long span design moment (kNm)	=H50*1.0E06
64	K-value	=H63/(H23*H10*H57^2)
65	Lever arm factors	=0.5+SQRT(0.25-H64/0.9)
66	Used laver arm	=MIN(H65,25)
67	Area of steel (mm^2)	=H63/(0.95*H54*H66*H57)
68	PROVIDE	
69	END DESIGN	

3.2.2   Analysis of ribs adopting the Rankine Grashoff Formulae

The Rankine Grashoff formulae were adopted for the analysis of the rib portion of the waffle slab, the formulae are very simple and they considered both the bending moments and shear forces at the ribs which are designed as flanged sections. This formulae were developed by Grashoff. (Krishna Raju, 1988).

The parameters for the equation are as follows;

- a_1 and b_1 = are the spacing of the ribs on the short and long span respectively.

- q = total load per unit area

- q_1 and q_2 = the load shared on the short and long span respectively

- a = Shorter dimension of grid

- b = Longer dimension of grid

- M_x and M_y are moments on the short - and long- span ribs respectively

- Q_x and Q_y are the shear forces on the short and long span ribs respectively.

Grashoff equations were derived as follows:

$$q_1 = q\left(\frac{b^4}{a^4 + b^4}\right) \text{ and } q_2 = q\left(\frac{a^4}{a^4 + b^4}\right) \quad \dots\dots\dots\dots\text{ 3.8}$$

The bending moment were given as shown below:

$$M_x = \left(\frac{q_1 b_1 a^2}{8}\right) \text{ and } M_y = \left(\frac{q_2 a_1 b^2}{8}\right) \quad \dots\dots\dots\dots\dots\text{3.9}$$

The shear force equation was given as shown below:

$$Q_x = \left(\frac{q_1 b_1 a}{2}\right) \text{ and } Q_y = \left(\frac{q_2 a_1 b}{2}\right) \quad \dots\dots\dots\dots\text{ 3.10}$$

3.3 Computer Program for the Analysis and Design of Waffle Slab.

The computer program YLRGT for the analysis and design of waffle slab was developed adopting the Microsoft excel package. The complete templates for the program are shown in Tables 3.1, 3.2 and 3.3.

3.4   Materials

3.4.1   Concrete

The ordinary Portland cement was used in the concrete, mixed with fine aggregate (sand) and coarse aggregate (granite), the maximum size of the granite was limited to 9.5 mm (3/8 in). This is because of the small models used and narrow dimensions between the sides of the form work.

The combined aggregate were prepared based on the recommendation of   BS 1881 Part 116; natural clean water without contamination was used in the concrete mix. Water cement ratio of 0.5 was selected to achieve the required 28-day concrete strength of 20 N/mm$^{2.}$, as was used in the case study; concrete samples were taken for the cube test (BS 1881, 1971).

3.4.2   Reinforcement

The steel reinforcements used in the case study were, 2Y20 mm for main ribs, 2Y12 mm for minor ribs, and Y10-200 mm c/c distribution steel in the slab for panels P1, P5, and for P3, (Figure 3.3), 2Y16mm was adopted for the main ribs and 2Y12 mm for minor ribs. When ratio 1:4 was adopted for the model, the main reinforcement became 1Y8 mm for the major ribs and 1R6 mm steel for the minor ribs, while plain 2.5 mm /D98 wire mesh was used in the slab as distribution steel.

For panels P2, P4 and P6, in the prototype, 2Y12 mm was used in the major and minor ribs respectively. And Y8-200 mm c/c distribution steel was used in the slab.
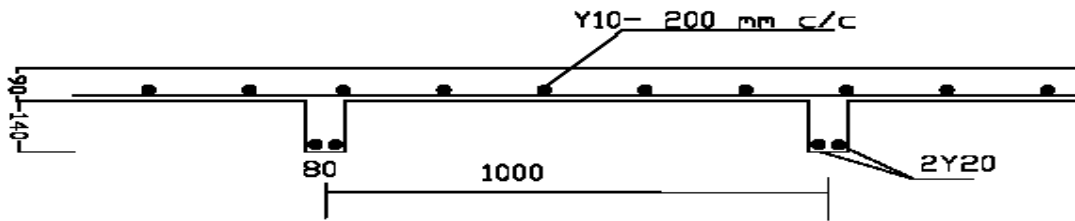
Figure 3.3: Reinforcement arrangement for slab and ribs in waffle slab

Adopting ratio 1:4, the major and minor ribs became 1R6 mm steel as main steel, while 2.5 mm/D98 wire mesh was also adopted as distribution steel in our models. The wires were cut to sizes and cleaned from rust using metal brush, in such a way to keep the bond forces that will be developed between the interface of concrete and steel. These bond forces are essential to prevent slip from occurring at the interfaces (Nilson and Winter, 1991).

3.5   Description of the Specimens

A total of sixty model samples were used for the experiment. Thirty samples each were used for the waffle and solid slabs respectively. These samples were subjected to axial loads, five samples per panel. Six types of panels were used for each of the waffle slabs and solid slabs respectively. Table 3.4 shows the geometric properties of waffle slab prototypes, while Tables 3.5 and 3.6 show the scaled down geometric description of the panel properties of the models for both the waffle and solid slabs respectively.

Different support conditions were adopted for all the slabs. Slabs W1, W3, S1 and S3 were supported on the two short sides, slabs W5, W6, S5 and S6 were all supported on three sides, i.e. two short and one long sides, while slabs W2, W4, S2 and S4 were supported on all the four sides. The reason for the different support conditions is to apply eccentric loading in the slabs.

Table 3.4: Panel properties for a waffle slab prototype.

Panel No	Length mm	Width mm	Slab thickness mm	Rib width (mm)	Rib spacing long span (mm)	Rib spacing Short span (mm)	Slab rein-forcement (spacing)	Longitudinal Rib rein-forcement	Transverse Ribs reinforcement
P1	5415	1720	90*	80	1040	460	Y10/200 mm	2Y20	2Y12
P2	3600	1200	50**	80	800	680	Y8/200 mm	2Y10	1Y12
P3	4340	1720	90*	80	1000	460	Y10/200 mm	1Y16	1Y12
P4	1635	1475	50**	80	760	1370	Y8/200 mm	2Y10	1Y12
P5	5250	1200	90*	80	1280	560	Y10/200 mm	2Y20	2Y12
P6	3280	1440	50**	80	1040	680	Y8/200 mm	2Y10	1Y12

 *Total Waffle Slab Thickness was 230 mm.

**Total waffle Slab Thickness was 200 mm

Table 3.5. Panel properties of the waffle models. (Scale ¼)

Panel No	Length mm	Width mm	Slab thickness (mm)	Rib width (mm)	Rib spacing Long span (mm)	Rib spacing short span (mm)	Slab rein-forcement (spacing)	Longitudinal Rib rein-forcement	Transverse Ribs reinforcement
W1	1353	430	22.5*	20	260	135	2.5/D98	1Y8	1R6
W2	900	300	12.5**	20	200	170	2.5/D98	1R6	1R6
W3	1085	430	22.5*	20	250	135	2.5/D98	1R6	1R6
W4	407	364	12.5**	20	190	344	2.5/D98	1R6	1R6
W5	1312	300	22.5*	20	320	140	2.5/D98	1Y8	1R6
W6	860	360	12.5**	20	260	170	2.5/D98	1R6	1R6

*Total Waffle slab Thickness was 58 mm.

**Total Waffle slab Thickness was 50 mm.

Table 3.6. Panel properties of the solid slab models.

Panel No	Length (mm)	Width (mm)	Slab thickness (mm)	Slab Reinforcement (spacing)
S1	1353	430	40	6R-75c/c
S2	900	300	40	6R-100c/c
S3	1085	430	40	6R-100c/c
S4	407	364	40	6R-100c/c
S5	1312	300	40	6R-75c/c
S6	860	360	40	6R-100c/c

3.5.1 Modelling of specimens

The areas of reinforcements for the models were obtained from equation 3.11, and the equation is derived below.

Adopting dimensional analysis, (Rajput, 1998: Bankole, 2007).

Using stresses relationship between the prototype and the model,

For Prototype.

$$\text{Stress } \sigma \ = \ \frac{Force}{Area} \qquad = M_p \cdot L_p^{-1} \cdot T_p^{-2}$$

And for the Model.

$$\text{Stress } \sigma \ = \ \frac{Force}{Area} \qquad = M_m \cdot L_m^{-1} \cdot T_m^{-2}$$

Where

$$M = \text{Mass}$$
$$L = \text{length}$$
$$T = \text{Time}$$

Subscript $_p$ and $_m$ are symbol for the prototype and model respectively.

At scale 1 : 4

$$4. \ M_m \cdot L_m^{-1} \cdot T_m^{-2} \ = 1. \ M_p \cdot L_p^{-1} \cdot T_p^{-2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad i$$

Substituting Mass as a function of reinforcement unit weight in dimensions symbol,

$$\text{Unit weight} \ = \ \text{weight / Volume} \ = \ \delta$$

$$\delta \ = \ M.L^{-2} \, T^{-2}$$
$$M \ = \ \delta \ L^2 \, T^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad ii$$

Substituting for M in equation i,

$$4. (\delta. L^2\ T^2)_m\ L_m^{-1}. T_m^{-2} \quad = \quad 1. (\delta. L^2\ T^2)_p\ L_p^{-1}. T_p^{-2} \quad \ldots\ldots\ldots\ iii$$

From equation iii:

$$4. \delta_m\ L_m \quad = \quad 1. \delta_p\ L_p$$

$$L_m \quad = \quad \frac{\delta_p.L_p.1}{\delta_m.4} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \quad 3.11$$

But L_p = Diameter of bars in the prototype

L_m = Diameter of bars in the model

δ_p = Characteristic strength of steel used in the prototype

δ_m = Characteristic strength of steel used in the model

For example, the area of the steel reinforcement for the major rib in panel P1 of the prototype is as follows

$2Y20 = 628$ mm^2

But for the model W1 using equation 3.11 above,

$$L_m \quad = \quad \frac{410.\ 20.\ 1}{410\ .\ 4} \qquad\qquad = \quad 5\ mm$$

The bar size is 5 mm diameter (area=19.64 mm^2), and since 2Y20 was required from the prototype, hence, 2 No.5 mm diameter bar will be obtained (area= 39.27 mm^2). Since there is no 5 mm diameter bar available in the market, then 1Y8 bar (area= 50 mm^2) was adopted for the main rib of the model W1.

For the solid slab models, the reinforcement was obtained as follows:

The reinforcement in the prototype was calculated to be   Y12- 275 mm c/c.

For panel S1,

L= 5415 mm, total length.

Hence, $\dfrac{5415}{275} = 19$

19 bars were used,

Total area of bars per length   =   113 x 19   = 2147 mm^2

At scale 1 : 4 , area required   =   2147 / 4   = 537 mm^2

Area of 5 mm bar   = 19.64 mm^2, (since Y12 will give 5 mm bar from scale 1:4)

No of bar required   = 537 / 19.64   = 27

Since there is no 5mm available in the market,

Adopt 6R- 75 mm c/c. (377 mm^2).

Hence, Length of model = 1354mm

$\therefore \dfrac{1354}{75} = 18.05$ Say 19

19 x 28.30 = 538 mm^2 area of bars required is   O.K

3.6 Casting of Specimens

All specimens were cast in forms made of plywood. The voids between the ribs were made by gluing Polystyrene blocks to the plywood form by following a marked pattern on the plywood. The reinforcement was then placed between the Polystyrene blocks in both directions. The reinforcement in the longer direction was placed first on small concrete blocks (biscuit) to keep it 5 mm from the bottom of the forms. This provided for the minimum cover required for the steel. The steel in the shorter direction was then placed on the top of the reinforcement in the longer direction. Both layers of

reinforcement were tied together with thin binding wire for stability during casting of the concrete. The wire mesh was placed and fixed properly.

The concrete was then cast, tamped and vibrated to prevent honey comb. After that, the top surface of the concrete was given a smooth final finish with the use of hand trowel. All specimens were moist cured for 28 days which helped the concrete to stabilize its own properties, like compressive strength and modulus of elasticity. To determine the compressive strength of the concrete, 150 x 150 mm concrete cubes were cast with the specimen. These cubes were submerged in water for curing and crushed at 7, 21 and 28 days (BS 1881, 1971).

3.7 Instrumentation

The deflections of the concrete specimens for the bending test were measured by means of a digital dial gauge with sensitivity of 0.01 mm. The dial gauges were placed at the top of the slab and held in place by the use of steel frame clamped on to the test machine. The crack widths were measured using a vernier caliper. Plate 1 showed both the dial gauge and the vernier caliper.

3.8 Experimental Set-up and Procedure

The slabs were loaded using Universal tensile testing machine. The slabs were placed on welded steel frame to give it a good support, and placed in between the loading machine. The machine loader applies the loads to the slabs from the top.

Deflections of the slabs were measured at load increment of 1 kN, and these were done during the loading of the concrete specimen. Crack widths were measured at failure load. The average mean value of the five results for each of the models samples was used.

The bending moments of each slab were obtained using the yield line theory formulae (equations 3.5 and 3.7.).

Plates 1 and 2 showed the precast waffle and solid slabs well stacked waiting transportation to the laboratory. Plate 3 showed the universal tensile testing machine while Plate 4 showed the test arrangements in the machine.

Plate 1: Stacked slabs.

Plate 2: Some samples of waffle slabs
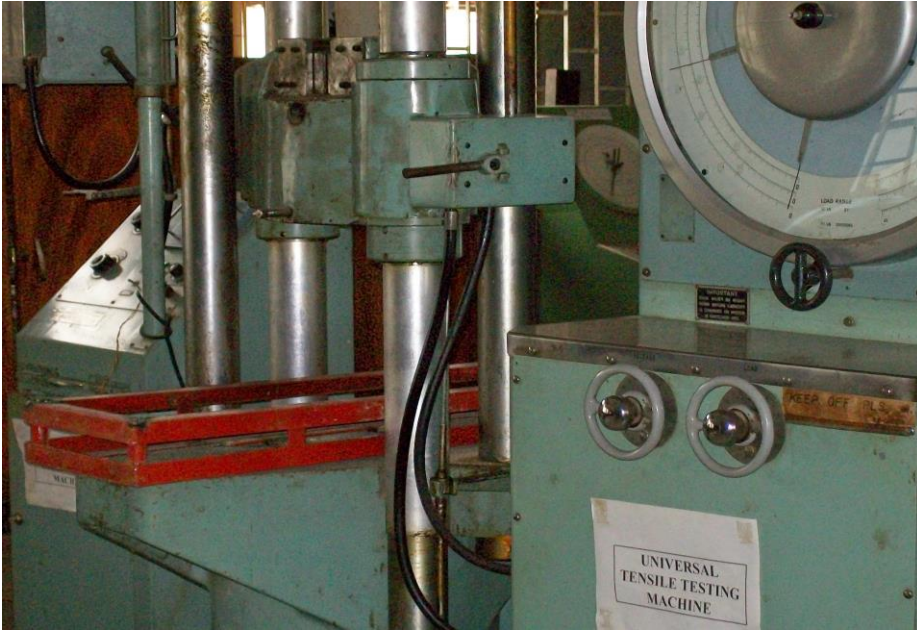
Plate 3: The Universal Tensile Testing Machine, with welded steel support for slabs.

Plate 4: Test arrangement for a slab sample

3.9 The Analysis of the Prototype

The Prototype was analysed by the Grillage analysis method which is mainly used in the analysis of bridges.

In grillage analysis, the longitudinal members are arranged to represent the main beams with the transverse members representing the deck slab and diaphragm beams. All loads are proportioned to the grillage members (longitudinal members only) and grillage joints before the moments, shear and torsion are calculated. The loads are applied to each longitudinal member as uniformly distributed loads (Childs, 2008). Hence the transverse members are neglected in this analytical method. This method may not be the most reliable for the analysis of waffle slabs because the transverse members in the waffle slab also carry some proportion of loads which should not be neglected. This might have led to some of the discrepancies observed in the laboratory results.

This study adopted the Rankine Grashoff theory which accommodated loading in both the longitudinal and transverse members of the waffle slab.

3.10 Analysis of Data

With the experimental set up above, coupled with the equations, some data were generated at the end of the work. The results obtained from both the waffle slabs and the solid slabs were subjected to Descriptive statistics and analysis of variance (ANOVA). Significant means were separated using least significant difference (LSD).

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1   Structural Response of Specimens to Loads

Waffle slabs and solid slabs were all simply supported, deflection was observed from the dial gauge immediately loading started. The cracks emerged from the middle of the slabs where the load was being applied and extended toward the slab edges, the crack width continued to expand until the failure load was reached .The cracks at the surface of the slab were very small compared to the ones formed at the slab bottom. This is as a result of the weakness of concrete in tension as the bottom parts of the slabs are under tension.

Deflection and the extent of cracking of a reinforced concrete slab are highly dependent on its support conditions, nonlinear and inelastic properties of concrete and the surrounding structure (Gilbert and Guo, 2005). The initial load at which deflection was observed, failure load and final deflection are showed in Table 4.1, for example, W1 started deflecting at 2 kN and the initial deflection was 0.10 mm, the final deflection was 1.19 mm at 18 kN failure load, while the load at which the first crack was observed was 9 kN and the final crack width was 0.60 mm, at a failure load of 18 kN. Table 4.2, showed the initial and final loads for the crack width of other slabs. The final crack patterns on the slab surface are showed in Plates 5 – 12. Each crack pattern for solid slabs depends on the support condition and loading orientation, whereas the crack pattern in waffle slabs did not show define pattern, Plate 13 showed the crushing of a waffle slab ribs.

Table 4.1: Deflection of slabs under load

Specimens	Initial Load (kN)	Failure load (kN)	Initial deflection (mm)	Deflection at failure load (mm)
W1	2.00	18.00	0.10	1.19
W2	1.00	9.00	0.34	3.64
W3	4.00	12.00	1.11	3.90
W4	1.00	6.00	1.45	6.60
W5	1.00	8.00	1.44	8.17
W6	2.00	12.00	0.43	3.28
S1	2.00	12.00	0.73	3.56
S2	1.00	7.00	1.10	9.28
S3	2.00	14.00	0.71	7..44
S4	1.00	6.00	1.19	6.44
S5	1.00	6.00	2.14	12.18
S6	2.00	10.00	0.14	3.89

Table 4.2: Crack properties of slabs under load

Specimens	Initial crack load (kN)	Final crack load (kN)	Crack width at failure (mm)	Crack patterns
W1	9.00	18.00	0.60	Plate 5
W2	6.00	9.00	0.35	Plate 6
W3	6.00	12.00	0.34	-
W4	3.00	6.00	0.25	Plate 7
W5	4.00	8.00	0.75	Plate10
W6	7.00	12.00	0.62	-
S1	5.00	12.00	1.20	-
S2	2.00	7.00	0.75	Plate 8
S3	6.00	14.00	0.90	-
S4	2.00	6.00	0.70	Plate 9
S5	3.00	6.00	1.10	Plate 11
S6	4.00	10.00	1.30	Plate 12

4.1.1 General observation of structural response of specimens to load

Generally, it was observed that the crack widths on all the waffle slabs were small compared to the corresponding solid slabs. The crack patterns in the solid slabs were more definite than those of waffle slabs.

This was due to the presence of ribs at the bottom and was also responsible for the small crack width and irregular crack pattern observed on the toping of the waffle slabs. The ribs were in the tension zone and were subjected to direct tensile stresses, unlike the solid slab where the bottom part was exposed directly to the tensile force.

In the waffle slabs, the ribs failed first before the slab portion started showing any sign of distress. The different crack patterns formed were as a result of the type of support (boundary) conditions that each slab was exposed to. Appendices 39 to 50 show details of the loads, deflections and crack widths of the slabs.

Plate 5: Crack pattern for waffle slab W1 supported on 2 short sides
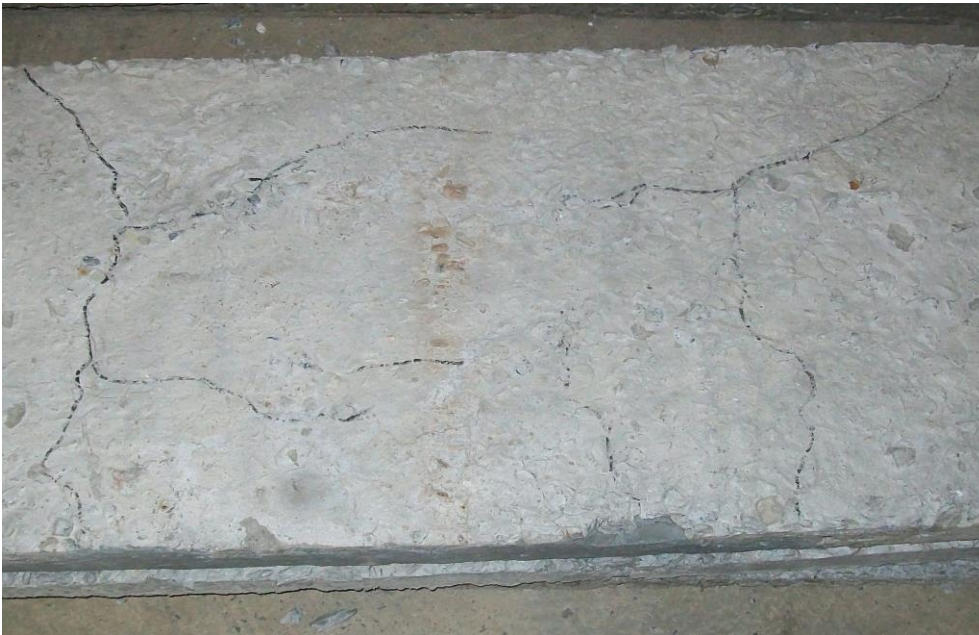
Plate 6: Crack pattern for waffle slab W2 Supported on all 4 sides

Plate 7: Crack pattern for waffle slab W4 Supported on all 4 sides

Plate 8: Crack pattern for Solid Slab S2 supported on all 4 sides

Plate 9: Crack pattern for solid slab S4 supported on all 4 sides

Plate 10: Crack pattern for waffle slab W5 Supported on 1 long side and 2 short sides

Plate 11: Crack pattern for Solid Slab S5 supported on 1 long side and 2 short sides

Plate 12: Crack pattern for Solid Slab S6 supported on 1 long side and 2 short sides
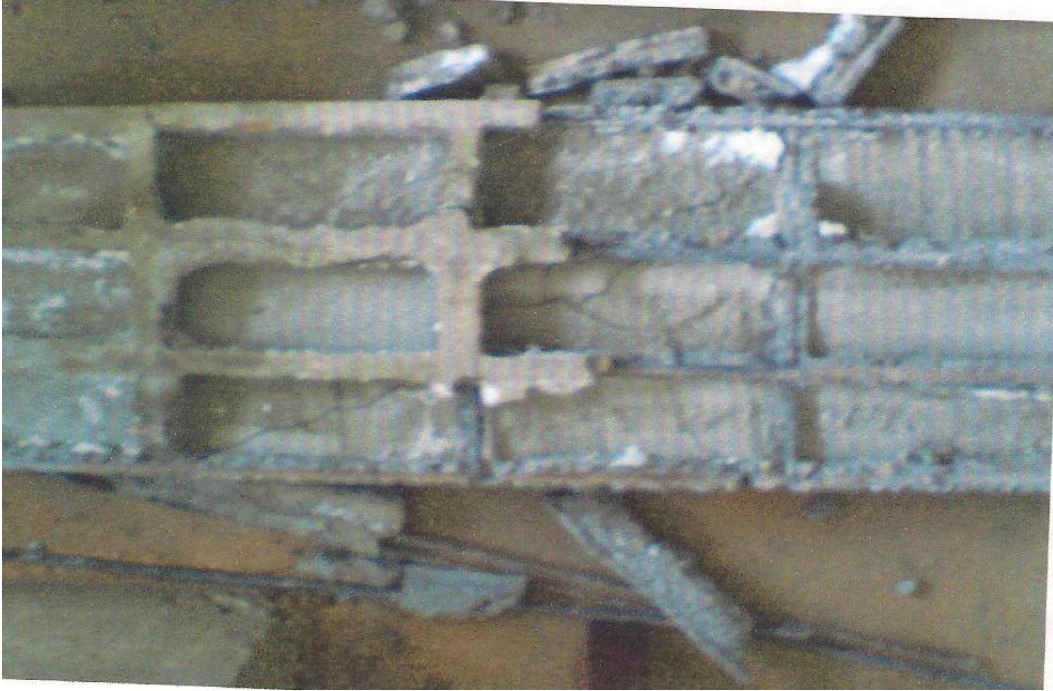
Plate 13: Crack pattern of a failed waffle slab bottom

4.2 Bending Moments of Specimens

The theoretical bending moments of the panels were compared with the experimental bending moments. The results for that of waffle and solid slabs are presented in Tables 4.3 and 4.4 respectively. The theoretical collapse load was obtained using the design load of 2.0 kN/m^2, Live load (for Hostels) and Dead load based on the depth of each slab, finishes and partition. The factors of safety 1.4 for dead load and 1.6 for live load were applied.

The experimental collapse load was obtained from the results of the laboratory work. The bending moments were obtained using the yield line theory for all the different supports conditions such as: simply supported on two, three, and four sides respectively.

From the results in Tables 4.3 and 4.4, it was observed that the experimental bending moments of the various panels were higher than the theoretical values (both for waffle and solid slabs). This is in contrast with the findings of Gudmand-Høyer in his work on yield line theory for concrete slabs subjected to axial forces, but concluded that the findings could be a rough estimate and an effectiveness factor could be introduced to make the findings reliable (Gudmand-Høyer 2003). Although it is expected that the experimental results be greater than the theoretical, because the final reinforcement provided is expected to be higher than the designed reinforcement area due entirely to the safety of the structure. But the difference between the theoretical results and the real results (experimental) should not be too high, since the economy of the structures is also very important. The differences may be as a result of the following factors:

- The design package used in the (Prototypes) experiment was different from the one used in the theoretical analysis.

Table 4.3: Comparison of theoretical and experimental bending moments (Waffle slab)

Panels	Theoretical		Experimental	Deflection
	Bending Moments (kNm)		Bending Moments (kNm)	(mm)
W1	2.157		5.526	1.19
W2	0.084		1.122	3.64
W3	1.653		2.880	3.90
W4	0.066		0.590	6.60
W5	0.436		0.947	8.17
W6	0.297		1.276	3.29

Table 4.4: Comparison of theoretical and experimental bending moments
(Solid slab)

Panels	Theoretical Bending Moments (kNm)	Experimental Bending Moments (kNm)	Deflection (mm)
S1	2.257	3.684	3.56
S2	0.0924	0.872	9.28
S3	1.452	3.360	7.44
S4	0.072	0.590	6.44
S5	0.459	0.710	12.18
S6	0.324	1.062	3.89

- The theoretical results are based on ideal situations that are assumed to be perfect, while the practical results are based on the situation on ground which may not be as perfect as expected due to various unforeseen conditions.

- The reinforcements provided to resist bending moments of reinforced concrete were sometimes over estimated. This was evident in all the reinforcements provided in the ribs of the prototypes. For example, panel P1 which had the largest dimensions of 5415 mm x 1720 mm, had reinforcement of 2Y20 (628 mm^2) in the long ribs and 2Y12 (226 mm^2) was provided in the short ribs. In the model, this was scaled down to ratio (1/4), and dimension 1353 mm x 430 mm was obtained. Using dimensional analysis, the reinforcements for the long ribs were 1Y8 (50 mm^2), and 1R6 (28 mm^2) for the short ribs (Tables 3.4 and 3.5). In the analysis of the ribs adopting the Rankine Grashoff theory (Appendix 1), the area of reinforcements for both long and short ribs were 3.320 mm^2 and 1.136 mm^2 respectively. This implies that if a reinforcement of 1R2.5 (5 mm^2) was used for both ribs, the bending moment would be supported without causing any structural problem. The use of 1R2.5 in the model suggested that 1Y10 (79mm^2) steel could be used in both ribs for the prototype P1. In order to reduce service stress in the steel, 2Y10 could also be used. The reinforcements used in the ribs of the prototype were very high and this was not economical. The reason for the high number of reinforcements was because the grillage analytical method that was used in the analysis of the prototype allowed excess load in the longitudinal ribs, the method did not distribute any load to the transverse ribs (nominal reinforcements were provided for the transverse ribs). This might have led to the

over reinforcement in the main ribs, thus affecting the bending moment of the structure. The grillage method is good for the analysis of bridge decks, not for precast waffle slab (Akinyele and Alade, 2011).

• The discrepancies in the results might also be due to the fact that concrete does not behave entirely as an elastic material (Mindes and Young, 1981; Neville, 1990). This is because, concrete was assumed to be elastic during the design stage. The analysis and design of other slabs can be seen in Appendices 2-24.

4.2.1 Comparison of bending moments of waffle slabs and solid slabs

The results for waffle W1 and solid S1 showed that the bending moments of the waffle slab were higher than that of the solid slabs by 33 %. Both slabs were supported on two short sides. For W2 and S2, the waffle was 22 % greater than that of solid slab in strength; both slabs were supported on four sides. Waffle slab W3 was 16.7% lower than solid slab S3, although they were supported on two sides. SlabsW4 and S4 gave the same results. However, these were supported on four sides.

The bending moments of W5 was 24.96 % higher than that of S5 while W6 was 16.67% greater than that of S6. These slabs were supported on three sides. Apart from the slabs that gave the same results (Tables 4.3 and 4.4); it was generally observed that waffle slabs had higher bending moments than solid slabs. This supports the advantages of waffle slabs over solid slabs in terms of ability to support heavy loads over a long span without increasing the depth.

This advantage was as a result of the presence of ribs in waffle slabs which act as flanged beams within the slab system. The bulk of the tensile load was carried by the ribs that spanned in both ways. This helped in the equal distribution of the load over the entire

span to the supports. The contribution of reinforcement to this advantage can not be ruled out also, because the presence of reinforcement in the slab portion and the ribs of waffle slabs gave it advantage over solid slabs which had reinforcement in it tensile zone only.

4.2.2 Effects of slab sizes and ribs on the bending moments of waffle slabs

The effects of the sizes of slabs on their bending moments were investigated. The sizes of waffle slabs that were set up under the same support conditions were compared with each other. The collapse loads (ultimate load) were used to determine the bending moments and the following observations were made.

4.2.2.1 Comparison of Bending moments of W1 and W3

Waffle slabs W1 and W3 were both simply supported on two sides, along the small spans while the large spans were free. From Table 3.5; it was observed that W1 has a larger dimension than W3. The collapse load of W1 was 18 kN, the bending moment obtained was 5.526 kNm, while for W3, the collapse load was 12 kN and bending moment was 2.88 kNm, W1 gave a 9 kN shear force at the supports while the shear force for W3 was 6 kN at the supports.

The above results showed that W1 has a higher bending moments than W3 despite it long span (Theoretically, slabs with long span are expected to fail earlier than those with short span for the same depth). The higher collapse load and moment may be attributed to the number of ribs in slab W1 which had four equally spaced ribs on the long axis while W3 had only three that were also equally spaced. They both had two ribs each on their short sides.

4.2.2.2 Evaluation of the bending moments of W2 and W4

Waffle slab W2 had larger dimensions than W4 (Table 3.5). The collapse load of W2 was 9 kN, and the bending moments was 1.122 kNm. The slab W4 had a collapse load of 6 kN, and its bending moments was 0.590 kNm. W4 was smaller in size; it had the lower collapse load and bending moments. This can be attributed to the single rib in it soffit, unlike W2 that had 3 short ribs and two long ribs in its soffit (plate 13), and both slabs were supported on all four sides.

4.2.2.3 Assessment of the bending moments of W5 and W6

The two waffle slabs W5 and W6 were supported on three sides, leaving one of the long span free. W5 had an ultimate load of 8 kN, and a bending moment of 0.947 kNm, while W6 had an ultimate load of 12 kN, and a bending moment of 1.276 kNm. It can be observed that although W5 had the higher dimension (Table 3.5), both the ultimate load and bending moment were lower than those of W6. While W5 had three ribs on the long side compared to the two of W6, they both had one rib each on the shorter side.

The reason for the low bending moment of W5 can be attributed to the type of support and the length of the long span. The length of W5 was about 1.5 times that of W6, this created a kind of in-balance in the slab. This also had effect on the load carrying ability of the slab since loads that were distributed to the unsupported long span area caused the early failure of the slab. The other area of the slab that were supported could not carry the excess load from the unsupported long span because the three ribs on the long span were responsible for the transfer of the load from the middle of the slab to the edge ribs. Since the edge ribs were to convey the loads to the supports, the edge rib on the unsupported side was under greater tensile stress as it had no supports under it. The three

mid ribs that transfered the load from the middle of the slab acted like a point load on this unsupported but long edge rib. The reinforcements on the edge rib could not withstand the excess load on its rib; therefore it yielded earlier, leading to the eventual failure of the slab system. Although W6 also reacted like W5 in terms of load distribution, it did not yield earlier than W5 because the edge rib (long span) was shorter than that of W6.

4.2.3 Effects of slab size on the bending moments of solid slabs

The sizes of solid slabs were also investigated to determine their contribution to the bending moment of solid slabs. Slabs with the same support conditions were compared with each other and the results discussed below:

4.2.3.1 Relationship between the bending moments of S1 and S3

The two solid slab samples were simply supported on the two short spans and the long spans were free. S1 was larger than S3, and both slabs were one way slab. It was observed that the collapse load for S1 was 12 kN, while the bending moment was 3.684 kNm; S3 had 14 kN for its collapse load, while bending moment of 3.36 kNm was generated, slab S1 had a shear force of 6 kN while S3 had 7 kN at the support respectively (Appendices 19 and 21).

From the above results, it was observed that both slabs had different collapse loads, but the bending moment generated in S1 was higher than S3 when the analytical method of a uniform load over part of a simply supported slab was used. This difference was because the span/ depth ratio of S1 was higher than that of S3, so that an appropriate reinforcement could be provided to resist the excess bending moment generated as a result of the longer span in the slab. If the result is compared to the corresponding waffle

slabs W1 and W3, the absence of ribs that would have provided additional rigidity to the solid slab also contributed to the lower collapse load in S1.

4.2.3.2 Difference in the bending moments between S2 and S4

The collapse load for S2 was 7 kN, and its bending moment was 0.872 kNm. Solid slab S4 gave a collapse load of 6 kN, and bending moment of 0.590 kNm. Both slabs were supported on four sides. The bending moment of S4 was 1.4 times smaller when compared to that of S2. The above results had shown that the size of a slab had a way of contributing to the amount of bending moment generated in the structure.

4.2.3.3 Comparison of the bending moments of S5 and S6

The results in slabs S5 and S6 showed that S5 had a collapse load of 6 kN and a bending moment of 0.710 kNm. The result of S6 gave a collapse load of 10 kN, while its bending moment was 1.062 kNm.

Both slabs were supported on three sides while one large side was free. They both had the same depth, and were both one way slabs. This condition gave rise to the lower collapse load of the longer slab S5. The in-balance was a result of one long side not supported, causing the tensile or bottom parts of the slabs to be over stressed, especially the non-supported slab edge. It can be recalled that all loads were transferred from the middle of the slabs where the load was being applied to the edges. Since there were no ribs on this edge as it was in waffle slabs, the bottom reinforcement in this part of the slab yielded easily, leading to the eventual failure of the entire slab system. It was the same failure mode that happened in slab S6, but the amount of failure was lower when compared with S5.

4.3 Assessment of the Rankine Grashoff and BS 8110 Slab Coefficients Factors

The BS 8110 slab coefficients factor was compared with the Rankine Grashoff theory in the analysis of waffle slab ribs. This factor was used to analyse a waffle slab with dimension 6000 mm by 5000 mm. Ribs spacing was 400 mm while the total depth of waffle slab was 200 mm. The topping was 60 mm depth and rib depth was 140 mm. This example was obtained from Mosley *et al* (1999) (Appendices 25 and 26).

The appropriate coefficient from BS 8110 (for slab analysis) was used for the ribs analysis, while minimum reinforcement was obtained by adopting 0.12bh (sectional area) for the toping or slab portion of the waffle slabs. But in this work, the ribs were analysed using the Rankine Grashoff theory of ribs analysis, while the toping was analyzed using the Yield line theory.

The section used was a waffle slab for an internal panel of a floor system. The characteristic material strengths for concrete and steel were f_{cu} = 30 N/mm^2 and f_y = 460 N/mm^2 respectively. The design ultimate load was 12.4 kN/m^2. The moment coefficients were taken from Table 3.14 of BS 8110 with ly/lx = 1.2. The results are shown in Table 4.5. From the results obtained, it was observed that the BS 8110 gave very low values for the moments generated in the ribs for each direction compared to the analysis using the Rankine theory. The moments obtained were 1.3 times greater than BS 8110 coefficients along the long span, and 1.4 times greater along the short span. The required area of steel from the Rankine theory was higher than the BS 8110 for both spans. The difference was between 1.8 – 2.0 times more than that of BS 8110 slab coefficients.

Table 4.5: Summary results for BS 8110 and Rankine theory

Analytical method	Moment in long span ribs (kNm)	Moment in short span ribs (kNm)	A_s required long span (mm^2)	A_s required short span (mm^2)	Shear force (kN)	Top Flange A_s required (mm^2)
BS 8110 Coefficients	2.98	3.97	45	60	5.20	72
Rankine Theory	3.13	4.50	80	122	3.60	462

Although Mosley *et al* (1999) provided 2T10 bars in each rib at the bottom (A_s =157 mm^2) which was adequate to cater for the required steel area and also adequate for the results from Rankine analysis. But the low analytical results from the use of BS 8110 coefficients may lead to under reinforcements of the ribs and hence allow for service stress in the steel provided. For the topping or slab portion, the use of 0.12bh gave the required area of steel to be 72 mm^2 which was the minimum required, and a wire mesh D98 (A_s=98 mm^2) was provided. If this is compared to the yield line analysis results of 462 mm^2 with the provision of Y8@100 mm c/c (503 mm^2), then the yield line results was adequate for the type of eccentric loading that the slabs in the prototypes were subjected to during loading and transportation to the site.

As discussed in chapter one, the reason for cracks and sometimes outright failures in the initial waffle slab panels was due to the use of wire mesh that can not withstand the eccentric load from cranes and transportation to site and eventual placements of the slabs to their positions on the site. The results obtained for the minimum reinforcements might be adequate for casting in-situ waffle slabs that were not exposed to eccentric forces, but not for precast waffle slabs that would be transported from the factory to the site and subjected to different types of loading conditions.

The reason for the cracks and failure in the precast slab of this research prototype was because the topping of the slab was subjected to both sagging and hogging moments during lifting. Since this portion of the slab was not designed to resist tensile moments (sagging / hogging), the forces generated as a result of lifting produced the tensile force that caused the slab topping to fail.

4.4 Comparison of YLRGT and ETABS

The computer program ETABS (developed by computers and structures Inc.) was also compared with the Yield line and Rankine Grashoff theory (YLRGT) in order to test the reliability of the proposed computer program. The result is showed in Table 4.6.

4.4.1 Outcome of W1 and W3

It was observed that the new program gave a higher moment of 2.157 kNm in the theoretical and 5.526 kNm in the experimental results of waffle W1 while ETABS results were 1.925 kNm and 4.729 kNm for theory and experiment respectively.

Also the new program gave the results for W3 as 1.653 kNm and 2.880 kNm for theory and experiment respectively whereas ETABS gave 1.613 kNm and 3.338 kNm for both theoretical and experimental result respectively for the same panel W3.

Both W1 and W3 were simply supported on the two short sides. The results showed that ETABS was conservative in the analysis of a simply supported waffle slabs on two short sides, although the difference in the results of both analysis was about 10% which is still reliable for any analysis.

4.4.2 Results of W2 and W4

The yield line result for W2 was 0.084 kNm for theory and 1.122 kNm for the experiment, while ETABS result was 0.502 kNm for theory and 0.581 kNm for experiment. Also W4 gave 0.066 kNm for theory and 0.590 kNm for experiment in yield line, whereas ETABS gave the theoretical result to be 0.472 kNm and experimental to be 0.733 kNm. Both slabs were simply supported on all four sides; yield line gave conservative results in the theory for both slab and the experimental result for W4, while ETAB gave conservative result in the experimental results of W2.

Table 4.6: Comparison of the bending moment of ETABS and YLRGT

	ETABS		YLRGT					
Panels	Theoretical (kNm)	Experimental (kNm)	Theoretical (kNm)			Experimental (kNm)		
			SP	L. Rib	S. Rib	SP	L. Rib	S. Rib
W1	1.925	4.729	2.157	0.0074	0.0378	5.526	0.9183	0.0345
W2	0.502	0.581	0.084	0.0031	0.0240	1.122	0.0365	0.2792
W3	1.613	3.338	1.653	0.0112	0.0370	2.880	0.6199	0.0272
W4	0.427	0.733	0.066	0.0200	0.0213	0.590	0.1609	0.1710
W5	0.568	0.851	0.436	0.0023	0.1898	0.947	0.0044	0.0370
W6	0.530	0.686	0.297	0.0102	0.0255	1.276	0.0362	0.0904

SP = Slab portion of waffle
L. Rib = Long rib of waffle
S. Rib = Short rib of waffle

4.4.3 Comparison of W5 and W6

The results for W5 were 0.436 kNm theoretical and 0.947 kNm experimental for yield line, while ETABS was 0.568 kNm theoretical and 0.851 kNm experimental. Waffle slab W6 gave theoretical result to be 0.297kNm and experimental result of 1.276 kNm for the yield line theory, while ETABS results were 0.530 kNm and 0.686 kNm for both theoretical and experimental results respectively. The experimental results showed that ETABS was conservative for both type of waffle slabs, whereas yield line was conservative in the theoretical results. The differences in all the values obtained varied and the variation had to do with the support conditions, size of slabs and the magnitude of load that caused the bending moments in each slab type. Both slabs were supported on three sides.

4.4.4 General observations of YLRGT and ETABS

Generally, it was observed that both analytical methods showed variation in the analysis of waffle slabs. The differences in their moments generated were very minimal and it all depended on the size of the slabs and support conditions. ETABS gave conservative results in slabs simply supported on two short sides, YLRGT gave conservative results in slabs supported on four sides, while the results in slabs supported on three sides varies between the two methods. These observations have showed that YLRGT method for the analysis of waffle slabs was reliable if compared with existing methods of analysis. Its advantage over ETABS is the ability to analyse the slab portion and ribs part of a waffle slab separately as shown in Table 4.6.

In the design of concrete elements, ETABS and other existing programs have a constant figure of 460 N/mm^2 for high yield steel and 250 N/mm^2 for low yield steel as recommended by BS 8110. Also, in most of these packages, the characteristic strength of

concrete had a minimum value of 30 N/mm^2 because they were developed to meet the environmental challenges of specific countries. Most of these programs are not suitable for the peculiar conditions that are prevalent in Nigeria.

In a research, Salau and Farayola (2006) carried out chemical and mechanical tests on some samples of locally available steel reinforcing bars in Nigeria, it was discovered that the chemical contents of all the steel did not conform to international standards. This non-conformity led to the poor performance of all the locally available steel bars under the tensile strength test. It was recommended that the characteristics yield strength of between 280 N/mm^2 and 300 N/mm^2 should be adopted during the design stage. Consequently, YLRGT has introduced some flexibility to all these parameters so that the designer can use any value desired, which must be within the recommendation of BS 8110.

This will therefore enhance the acceptability of the proposed method in countries like Nigeria where most of the steel that are used as reinforcement in a good number of sites have yield strength that are lower than the BS 8110 recommended values. Meanwhile, the common concrete characteristic strength that is used in Nigeria is between 20 N/mm^2 and 30 N/mm^2. Hence, this program is friendly to the Nigerian environment.

4.5 Deflections in Specimens

The deflection of each slab was obtained, the variation in the results of waffle slabs and the corresponding solid slabs were compared with each other and the graph of load against deflection was plotted (Figures 4.1 to 4.6).
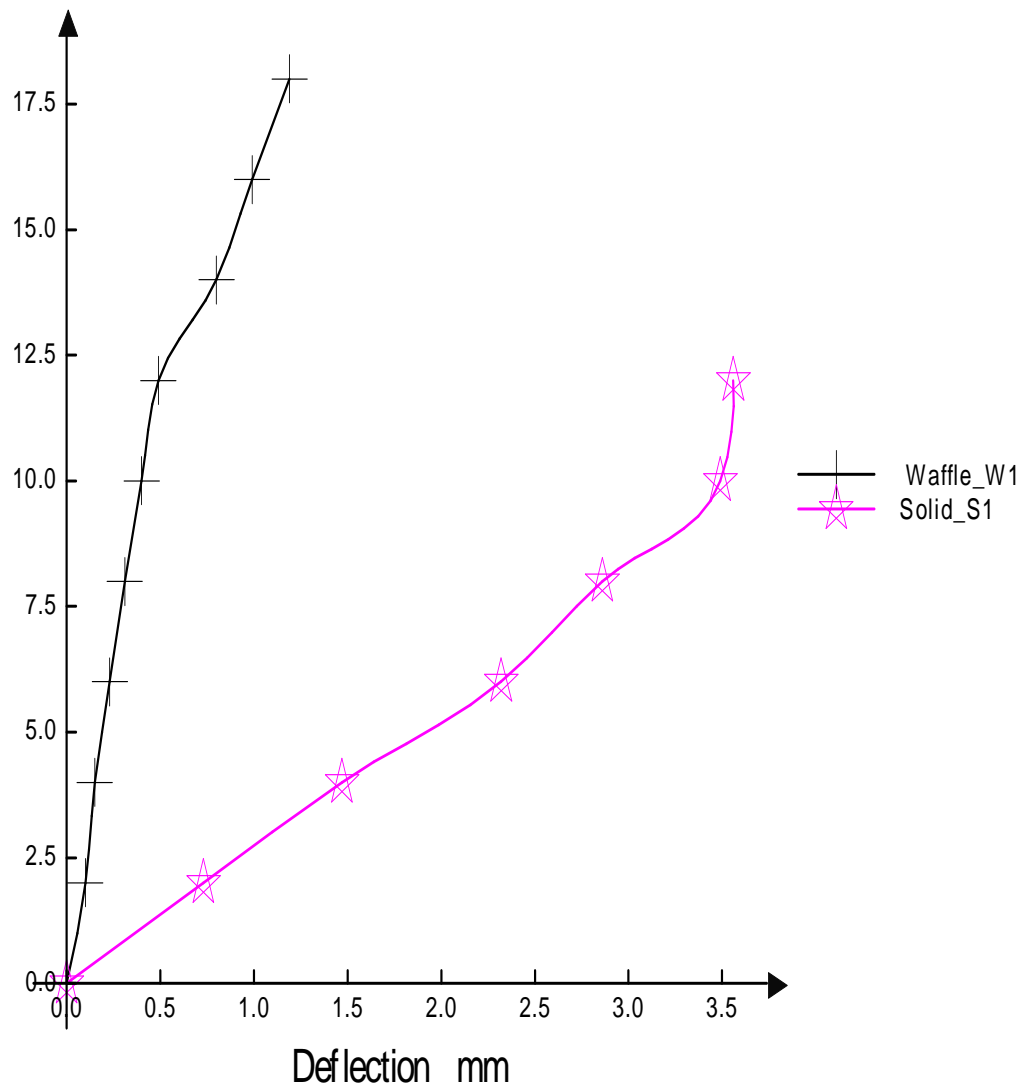
Figure 4.1: Variation in the deflection of W1 and S1
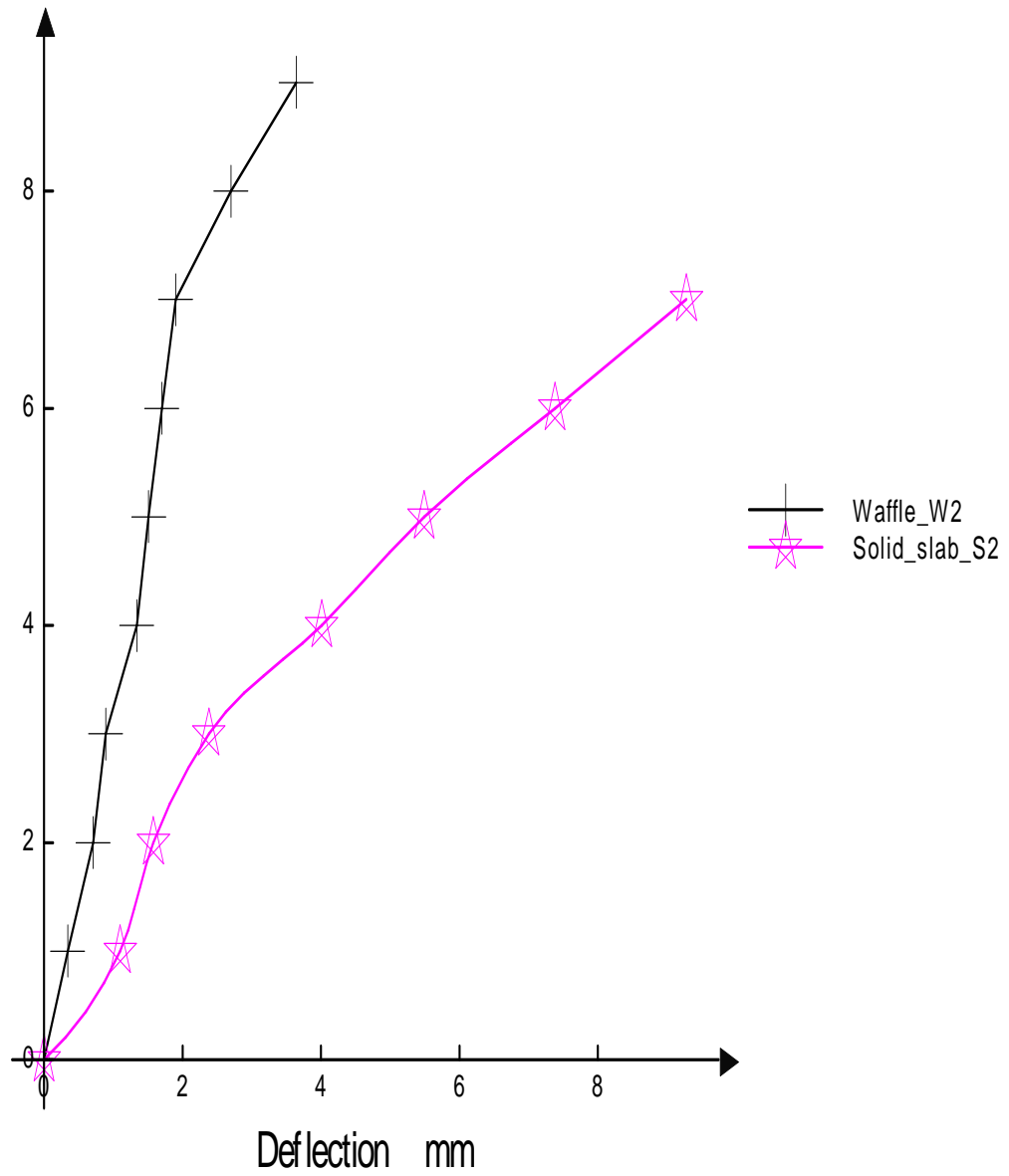
Figure 4.2: Variation in the deflection of W2 and S2
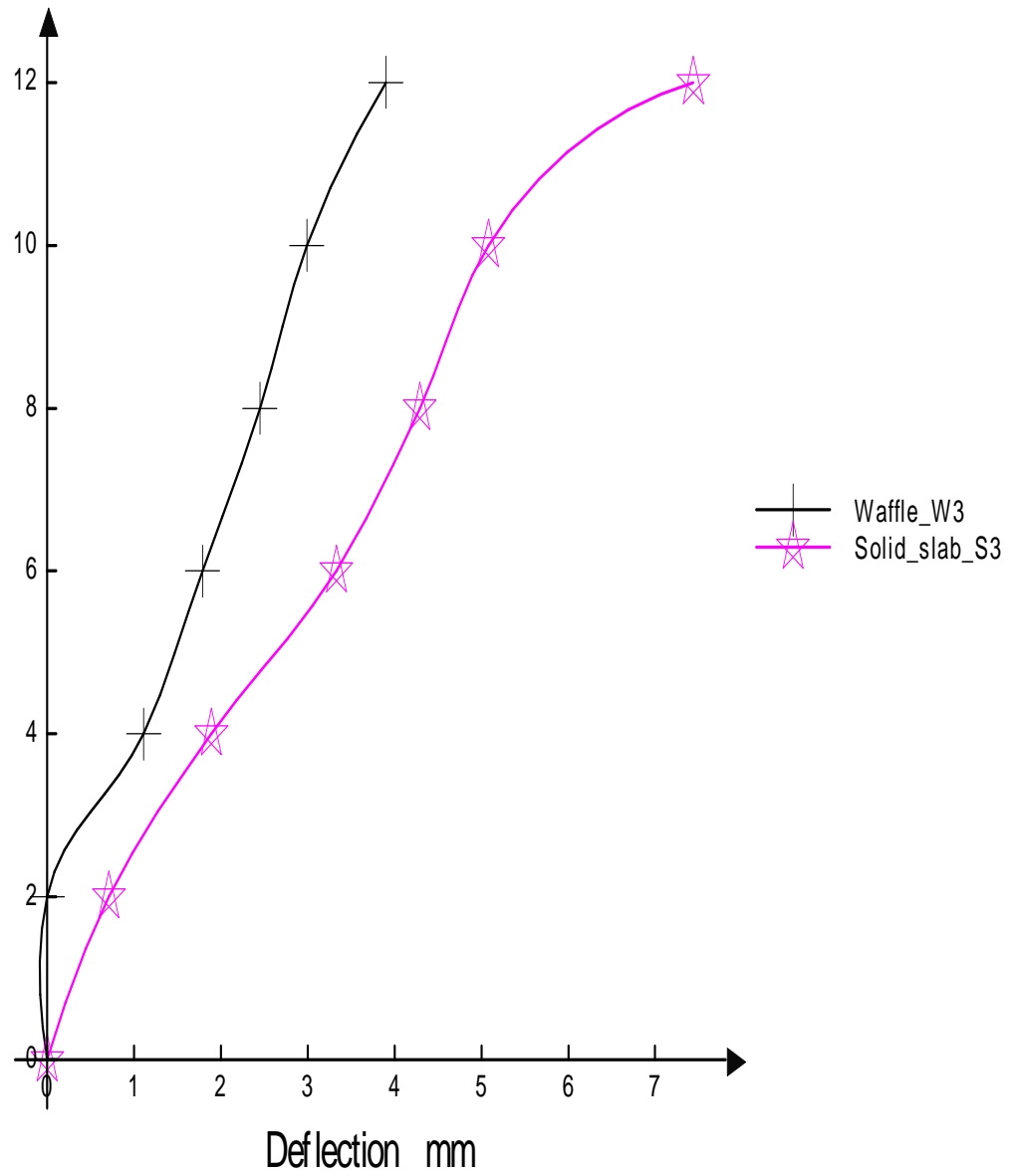
Figure 4.3: Variation in the deflection of W3 and S3
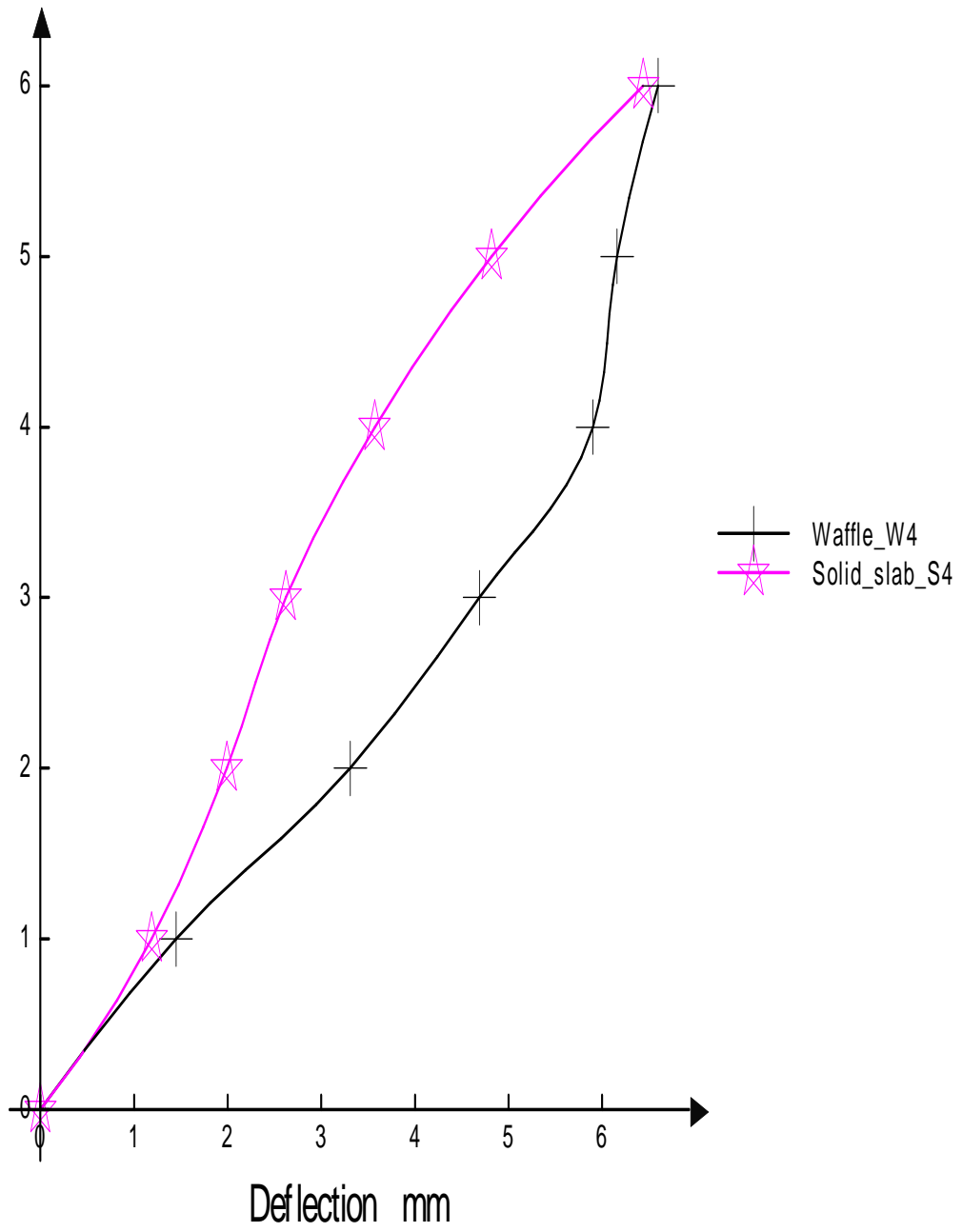
Figure 4.4: Variation in the deflection of W4 and S4
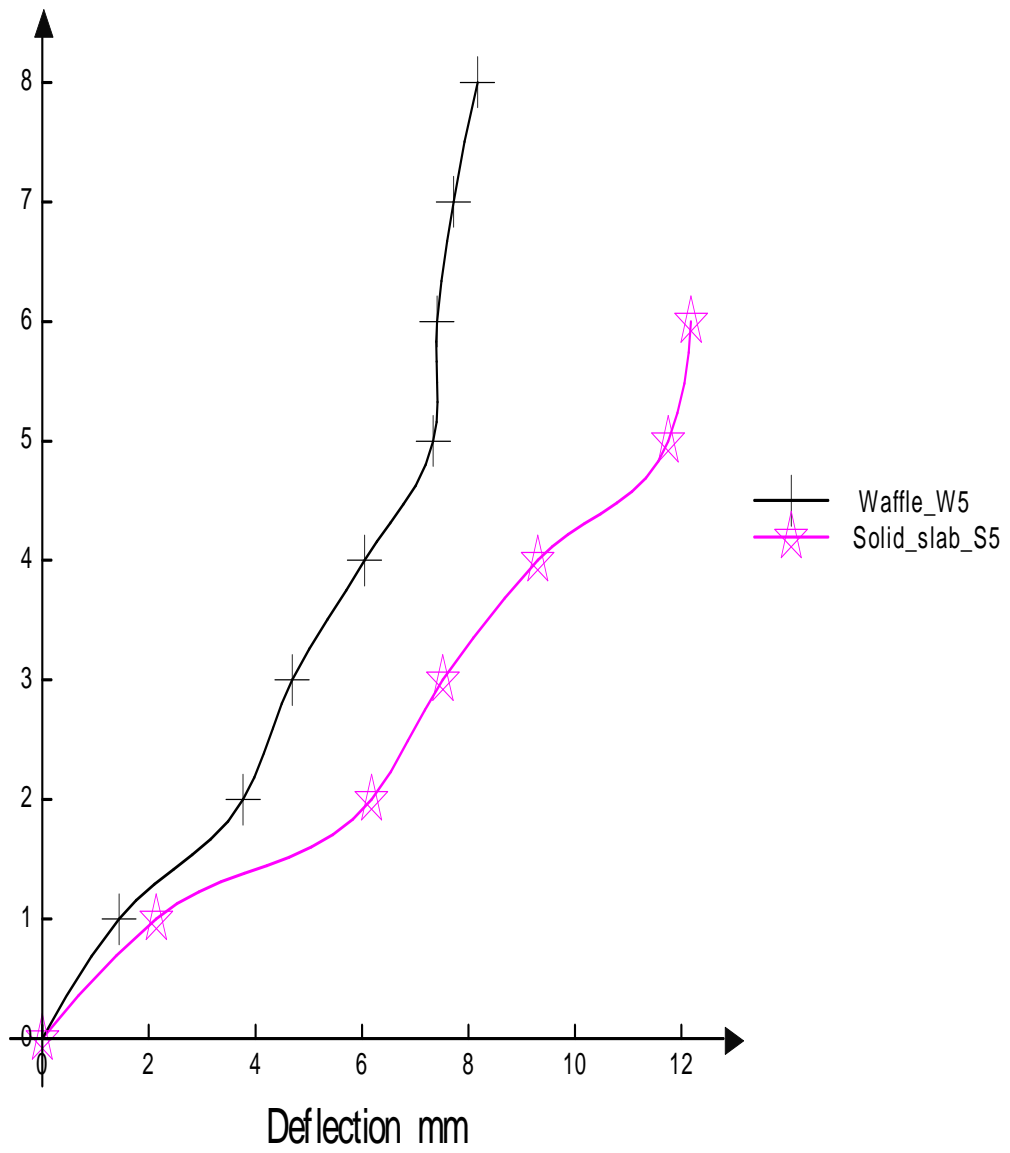
Figure 4.5: Variation in the deflection of W5 and S5
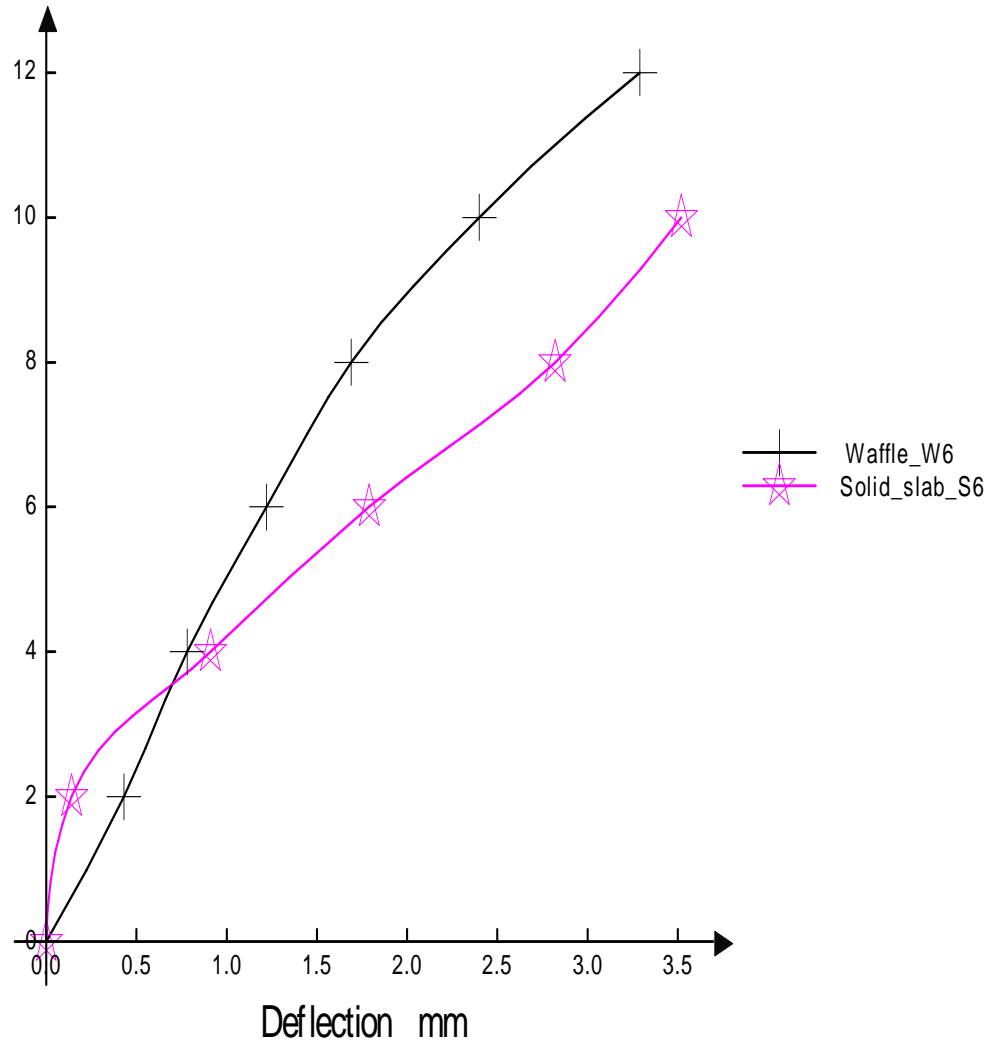
Figure 4.6: Variation in the deflection of W6 and S6

It was observed from Figure 4.1, that solid slab S1 deflected more than waffle slab W1. The deflection of S1 continued gradually forming a curve until there was a sudden change in direction at 10 kN. This continued on a straight line until failure at 12 kN. The waffle W1 curve was gradual; in fact it maintained a parallel deflection below 1mm for a long time until it got to 12kN where the elastic limit was exceeded. The same condition applied to S1, when it got to 10 kN, the elastic limit was exceeded. The rate at which slab S1 deflected showed that its ability to fail rapidly under overload was higher compared with W1 and it actually failed earlier. This confirmed that it can not withstand as much load as the corresponding waffle slab. Both slabs were simply supported on two sides.

Figure 4.2, showed the graph of waffle slab W2 and solid slab S2, it was observed that there was gradual deflection in W2. The curve formed between 1 kN and 7 kN was gradual until a sudden change at 7 kN which was the elastic limit. S2 also showed a gradual curve from the beginning until it got to 3 kN where it maintained a straight line to 7 kN which was the failure point. The elastic limit was exceeded at the 3 kN. The amount of deflection in S2 was greater than that of W2, hence it showed signs of failure earlier than W2. Both slabs were supported on four sides.

The graph of W3 and S3 (Figure 4.3) showed that S3 failed at 14 kN, and the amount of deflection in S3 is greater than that of W3. Both graphs showed gradual deflection from the beginning but as soon as W3 got to 12 kN it collapsed. The elastic limit for S3 was exceeded at 10 kN. Both slabs were simply supported on two sides.

The graph of W4 and S4 showed some differences when compared with the previous graphs Figure 4.4. In this case, W4 showed a slightly higher amount of

deflection over S4. The curve at W4 was constant until it got to 4 kN where it maintained a straight line to 6 kN. The elastic limit was exceeded after the 4 kN load point. S4 was able to maintain its curve to the 2 kN before its changes it directions. Both slabs failed at the 6 kN, they were supported on four sides.

In the W5 and S5 graph Figure 4.5, the slab S5 showed a gradual rise to 1 kN and maintain a curve to 2 kN, and change the direction of the curve up to 4 kN. The elastic limit was exceeded after the 4 kN. The amount of deflection in this slab was higher than that of its W5 counterpart. The W5 waffle slabs also showed a gradual rise from the beginning to 2 kN, it then sustained its curve between the 2 kN and 5 kN, after which a straight line was formed to the 8 kN failure point. The elastic limit was exceeded after the 5 kN load. Both slabs were supported on three sides during the test.

Figure 4.6 showed that, Slab W6 maintained a good curve from the beginning to the 12 kN load point. The amount of deflection in this slab is lower than that of S6. The shape of S6 showed that it maintained a curve from the beginning to 4 kN and the elastic limit was exceeded at this point and it eventually failed at 10 kN. Both slabs were supported on three sides.

All slabs with higher amount of displacements have lower slab stiffness when compared with those with lower amount of displacements. Waffle W1 had higher slab stiffness than solid S2; waffle W2 had better stiffness than solid S2, while waffle W3 showed good slab stiffness over solid S3. However, waffle W4 had a lower stiffness when compared with solid slab S4. Waffle W5 had better slab stiffness than solid S5 while waffle W6 also showed good slab stiffness over solid S6. All the slabs maintained their stiffnesses within the elastic limit stage, but lost it when ever the elastic limit was

exceeded, if the loads were removed before the loss of stiffnesses, there is tendency for the slab to return to it initial stage, but after losing its stiffnesses, the slab will never recover it pre-loading stage because the elastic limit was exceeded.

In general, it was observed that all the waffle slab specimens, with the exception of waffle W4, had better slab stiffness over corresponding solid slab. This must be as a result of the presence of ribs that was under tension in the various waffle slabs, and also the slab thickness.

If the corresponding solid slab was to have the same amount of slab stiffness like the waffle slab, the depth of such solid slab and the reinforcement provided would have to be increased. This solution was not the best and it was not economical. The nature of all the curves (both for waffle and solid slabs) had shown that concrete is ductile, and it can be classified as an elastoplastic material. Such material behave in an elastic manner until the elastic limit is reached after which they behave plastically (Megson, 2002).

4.6   Mean Deflection in Waffle Slabs and Solid Slabs

Mean deflections in waffle and solid slabs were compared using analysis of variance (ANOVA) (Miller and Freund, 1987). Mean deflection for waffle slab W1 and solid slab S1 were significantly different at 5% probability level (Appendix 51). Least significant difference was used to separate the means (Table 4.7). The results showed that S1 had the higher mean value of 2.405 mm.

The results of W2 and S2 (Appendix 52) showed that the mean deflection for both slabs were significantly different at 5% probability level. Least significant difference was also used to separate the means (Table 4.8). However, S2 had the higher mean of 4.46 mm.

Table 4.7: Least significant difference table for S1 and W1

Parameters	Deflection Means	LSD	P 0.05
S1	2.405a	1.885	0.955
W1	0.520b		

Means with the same letter under each column are not different at P=0.05.

Means with different letters under each column are different at P=0.05

Table 4.8: Least significant difference table for S2 and W2

Parameters	Deflection Means	LSD	P= 0.05
S2	4.46a	2.970	2.520
W2	1.49b		

Means with the same letter under each column are not different at P=0.05.

Means with different letters under each column are different at P=0.05

The W3 and S3 analysis (Appendix 53) showed that there was no significant difference in the deflections of both slab at the 5% probability level, since the calculated variance ratio (F) 3.637 was less than the tabulated (F) 4.96, S3 gave a mean value of 4.15 mm, which was higher than that of W3 (2.04 mm).

In the analysis of W4 and S4 (Appendix 54), the results also showed that there was no significant difference in the mean deflection for both slabs at the 5% probability level. The calculated variance ratio (F) 0.591 is less than the tabulated (F) 4.60. W4 had the higher mean value of 5.68 mm compared to the 4.65 mm of S4.

From Appendix 55, the results also showed that there was no significant difference in the average mean values of both slab W5 and S5, as the calculated (F) 2.076 is less than the tabulated (F) 4.75. Solid slab S5 showed a higher mean value of 8.18 mm, compared with 5.82 mm of W5.

Waffle slab W6 and solid slab S6 also did not reflect any significant difference in their mean values. This is because the calculated (F) 0.000 is far below the tabulated (F) 4.96. The mean value of both slab was 1.91 mm, this same mean value contributed to the 0.00 variance ratio obtained (F). (Appendix 56)

Apart from slabs W1, S1, W2 and S2 that showed significant difference in their mean deflections with standard deviations of ±1.131, ±0.388, ±3.080, ±1.025 respectively, all other slab samples showed that there was no significant difference in their mean value at $p < 0.05$ level. It can be generally concluded that despite the ability of solid slabs to fail earlier than waffle slabs, due to its low stiffness which resulted in higher deflection value, it can still maintain its stiffness up to $p < 0.05$ level with its corresponding waffle slabs in

the case of solid slabs S1 and S2. The remaining results showed that there was minimal difference between the mean deflection of solid and waffle slabs.

4.7 Correlation between the Deflections in Slabs of Similar Support Condition

The Pearson's correlation analysis method was used to determine the correlation of the deflections of slabs that are subjected to the same boundary conditions but of different sizes. From Appendix 57, the result of the analysis shows that the correlation coefficient in the deflection of both slab W1 and W3 is 0.984 and it is significant at 0.01 levels.

For slabs W2 and W4, the correlation coefficient is 0.988, and it is significant at 0.01 levels. See Appendix 58. Waffle slabs W5 and W6 showed a correlation coefficient of 0.919 and it is also significant at 0.01 levels (Appendix 59). The results for S1 and S3 also gave a correlation coefficient of 0.983, and it is significant at 0.01 levels as well. See Appendix 60. S2 and S4 gave a coefficient of 0.997 and were significant at the 0.01 levels (Appendix 61). From Appendix 62, solid slabs S5 and S6 gave a coefficient of 0.974 values, and it is also significant at 0.01 levels.

It can be generally concluded that the results obtained in the comparison showed that the deflections were within the same range, indicating a good relationship between the displacements values and positive correlation results. All results were significant at the 0.01 levels. This can imply that slabs of the same boundary conditions have corresponding displacement, not withstanding the sizes of the slabs.

4.8   Flexural Cracking

Members subjected to bending generally exhibit a series of distributed flexural cracks, even at service load. These cracks are harmless and can not cause any damage to the structure unless the widths become excessive (Mosley *et al*, 1999). Excessively wide cracks can be unsightly and spoil the appearance of an exposed concrete surface. They can allow the ingress of moisture and accelerate corrosion of the reinforcement and durability failure. In exceptional cases; they can reduce the contribution of the concrete to the shear strength of a member (Gilbert, 2001).

The crack width at service for both the waffle and solid slabs were estimated, Table 4.9, the example in Appendix 63 is the estimated crack width for solid slab S1 at service. The results showed that all the solid slabs have lower crack widths at service, when compared with corresponding waffle slabs. This is because the estimated crack width in waffle at service was based on the slab portions only, the ribs were not considered. The depth of the slab portion was small if compared with that of the solid slab, also the size of reinforcement used (wire mesh) in the slab portion was small if compared to the steel reinforcement used in the solid slabs and these were the parameters used to determine the estimated crack width. In the experiment, the crack widths at failure in waffle slabs were smaller than those of solid slabs. The presence of ribs in the waffle reduced the effect of load on the slab portion, since the ribs were directly exposed to tensile forces instead of the slab portion. The solid slabs were however exposed to the direct tensile forces which resulted in the flexural cracks that were formed.

Table 4.9: Crack width at failure and service

Specimens	Crack width at failure load (mm)	Estimated crack width at service (mm)
W1	0.60	0.425
W2	0.35	0.065
W3	0.34	0.232
W4	0.25	0.063
W5	0.35	0.054
W6	0.62	0.283
S1	1.20	0.035
S2	0.75	0.002
S3	0.90	0.049
S4	0.70	0.003
S5	1.10	0.006
S6	1.30	0.009

4.8.1   Comparison of mean crack width between waffle slab and solid slab

Crack width mean at failure load in waffle and solid slabs were compared using analysis of variance. The crack width mean for waffle slab and solid slab were significantly different at 5% probability level (Appendix 64). Least significant difference was used to separate the means, and the results showed that the differences in means were very minimal (Table 4.10). The results showed that solid slab had the higher mean value of 0.99 mm, compared to that of waffle slab which is 0.48 mm. These results confirmed that the crack widths formed in solid slabs were wider than those of waffle slabs in the experiment.

 Consequently, the presence of excess crack width in solid slabs samples must have contributed to their earlier failure when compared with corresponding waffle slabs.

Table 4.10: Least significant difference table of crack width for Waffle and Solid slabs

Parameters	Crack width Means	LSD	P= 0.05
Solid slab	0.99a	0.507	0.907
Waffle slab	0.48a		

Means with the same letter under each column are not different at P=0.05.

Means with different letters under each column are different at P=0.05

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The structural characteristics of waffle slabs have been determined through the investigation of the slabs and its response to applied loads. The bending moments of the slabs were determined using YLRGT.

i.   The response of waffle slabs to load afforded their advantages over solid slabs because the crack width formed as a result of loading to failure is generally small when compared to corresponding solid slabs. The crack patterns formed in solid slabs follow the effects of the types of support conditions which the slabs were   exposed   to, unlike waffle slabs where the crack patterns were not precise and did   not   follow   a particular pattern. This was due to the presence of ribs in the waffle slabs.

ii. The bending moments of waffle slabs are generally higher than that of solid slabs in all the tests carried out except in two cases where the collapse load of both slabs were equal. This implied that the stiffness of waffle slab is higher than that of solid slabs in its response to failure loads.

iii. The grillage analytical method that was used in the analysis of the ribs in the prototype was defective because it considered moments in the longitudinal ribs   only while neglecting the transverse ribs, although nominal reinforcements were provided for the transverse ribs, this eventually led to over reinforcement of the ribs.   The   Rankine Grashoff method adopted in the analysis of the model was able   to   overcome   this defect in the prototype by providing appropriate reinforcement   for   both   transverse and longitudinal ribs.

iv. Statistical analysis revealed that solid slabs generally have higher mean deflections than waffle slabs. However, solid slab S1 and S2 are significantly different in mean deflections at (p=0.05) over the corresponding waffle slabs W1 and W2 respectively. The significant difference between the other solid slabs and corresponding waffle slabs are less than 1%. This implied that solid slabs deflect more than waffle slabs under failure loads.

v. The graph of load against deflection also showed the gradual deflection of both waffle and solid slabs, but solid slabs generally deflect more than waffle slabs.

vi. The Pearson's correlation analysis method revealed that slabs that are subjected to the same boundary conditions showed positive correlation results in their deflections and are all significant at 0.01 levels.

vii. The crack width formed in solid slab is significantly different from that of waffle slabs at the (p=0.05) probability level, although the difference is very minimal. This implied that the average crack width in solid slab is wider than that of waffle

slab at failure load. This must have contributed to the low flexural strength of solid slabs if compared with waffle slabs.

viii. YLRGT also gave more dependable results when it was compared with the BS 8110 slab coefficients method that some engineers have adopted in the analysis of the waffle slab ribs.

ix. The study has showed that the reason for the failures observed in the prototype was as a result of approximate analytical procedures adopted in the design of the waffle slabs and the provision of inadequate reinforcements at the slab portions while the ribs were over reinforced.

x. YLRGT developed in this study for solving the problem was validated in the case study; by comparing the results of YLRGT analytical method with the results obtained from ETABS which is based on the finite element method (FEM). The results obtained showed good relationship between the two methods. However, the advantage of YLRGT over ETABS is the analysis of the slab portion separately using the Yield line theory and the rib portion by adopting the Rankine Grashoff theory in waffle slabs, the program can also be effectively used for the analysis of solid slabs.

xi. The introduction of YLRGT has facilitated the improved analysis of pre-cast waffle slabs. The use of YLRGT will enhance the structural integrity of pre-cast waffle slab as revealed when compared with the finite element method. The program is safe and it is very easy to use.

5.2 Contribution to Knowledge

i. This research has provided an improved method of analysing and designing separately, the slab and rib of waffle slabs. Adequate reinforcements can therefore be put in the appropriate sections to ensure structural integrity and reliability.

ii. The new method of analysis (YLRGT) can also be used in solving the problems of solid slabs.

5.4 Recommendations for Further Research

The following suggestions are recommended for further research as extension of this investigation:

(i.) Equipment that can give incremental results for every crack width measured is recommended.

(ii) The use of crack measuring meter or crack detecting microscope instead of Vernier caliper is suggested to give correct values of all measured crack widths.

(iii). During the conception and design of civil engineering structures, efforts should be made by the design engineer to adopt effective and reliable analytical methods in order to avoid the type of problems encountered in this study.

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APPENDICES

Appendix 1: Theoretical result of waffle slab W1.

ANALYSIS OF WAFFLE SLABS USING YIELD LINE THEORY

CODE B.S.8110 PART1 1997

Designed by Akinyele J.O

simply supported on two sides

Slab Long span (ly) (m)	1.353
Slab Short span (lx) (m)	0.43
Depth of Slab part (mm)	27
Distance between ribs on Long Span (m)	0.26
Distance between ribs on Short Span (m)	0.135
Rib width (mm)	20
Depth of ribs (mm)	33
Total depth of slab (mm)	60
Cover to reinforcement (mm)	5
Effective depth of slab (mm)	22
Slab dead load (kN/m^2)	0.9072
Slab imposed load (kN/m^2)	3.2
Finishes and Partitions	5.32
Total load on slab (kN/m^2)	9.4272
I_1 fixity at 1st support	0
I_2 fixity at 2nd support	0
Bending moment in Slab (KNm)	2.15719

DESIGN OF SLAB

Concrete characteristic strength (N/mm^2)	20
Steel characteristic strength (N/mm^2)	250
Maximum Lever arm factor	0.95
Maximum K- value	0.156
Moment M	2157190
K-value	0.22285
Lever arm Factor	0.54887
Used Lever arm	0.54887
Area of Steel (mm^2)	752.192
Provide	B785 structural mesh@100mmc/c bottom (785mm^2)

ANALYSIS AND DESIGN OF RIBS

Adopting the Rankine Grashoff Theory.

Number of Ribs on Long Span (ly)	4
Number of Ribs on Short Span (lx)	2
Weight of Slabs (kN/m^2)	9.4272
Total Load on Slab (KN)	5.48465
Weight of Ribs (kN/m)	0.01584
Total weight of Ribs on long span (ly), (kN)	0.10715
Total weight of Ribs on Short span (lx), (kN)	0.01362
Total weight of Finishes (kN)	0.58179
Total Live Load (kN)	0.93086
Total Load on grid floor (kN)	7.11808
q=Load per Area (m^2)	12.2348
Denominator for qx and qy	3.38531
Load sheared on short span (qx), (KN/m^2)	12.1112
Load sheared on Long span (qy), (KN/m^2)	0.12355

Moment on short span Mx (kN.m)	0.03778
Moment on long span My (kN.m)	0.00735
Shear force on short span, Vx, (kN)	0.67701
Shear force on long span, Vy, (kN)	0.01128

<div align="center">DESIGN</div>

Steel characteristic strength (N/mm^2)	250
Short span design moment (Nmm)	37789.3
Cover to ribs (mm)	5
Effective depth of ribs	28
K.value	0.12050
Lever arm Factor	0.84074
Used Lever arm	0.84074
Area of Steel (mm^2)	3.32020
Provide 1R6@Bottom-28mm^2	
Long span design moment (Nmm)	7351.06
K.value	0.02344
Lever arm Factor	0.97323
Used Lever arm	0.97323
Area of Steel (mm^2)	1.13582
Provide 1R6@Bottom-28mm^2	
END DESIGN	

Appendix 2: Theoretical results of waffle slab W2.

ANALYSIS AND DESIGN OF WAFFLE SLABS USING THE YIELD LINE THEORY.
CODE B.S. 8110: PART 1: 1997.
Designed By : Engr. Akinyele J.O

<div align="center">Simply supported on all four sides.</div>

Slab Length(Long span) (ly) (m)	0.9
Slab Width (Short span) (lx) (m)	0.3
Rib width (mm)	20
Distance between ribs on Long span (m)	0.2
Distance between ribs on Short span (m)	0.17
Depth of slab portion (mm)	15
Depth of ribs (mm)	35
Total depth of slab (mm)	50
Effective depth of slab (mm)	10
Slab dead load kN/m^2	0.504
Imposed load on slab kN/m^2	5.32
Finishes and Partition load	3.2
Total load on slab (kN/m^2)	9.024
ANALYSIS OF SLABS.	
Fixity ratio for continuous supports on any side	
Fixity ratio short span left side I_1	0
Fixity ratio long span bottom I_2	0
Fixity ratio short span right side I_3	0
Fixity ratio long span top I_4	0
Reduced sides ar'	0.3
Reduced sides br'	0.9
Bending moments in slab (KNm)	0.08434

<div align="center">DESIGN</div>

Cover to reinforcement. (mm)	5
Concrete characteristic stress (N/mm^2)	30
Steel Characteristic Stress (N/mm^2)	250
Maximum lever arm factor	0.95
Maximum K- Value	0.156
Moment M	84339.69
K- value	0.028113
Lever arm factor	0.967721
Used Lever arm	0.95
Area of steel (mm^2)	37.38047
Provide R2.5@100mmc/c(49mm^2)	

Appendix 3: Theoretical results of waffle slab W3.

ANALYSIS OF WAFFLE SLABS USING YIELD LINE THEORY

CODE B.S.8110 PART1 1997

Designed by Akinyele J.O

SUPPORTED ON TWO SIDE (simply supported)

Slab Long span (ly) (m)	1.085
Slab Short span (lx) (m)	0.43
Depth of Slab part (mm)	25
Distance between ribs on Long Span (m)	0.26
Distance between ribs on Short Span (m)	0.135
Rib width (mm)	20
Depth of ribs (mm)	33
Total depth of slab (mm)	58
Cover to reinforcement (mm)	5
Effective depth of slab (mm)	20
Slab dead load (KN/m^2)	0.84
Slab imposed load (KN/m^2)	5.32
Finishes and Partitions	3.2
Total load on slab (KN/m^2)	9.36
I_1 fixity at 1st support	0
I_2 fixity at 2nd support	0
Bending moment in Slab (KNm)	1.65282

DESIGN OF SLAB

Concrete characteristic stress (N/mm^2)	20
Steel characteristic stress (N/mm^2)	250
Maximum Lever arm factor	0.95
Maximum K- value	0.156
	165282
Moment M	4
K-value	0.20660
Lever arm Factor	0.64297
Used Lever arm	0.64297
Area of Steel (mm^2)	541.178
Provide	R6@50mmc/c bottom ($566mm^2$)

Appendix 4: Theoretical results of waffle slab W4.

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY.
CODE B.S. 8110: PART 1: 1997.
Designed By : Engr. Akinyele J.O

Simple supported on all four sides.

Slab Length(Long span) (ly) (m)	0.407
Slab Width (Short span) (lx) (m)	0.364
Rib width (mm)	20
Distance between ribs on Long span (m)	0.2
Distance between ribs on Short span (m)	0.17
Depth of slab portion (mm)	15
Depth of ribs (mm)	35
Total depth of slab (mm)	50
Effective depth of slab (mm)	10
Slab dead load kN/m^2	0.504
Imposed load on slab kN/m^2	5.32
Finishes and Partition load	3.2
Total load on slab (kN/m^2)	9.024

ANALYSIS OF SLABS.
Fixity ratio for continuous supports on any side

Fixity ratio short span left side I_1	0
Fixity ratio long span bottom I_2	0
Fixity ratio short span right side I_3	0
Fixity ratio long span top i4	0
Reduced sides ar'	0.364
Reduced sides br'	0.407
Bending moments in slab (KNm)	0.066567

DESIGN

Cover to reinforcement. (mm)	5
Concrete characteristic stress (N/mm^2)	20
Steel Characteristic Stress (N/mm^2)	250
Maximum lever arm factor	0.95
Maximum K- Value	0.156
Moment M	66567.44
K- value	0.033284
Lever arm factor	0.961539
Used Lever arm	0.95
Area of steel (mm^2)	29.50357
Provide	R2.5@100mmc/c(49mm^2)

Appendix 5: Theoretical results of waffle slab W5.

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY
CODE: B.S. 8110: Part 1
1997
Designed By : Akinyele
J.O

<div align="center">Simply supported on 3 sides</div>

Slab Long Span (ly) (m)	1.312
Slab Short Span (lx) (m)	0.3
Depth of Slab part (mm)	25
Distance between ribs on long span (m)	0.32
Distance between ribs on short span (m)	0.14
Rib width (mm)	20
Rib depth (mm)	33
Total depth of slab (mm)	58
Cover to reinforcement (mm)	5
Effective depth of slab (mm)	20
Slab dead load (kN/m^2)	0.84
Slab imposed load (kN/m^2)	5.32
Finishes and Partition load	3.2
Total load on Slab (kN/m^2)	9.36
For case A, = $h_1 + h_3$ <b	
l_1= fixity1 for left side of short span	0
l_2= fixity2 for supported side of long span	0
l_3= fixity3 for right side of short span	0
Reduced span br (m)	1.312
K'= redistribution factor	0.152439
h= factor to determine yield length	0.236725
1st Bending moment in slab (kNm)	0.436058
For case B, = $h_1 + h_3$ =b	
b' = dimensions	0.6
2nd.Bending moment in slab (kNm)	0.303302

<div align="center">DESIGN</div>

Concrete characteristic stress (N/mm^2)	20
Steel Characteristic Stress (N/mm^2)	250
Maximum Lever arm factor	0.95
Maximum K- value	0.156
Used Bending moment (KNm)	0.436058
Moment M	436058.3
K-value	0.054507
Lever arm factor	0.935243
Used Lever arm	0.935243
Area of Steel (mm^2)	98.15818
Provide	R5@B196Struct.mesh100mmc/c bottom(196mm^2)

Appendix 6: Theoretical result of waffle slab W6.

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY
CODE: B.S. 8110: Part 1
1997
Designed By : Akinyele
J.O

<center>Simply supported on 3 sides</center>

Slab Long Span (ly) (m)	0.86
Slab Short Span (lx) (m)	0.36
Depth of Slab part (mm)	15
Distance between ribs on long span (m)	0.32
Distance between ribs on short span (m)	0.14
Rib width (mm)	20
Rib depth (mm)	35
Total depth of slab (mm)	50
Cover to reinforcement (mm)	5
Effective depth of slab (mm)	10
Slab dead load (kN/m^2)	0.504
Slab imposed load (kN/m^2)	5.32
Finishes and Partition load	3.2
Total load on Slab (kN/m^2)	9.024
For case A, $= h_1+h_3 <b$	
$l_1=$ fixity1 for left side of short span	0
$l_2=$ fixity2 for supported side of long span	0
$l_3=$ fixity3 for right side of short span	0
Reduced span br (m)	0.86
K'= redistribution factor	0.27907
H= factor to determine yield length	0.255007
1st Bending moment in slab (kNm)	0.296852
For case B, $= h_1+h_3 =b$	
b' = dimensions	0.72
2nd.Bending moment in slab (kNm)	0.276466

<center>DESIGN</center>

Concrete characteristic stress (N/mm^2)	20
Steel Characteristic Stress (N/mm^2)	250
Maximum Lever arm factor	0.95
Maximum K- value	0.156
Used Bending moment (KNm)	0.296852
Moment M	296852.4
K-value	0.148426
Lever arm factor	0.791688
Used Lever arm	0.791688
Area of Steel (mm^2)	157.8784
Provide	R5/B196Struct.mesh100mmc/c

bottom(196mm^2)

Appendix 7: Theoretical result of solid slab S1.

ANALYSIS OF WAFFLE SLABS USING YIELD LINE THEORY

CODE B.S.8110 PART1 1997

Designed by Akinyele J.O

simply supported on two short sides

Slab Long span (ly) (m)	1.353
Slab Short span (lx) (m)	0.43
Depth of Slab part (mm)	40
Distance between ribs on Long Span (m)	0
Distance between ribs on Short Span (m)	0
Rib width (mm)	0
Depth of ribs (mm)	0
Total depth of slab (mm)	40
Cover to reinforcement (mm)	5
Effective depth of slab (mm)	35
Slab dead load (kN/m^2)	1.344
Slab imposed load (kN/m^2)	3.2
Finishes and Partitions	5.32
Total load on slab (kN/m^2)	9.864
I_1 fixity at 1st support	0
I_2 fixity at 2nd support	0
Bending moment in Slab (kNm)	2.25714

DESIGN OF SLAB

Concrete characteristic strength (N/mm^2)	20
Steel characteristic strength (N/mm^2)	250
Maximum Lever arm factor	0.95
Maximum K- value	0.156
Moment M	225714
K-value	0.09212
Lever arm Factor	0.88423
Used Lever arm	0.88423
Area of Steel (mm^2)	307.086
Provide	R6@75mmc/c bottom (377mm^2)

Appendix 8: Theoretical result of solid slab S2.

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY.
CODE B.S. 8110: PART 1: 1997.
Designed By : Engr. Akinyele J.O
Simply supported on all four sides.

Slab Length(Long span) (ly) (m)	0.9
Slab Width (Short span) (lx) (m)	0.3
Rib width (mm)	0
Distance between ribs on Long span (m)	0
Distance between ribs on Short span (m)	0
Depth of slab portion (mm)	40
Depth of ribs (mm)	0
Total depth of slab (mm)	40
Effective depth of slab (mm)	35
Slab dead load kN/m^2	1.344
Imposed load on slab kN/m^2	3.2
Finishes and Partition load	5.32
Total load on slab (kN/m^2)	9.864
ANALYSIS OF SLABS.	
Fixity ratio for continuous supports on any side	
Fixity ratio short span left side i_1	0
Fixity ratio long span bottom i_2	0
Fixity ratio short span right side i_3	0
Fixity ratio long span top i_4	0
Reduced sides ar'	0.3
Reduced sides br'	0.9
Bending moments in slab (kN.m)	0.092425

DESIGN

Cover to reinforcement. (mm)	5
Concrete characteristic strength (N/mm^2)	20
Steel Characteristic Strength (N/mm^2)	250
Maximum lever arm factor	0.95
Maximum K- Value	0.156
Moment M	92425.38
K- value	0.003136
Lever arm factor	0.996504
Used Lever arm	0.95
Area of steel (mm^2)	9.728581
Provide R6@100mmc/c($283mm^2$)	

Appendix 9: Theoretical result of solid slab S3.

ANALYSIS OF WAFFLE SLABS USING YIELD LINE THEORY
<center>CODE B.S.8110 PART1 1997</center>

Designed by Akinyele J.O
<center>simply supported on two short sides</center>

Slab Long span (ly) (m)	1.085
Slab Short span (lx) (m)	0.43
Depth of Slab part (mm)	40
Distance between ribs on Long Span (m)	0
Distance between ribs on Short Span (m)	0
Rib width (mm)	0
Depth of ribs (mm)	0
Total depth of slab (mm)	40
Cover to reinforcement (mm)	5
Effective depth of slab (mm)	35
Slab dead load (kN/m^2)	1.344
Slab imposed load (kN/m^2)	3.2
Finishes and Partitions	5.32
Total load on slab (kN/m^2)	9.864
I_1 fixity at 1st support	0
I_2 fixity at 2nd support	0
	1.4515
Bending moment in Slab (kNm)	1

<center>DESIGN OF SLAB</center>

Concrete characteristic strength (N/mm^2)	20
Steel characteristic strength (N/mm^2)	250
Maximum Lever arm factor	0.95
Maximum K- value	0.156
Moment M	145151
K-value	0.0592
Lever arm Factor	0.9291
Used Lever arm	0.9291
Area of Steel (mm^2)	187.93
Provide	R6@100mmc/c bottom (283mm^2)

Appendix 10: Theoretical result of solid slab S4.

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY.
CODE B.S. 8110: PART 1: 1997.
Designed By : Engr. Akinyele J.O
<div align="center">Simply supported on all four sides.</div>

Slab Length(Long span) (ly) (m)	0.407
Slab Width (Short span) (lx) (m)	0.364
Rib width (mm)	0
Distance between ribs on Long span (m)	0
Distance between ribs on Short span (m)	0
Depth of slab portion (mm)	40
Depth of ribs (mm)	0
Total depth of slab (mm)	40
Effective depth of slab (mm)	35
Slab dead load kN/m^2	1.344
Imposed load on slab kN/m^2	3.2
Finishes and Partition load	5.32
Total load on slab (KN/m^2)	9.864
ANALYSIS OF SLABS.	
Fixity ratio for continuous supports on any side	
Fixity ratio short span left side i_1	0
Fixity ratio long span bottom i_2	0
Fixity ratio short span rigth side i_3	0
Fixity ratio long span top i_4	0
Reduced sides ar'	0.364
Reduced sides br'	0.407
Bending moments in slab (kN.m)	0.071637

<div align="center">DESIGN</div>

Cover to reinforcement. (mm)	5
Concrete characteristic strength (N/mm^2)	20
Steel Characteristic Strength (N/mm^2)	250
Maximum lever arm factor	0.95
Maximum K- Value	0.156
Moment M	71637.56
K- value	0.002475
Lever arm factor	0.997242
Used Lever arm	0.95
Area of steel (mm^2)	7.678552
Provide R6@100mmc/c(283mm^2)	

Appendix 11: Theoretical result of solid slab S5.

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY
CODE: B.S. 8110: Part 1
1997
Designed By : Akinyele J.O

<div align="center">Simply supported on 3 sides</div>

Slab Long Span (ly) (m)	1.312
Slab Short Span (lx) (m)	0.3
Depth of Slab part (mm)	40
Distance between ribs on long span (m)	0
Distance between ribs on short span (m)	0
Rib width (mm)	0
Rib depth (mm)	0
Total depth of slab (mm)	40
Cover to reinforcement (mm)	5
Effective depth of slab (mm)	35
Slab dead load (kN/m^2)	1.344
Slab imposed load (kN/m^2)	3.2
Finishes and Partition load	5.32
Total load on Slab kN/m^2	9.864
For case A, = h_1+h_3 <b	
l_1= fixity1 for left side of short span	0
l_2= fixity2 for supported side of long span	0
l_3= fixity3 for right side of short span	0
Reduced span br (m)	1.312
K'= redistribution factor	0.152439
h= factor to determine yield length	0.236725
1st Bending moment in slab (KNm)	0.458949
For case B, = h1+h3 =b	
b' = dimensions	0.6
2nd.Bending moment in slab (KNm)	0.3196344

<div align="center">DESIGN</div>

Concrete characteristic strength (N/mm^2)	20
Steel Characteristic Strength (N/mm^2)	250
Maximum Lever arm factor	0.95
Maximum K- value	0.156
Used Bending moment (kNm)	0.458949
Moment M	458949.6
K-value	0.015631
Lever arm factor	0.98232
Used Lever arm	0.95
Area of Steel (mm^2)	48.49369
Provide	R6@100mmc/c bottom(283mm^2)

Appendix 12: Theoretical result of solid slab S6

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY
CODE: B.S. 8110: Part 1
1997
Designed By : Akinyele J.O

<center>Simply supported on 3 sides</center>

Slab Long Span (ly) (m)	0.86
Slab Short Span (lx) (m)	0.36
Depth of Slab part (mm)	40
Distance between ribs on long span (m)	0
Distance between ribs on short span (m)	0
Rib width (mm)	0
Rib depth (mm)	0
Total depth of slab (mm)	40
Cover to reinforcement (mm)	5
Effective depth of slab (mm)	35
Slab dead load (kN/m^2)	1.344
Slab imposed load (kN/m^2)	3.2
Finishes and Partition load	5.32
Total load on Slab kN/m^2	9.864
For case A, = h_1+h_3 <b	
l_1= fixity1 for left side of short span	0
l_2= fixity2 for supported side of long span	0
l_3= fixity3 for right side of short span	0
Reduced span br (m)	0.86
K'= redistribution factor	0.27907
h= factor to determine yield length	0.255007
1st Bending moment in slab (kNm)	0.3240404
For case B, = h_1+h_3 =b	
b' = dimensions	0.72
2nd.Bending moment in slab (kNm)	0.3012834

<center>DESIGN</center>

Concrete characteristic strength (N/mm^2)	20
Steel Characteristic Strength (N/mm^2)	250
Maximum Lever arm factor	0.95
Maximum K- value	0.156
Used Bending moment (kNm)	0.3240404
Moment M	324040.1
K-value	0.011037
Lever arm factor	0.987583
Used Lever arm	0.95
Area of Steel (mm^2)	34.24191
Provide R6@100mmc/c bottom(283mm^2)	

Appendix 13: Experimental result of waffle slab W1

ANALYSIS OF WAFFLE SLABS USING YIELD LINE THEORY
                    CODE B.S.8110 PART1
                    1997
Designeded by Akinyele J.O
                    SUPPORTED ON TWO SIDE (simply supported)

Slab Long span (Ly) (m)	1.353
Slab Short span (Lx) (m)	0.43
Depth of Slab part (mm)	0
Distance between ribs on Long Span (m)	0
Distance between ribs on Short Span (m)	0
Rib width (mm)	0
Depth of ribs (mm)	0
Total depth of slab (mm)	0
Cover to reinforcement (mm)	0
Effective depth of slab (mm)	0
Slab dead load (KN/m^2)	0
Slab imposed load (KN/m^2)	288
Total load on slab (KN/m^2)	288
Load distance from 1st Support	0.5515
Load distance from 2nd support	0.5515
Length of UDL	0.25
UDL obtained kN/m	72
Reactions at supports kN	9
Bending moments in slab kNm	5.526

Appendix 14: Experimental result of waffle slab W2

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY.
CODE B.S. 8110: PART 1: 1997.
Designed By : Engr. Akinyele
J.O

	Simply supported on all four sides.	
Slab Length(Long span) (ly) (m		0.9
Slab Width (Short span) (lx) (m)		0.3
Rib width (mm)		0
Distance between ribs on Long span (m)		0
Distance between ribs on Short span (m)		0
Depth of slab portion (mm)		0
Depth of ribs (mm)		0
Total depth of slab (mm)		0
Effective depth of slab (mm)		0
Slab dead load kN/m^2		0
Imposed load on slab kN/m^2		144
Finishes and Partition load		0
Total load on slab (kN/m^2)		1
ANALYSIS OF SLABS.		
Fixity ratio for continuous supports on any side		
Fixity ratio short span left side i_1		0
Fixity ratio long span bottom i_2		0
Fixity ratio short span right side i3		0
Fixity ratio long span top i4		0
Reduced sides ar'		0.3
Reduced sides br'		0.9
Bending moments in slab (kN.m)		1.121538

Appendix 15: Experimental result of waffle slab W3

ANALYSIS OF WAFFLE SLABS USING YIELD LINE THEORY
              CODE B.S.8110 PART1
              1997
Designed by Akinyele J.O
              SUPPORTED ON TWO SIDE (simply supported)

Slab Long span (Ly) (m)	1.085
Slab Short span (Lx) (m)	0.43
Depth of Slab part (mm)	0
Distance between ribs on Long Span (m)	0
Distance between ribs on Short Span (m)	0
Rib width (mm)	0
Depth of ribs (mm)	0
Total depth of slab (mm)	0
Cover to reinforcement (mm)	0
Effective depth of slab (mm)	0
Slab dead load (kN/m^2)	0
Slab imposed load (kN/m^2)	192
Total load on slab (kN/m^2)	192
Load distance from 1st Support	0.4175
Load distance from 2nd support	0.4175
Length of UDL	0.25
UDL obtained kN/m	48
Reactions at supports kN	6
Bending moments in slab kNm	2.88

Appendix 16 Experimental result of waffle slab W4

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY.
CODE B.S. 8110: PART 1: 1997.
Designed By : Engr. Akinyele J.O
                Simple supported on all four
                sides.

Slab Length(Long span) (Ly) (m)	0.407
Slab Width (Short span) (Lx) (m)	0.364
Rib width (mm)	20
Distance between ribs on Long span (m)	0.4
Distance between ribs on Short span (m)	0.4
Depth of slab portion (mm)	0
Depth of ribs (mm)	0
Total depth of slab (mm)	0
Effective depth of slab (mm)	0
Slab dead load kN/m^2	0
Imposed load on slab kN/m^2	96
Finishes and Partition load	0
Total load on slab (kN/m^2)	96
ANALYSIS OF SLABS.	
Fixity ratio for continuos supports on any side	
Fixity ratio short span left side i1	0
Fixity ratio long span bottom i2	0
Fixity ratio short span rigth side i3	0
Fixity ratio long span top i4	0
Reduced sides ar'	0.364
Reduced sides br'	0.407
Bending moments in slab (kN.m)	0.590137

Appendix 17: Experimental result of waffle slab W5

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY
CODE: B.S. 8110: Part 1 1997
Designed By : Akinyele J.O
                    Simply supported on 3 sides

Slab Long Span (ly) (m)	1.312
Slab Short Span (lx) (m)	0.3
Depth of Slab part (mm)	0
Distance between ribs on long span (m)	0
Distance between ribs on short span (m)	0
Rib width (mm)	0
Rib depth (mm)	0
Total depth of slab (mm)	0
Cover to reinforcement (mm)	0
Effective depth of slab (mm)	0
Slab dead load (kN/m^2)	0
Slab imposed load (kN/m^2)	15.01
Finishes and Partition load	5.32
Total load on Slab (kN/m^2)	20.33
For case A, = h_1+h_3 <b	
I_1= fixity1 for left side of short span	0
I_2= fixity2 for supported side of long span	0
I_3= fixity3 for right side of short span	0
Reduced span br (m)	1.312
K'= redistribution factor	0.152439
h= factor to determine yield length	0.236725
1st Bending moment in slab (kNm)	0.947269
For case B, = h_1+h_3 =b	
b' = dimensions	0.6
2nd.Bending moment in slab (kNm)	0.657679

Appendix 18: Experimental result of waffle slab W6

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY
CODE: B.S. 8110: Part 1 1997
Designed By : Akinyele J.O

Simply supported on 3 sides

Slab Long Span (ly) (m)	0.86
Slab Short Span (lx) (m)	0.36
Depth of Slab part (mm)	0
Distance between ribs on long span (m)	0
Distance between ribs on short span (m)	0
Rib width (mm)	0
Rib depth (mm)	0
Total depth of slab (mm)	0
Cover to reinforcement (mm)	5
Effective depth of slab (mm)	0
Slab dead load (kN/m^2)	0
Slab imposed load (kN/m^2)	33.44
Finishes and Partition load	5.32
Total load on Slab (kN/m^2)	38.76
For case A, = $h_1 + h_3$ <b	
l_1= fixity1 for left side of short span	0
l_2= fixity2 for supported side of long span	0
l_3= fixity3 for right side of short span	0
Reduced span br (m)	0.86
K'= redistribution factor	0.27907
h= factor to determine yield length	0.255007
1st Bending moment in slab (kNm)	1.27624
For case B, = h1+h3 =b	
b' = dimensions	0.72
2nd.Bending moment in slab (kNm)	1.1147

Appendix 19: Experimental result of solid slab S1

ANALYSIS OF WAFFLE SLABS USING YIELD LINE THEORY
CODE B.S.8110 PART1
1997

Designed by Akinyele J.O

SUPPORTED ON TWO SIDE (simply supported)

Slab Long span (Ly) (m)	1.353
Slab Short span (Lx) (m)	0.43
Depth of Slab part (mm)	0
Distance between ribs on Long Span (m)	0
Distance between ribs on Short Span (m)	0
Rib width (mm)	0
Depth of ribs (mm)	0
Total depth of slab (mm)	0
Cover to reinforcement (mm)	0
Effective depth of slab (mm)	0
Slab dead load (kN/m^2)	0
Slab imposed load (kN/m^2)	192
Total load on slab (kN/m^2)	192
Load distance from 1st Support	0.5515
Load distance from 2nd support	0.5515
Length of UDL	0.25
UDL obtained kN/m	48
Reactions at supports kN	6
Bending moments in slab kNm	3.684

Appendix 20: Experimental result of solid slab S2

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY.
CODE B.S. 8110: PART 1: 1997.
Designed By : Engr. Akinyele J.O
<div align="center">Simply supported on all four sides.</div>

Slab Length(Long span) (ly) (m)	0.9
Slab Width (Short span) (lx) (m)	0.3
Rib width (mm)	0
Distance between ribs on Long span (m)	0
Distance between ribs on Short span (m)	0
Depth of slab portion (mm)	0
Depth of ribs (mm)	0
Total depth of slab (mm)	0
Effective depth of slab (mm)	0
Slab dead load kN/m^2	0
Imposed load on slab kN/m^2	112
Finishes and Partition load	0
Total load on slab (kN/m^2)	112
ANALYSIS OF SLABS.	
Fixity ratio for continuous supports on any side	
Fixity ratio short span left side i_1	0
Fixity ratio long span bottom i_2	0
Fixity ratio short span right side i_3	0
Fixity ratio long span top i_4	0
Reduced sides ar'	0.3
Reduced sides br'	0.9
Bending moments in slab (kN.m)	0.872308

Appendix 21: Experimental result of solid slab S3

ANALYSIS OF WAFFLE SLABS USING YIELD LINE THEORY
CODE B.S.8110 PART1
1997
Designed by Akinyele J.O
SUPPORTED ON TWO SIDE (simply supported)

Slab Long span (Ly) (m)	1.085
Slab Short span (Lx) (m)	0.43
Depth of Slab part (mm)	0
Distance between ribs on Long Span (m)	0
Distance between ribs on Short Span (m)	0
Rib width (mm)	0
Depth of ribs (mm)	0
Total depth of slab (mm)	0
Cover to reinforcement (mm)	0
Effective depth of slab (mm)	0
Slab dead load (kN/m^2)	0
Slab imposed load (kN/m^2)	244
Total load on slab (kN/m^2)	244
Load distance from 1st Support	0.4175
Load distance from 2nd support	0.4175
Length of UDL	0.25
UDL obtained kN/m	56
Reactions at supports kN	7
Bending moments in slab kNm	3.360

Appendix 22: Experimental result of solid slab S4

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY.
CODE B.S. 8110: PART 1: 1997.
Designed By : Engr. Akinyele J.O

<center>Simply supported on all four sides.</center>

Slab Length(Long span) (ly) (m)	0.407
Slab Width (Short span) (lx) (m)	0.364
Rib width (mm)	0
Distance between ribs on Long span (m)	0
Distance between ribs on Short span (m)	0
Depth of slab portion (mm)	0
Depth of ribs (mm)	0
Total depth of slab (mm)	0
Effective depth of slab (mm)	0
Slab dead load kN/m^2	0
Imposed load on slab kN/m^2	96
Finishes and Partition load	0
Total load on slab (KN/m^2)	96
ANALYSIS OF SLABS.	
Fixity ratio for continuous supports on any side	
Fixity ratio short span left side i_1	0
Fixity ratio long span bottom i_2	0
Fixity ratio short span right side i_3	0
Fixity ratio long span top i_4	0
Reduced sides ar'	0.364
Reduced sides br'	0.407
Bending moments in slab (kN.m)	0.590152

Appendix 23: Experimental result of solid slab S5

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY
CODE: B.S. 8110: Part 1 1997
Designed By : Akinyele J.O
Simply supported on 3 sides

Slab Long Span (ly) (m)	1.312
Slab Short Span (lx) (m)	0.3
Depth of Slab part (mm)	0
Distance between ribs on long span (m)	0
Distance between ribs on short span (m)	0
Rib width (mm)	0
Rib depth (mm)	0
Total depth of slab (mm)	0
Cover to reinforcement (mm)	5
Effective depth of slab (mm)	0
Slab dead load (kN/m^2)	0
Slab imposed load (kN/m^2)	15.24
Finishes and Partition load	0
Total load on Slab (kN/m^2)	15.24
For case A, = h_1+h_3 <b	
I_1 = fixity1 for left side of short span	0
I_2 = fixity2 for supported side of long span	0
I_3 = fixity3 for right side of short span	0
Reduced span br (m)	1.312
K'= redistribution factor	0.152439
H= factor to determine yield length	0.236725
1st Bending moment in slab (KNm)	0.71066
For case B, = h_1+h_3 =b	
B' = dimensions	0.6
2nd.Bending moment in slab (KNm)	0.4932

Appendix 24: Experimental result of solid slab S6

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY
CODE: B.S. 8110: Part 1 1997
Designed By : Akinyele J.O
           Simply supported on 3 sides

Slab Long Span (ly) (m)	0.86
Slab Short Span (lx) (m)	0.36
Depth of Slab part (mm)	0
Distance between ribs on long span (m)	0
Distance between ribs on short span (m)	0
Rib width (mm)	0
Rib depth (mm)	0
Total depth of slab (mm)	0
Cover to reinforcement (mm)	5
Effective depth of slab (mm)	5
Slab dead load (kN/m^2)	0
Slab imposed load (kN/m^2)	32.3
Finishes and Partition load	0
Total load on Slab (kN/m^2)	32.3
For case A, = h_1+h_3 <b	
I_1 = fixity1 for left side of short span	0
I_2 = fixity2 for supported side of long span	0
I_3 = fixity3 for right side of short span	0
Reduced span br (m)	0.86
K'= redistribution factor	0.27907
H= factor to determine yield length	0.255007
1st Bending moment in slab (kNm)	1.062447
For case B, = h1+h3 =b	
B' = dimensions	0.72
2nd.Bending moment in slab (kNm)	0.988246

Appendix 25: Result of waffle slab using the example in Mosley et all

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY.
CODE B.S. 8110: PART 1: 1997.
Designed By : Engr. Akinyele J.O

<div align="center">Simple supported on all four sides.</div>

Slab Length(Long span) (ly) (m)	6
Slab Width (Short span) (lx) (m)	5
Rib width (mm)	125
Distance between ribs on Long span (m)	0.4
Distance between ribs on Short span (m)	0.4
Depth of slab portion (mm)	60
Depth of ribs (mm)	140
Total depth of slab (mm)	200
Effective depth of slab (mm)	40
Slab dead load kN/m^2	2.016
Imposed load on slab kN/m^2	10.384
Finishes and Partition load	0
Total load on slab (kN/m^2)	12.4

ANALYSIS OF SLABS.

Fixity ratio for continuous supports on any side	
Fixity ratio short span left side i1	1
Fixity ratio long span bottom i2	1
Fixity ratio short span right side i3	1
Fixity ratio long span top i4	1
Reduced sides ar'	3.535534
Reduced sides br'	4.242641
Bending moments in slab (KN.m)	7.664835

<div align="center">DESIGN</div>

Cover to reinforcement. (mm)	20
Concrete characteristic stress (N/mm^2)	30
Steel Characteristic Stress (N/mm^2)	460
Maximum lever arm factor	0.95
Maximum K- Value	0.156
Moment M	7664835
K- value	0.159684
Lever arm factor	0.769394
Used Lever arm	0.769394
Area of steel (mm^2)	461.5702
Provide	T8@100mmc/c(503mm2)

ANALYSIS AND DESIGN OF RIBS.

Adopting the Rankine Grashoff Theory.

Number of Ribs on Long Span (ly)	10
Number of Ribs on Short Span (lx)	8

Weight of Slabs (kN/m^2)	1.44
Total Load on Slab (kN)	43.2
Weight of Ribs (kN/m)	0.42
Total weight of Ribs on long span (ly), (kN)	25.2
Total weight of Ribs on Short span (lx), (kN)	16.8
Total weight of Finishes (kN)	30
Total Live Load (kN)	45
Total Load on grid floor (KN)	160.2
Q=Load per Area (m2)	5.34
Denominator for qx and qy	1921
Load sheared on short span (qx), (kN/m^2)	3.602624
Load sheared on Long span (qy), (kN/m^2)	1.737376
Moment on short span Mx (kN.m)	4.50328
Moment on long span My (kN.m)	3.127277
Shear force on short span, Vx, (kN)	3.602624
Shear force on long span, Vy, (kN)	2.084852

<div align="center">DESIGNED</div>

Steel characteristic stress (N/mm^2)	460
Short span Designed moment (Nmm)	4503280
Cover to ribs (mm)	40
Effective depth of ribs	100
K.value	0.120087
Lever arm Factor	0.841423
Used Lever arm	0.841423
Area of Steel (mm^2)	122.471
Provide 2T10@ bottom(157mm^2)	
Long span Designed moment (Nmm)	3127277
K.value	0.083394
Lever arm Factor	0.896661
Used Lever arm	0.896661
Area of Steel (mm^2)	79.80989
Provide 2T10 Near bottom (157mm^2)	
END DESIGNED	

Appendix 26: Waffle slab result of Mosley et all. Adopting BS 8110 coefficients.

Length = 6m

Breadth = 5m

Total depth of waffle slab = 200mm

Depth of slab portion= 60mm

Depth of ribs= 140mm

Cover to reinforcements= 40mm

Effective depth = 160mm

Spacing between ribs in both axis= 400mm

Designed ultimate load = 12.4kN/m^2

Slab is an internal panel of a floor system

Using the slab coefficient of Table 3.14 of BS 8110.

Ly/lx   = 6/5 = 1.2

βsx = +0.032

βsy = + 0.024

Moment at mid span along short span = 0.032 x 12.4 x 5^2 = 9.92kN/m/m

Moment carried by each rib= 0.4 x 9.92 = 3.97kNm

$$\frac{m}{bd^2 f_{cu}} = \frac{3.97x10^6}{400x160^2 x30} = 0.013$$

$$A_s = \frac{m}{0.95 f_y z} = \frac{3.97x10^6}{0.95x460x0.95x160} = 60mm^2$$

Provide Two T10 bars in each rib along the small span.

Moment at Mid span along long span = 0.024 x 12.4 x 5^2=7.44kNm/m

Moment carried by each rib = 0.4 x 7.44= 2.98kNm

$$\frac{m}{bd^2 f_{cu}} = \frac{2.98x10^6}{400x160^2 x30} = 0.009$$

$$A_s = \frac{m}{0.95 f_y z} = \frac{2.98x10^6}{0.95x460x0.95x160} = 45mm$$

Provide Two T10 bars in each rib along the Long span.

Reinforcements at the top flange (slab portion)

As= 0.12 x b x h/100 = 0.12 x 1000 x 60/100 = 72mm^2 /m.

Provide D98 Mesh (98mm^2).

Shear

From table 3.15 of BS 8110, the shear force coefficient for a continuous edge supports is 0.33. Hence, for one rib, the shear at supports

$V_{sx} = \beta_{vx}nl_x$ x b = 0.33 x 12.4 x 5 x 0.4 = 8.18kN.

Maximum shear in the rib 0.6m from the centre line is

V = 8.18 – 0.6 x 12.4 x 0.4 = 5.20kN.

Appendix 27: Result of waffle W1 Theoretical (ETABS)

```
ETABS v9.0.0  File:WAFFLE W1THEORY  Units:kN-m

PROJECT INFORMATION

 Company Name                      = eng

ETABS v9.0.0  File:WAFFLE W1THEORY  Units:kN-m

S T O R Y   D A T A

STORY        SIMILAR TO        HEIGHT   ELEVATION

STORY1      None                 0.750       0.750
BASE        None                             0.000

ETABS v9.0.0  File:WAFFLE W1THEORY  Units:kN-m

S T A T I C   L O A D   C A S E S

STATIC      CASE        AUTO LAT         SELF WT
CASE        TYPE        LOAD             MULTIPLIER

DEAD        DEAD        N/A              0.1000
LIVE        REDUCE LIVE N/A              0.0000

ETABS v9.0.0  File:WAFFLE W1THEORY  Units:kN-m
M A S S   S O U R C E   D A T A

MASS        LATERAL     LUMP MASS
FROM        MASS ONLY   AT STORIES

Masses      Yes         Yes

ETABS v9.0.0  File:WAFFLE W1THEORY  Units:kN-m

 D I A P H R A G M   M A S S   D A T A

STORY  DIAPHRAGM MASS-X    MASS-Y        MMI         X-M          Y-M

STORY1    D1   3.0170     3.0170      1.1743      0.677        0.215


 ETABS v9.0.0  File:WAFFLE W1THEORY  Units:kN-m
A S S E M B L E D   P O I N T   M A S S E S

STORY   UX           UY          UZ        RX          RY           RZ

STORY1 3.017041   3.017041    0.0000     0.0000    0.000000    1.174264
BASE  0.752728   0.752728    0.000000   0.0000    0.000000    0.000000
Totals 3.769769   3.769769    0.000000   0.0000    0.000000    1.174264
```

ETABS v9.0.0 File:WAFFLE W1THEORY Units:kN-m May 9, 2009 4:47

C E N T E R S   O F   C U M U L A T I V E   M A S S   &   C E N T E R
S   O F   R I G I D I T Y

STORY   DIAPHRAGM   /----------CENTRE OF MASS----------//--CENTRE
OF RIGIDITY--/
LEVEL   NAME   MASS ORDINATE-X ORDINATE-Y
ORDINATE-X   ORDINATE-Y

STORY1   D1   3.0170   0.677   0.215
0.676   0.215

ETABS v9.0.0 File:WAFFLE W1THEORY Units:kN-m May 9, 2009 4:47

M O D A L   P E R I O D S   A N D   F R E Q U E N C I E S

MODE   PERIOD   FREQUENCY   CIRCULAR FREQ
NUMBER   (TIME)   (CYCLES/TIME)   (RADIANS/TIME)

Mode 1 0.01249   80.04585   502.94288
Mode 2 0.01096   91.26274   573.42070
Mode 3 0.00990   101.02017   634.72843

ETABS v9.0.0 File:WAFFLE W1THEORY Units:KN-m
M O D A L   P A R T I C I P A T I N G   M A S S   R A T I O S

MODE   X-TRANS   Y-TRANS   Z-TRANS   RX-
ROTN   RY-ROTN   RZ-ROTN
NUMBER   %MASS <SUM>   %MASS <SUM>   %MASS <SUM>   %MASS
<SUM>   %MASS <SUM>   %MASS <SUM>

Mode 1   0.00 < 0>   100.00 <100>   0.00 < 0>   100.00
<100>   0.00 < 0>   0.00 < 0>
Mode 2   100.00 <100>   0.00 <100>   0.00 < 0>   0.00
<100>   100.00 <100>   0.00 < 0>
Mode 3   0.00 <100>   0.00 <100>   0.00 < 0>   0.00
<100>   0.00 <100>   100.00 <100>

ETABS v9.0.0 File:WAFFLE W1THEORY Units:kN-m

M O D A L   L O A D   P A R T I C I P A T I O N   R A T I O S
(STATIC AND DYNAMIC RATIOS ARE IN PERCENT)
TYPE   NAME   STATIC   DYNAMIC
Load   DEAD   0.0000   0.0000
Load   LIVE   0.0000   0.0000
Accel   UX   100.0000   100.0000
Accel   UY   100.0000   100.0000
Accel   UZ   0.0000   0.0000
Accel   RX   100.0000   100.0000

```
Accel        RY                     100.0000    100.0000
Accel        RZ                     100.0000    100.0000


     ETABS v9.0.0   File:WAFFLE W1THEORY   Units:KN-m

     TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

     LOAD        FX           FY          FZ          MX          MY          MZ

     DEAD        0.00        0.00        7.10        1.525      -4.800       0.00
     LIVE        0.00        0.00        1.86        0.400      -1.259       0.000
```

Appendix 28: Result of waffle W2 Theoretical (ETABS)

```
ETABS v9.0.0  File:WAFFLE W2THEORY  Units:kN-m
 PROJECT INFORMATION

  Company Name                      = eng

 ETABS v9.0.0  File:WAFFLE W2THEORY  Units:kN-m
 S T O R Y   D A T A

  STORY        SIMILAR TO        HEIGHT    ELEVATION

  STORY1      None                 0.750       0.750
  BASE        None                             0.000

 ETABS v9.0.0  File:WAFFLE W2THEORY  Units:kN-m

 S T A T I C   L O A D   C A S E S

  STATIC       CASE         AUTO LAT         SELF WT
  CASE         TYPE         LOAD          MULTIPLIER

  DEAD         DEAD         N/A              0.1000
  LIVE         REDUCE LIVE N/A              0.0000

 ETABS v9.0.0  File:WAFFLE W2THEORY  Units:kN-m

 M A S S   S O U R C E   D A T A

  MASS         LATERAL      LUMP MASS
  FROM         MASS ONLY    AT STORIES

  Masses       Yes          Yes

 ETABS v9.0.0  File:WAFFLE W2THEORY  Units:kN-m

 D I A P H R A G M   M A S S   D A T A

  STORY DIAPHRAGM  MASS-X  MASS-Y     MMI          X-M          Y-M

  STORY1  D1       2.2222  2.2222    0.4092       0.450        0.150

 ETABS v9.0.0  File:WAFFLE W2THEORY  Units:kN-m

 A S S E M B L E D   P O I N T   M A S S E S

  STORY     UX        UY         UZ        RX          RY          RZ

  STORY1 2.222178  2.222178  0.000000  0.000000    0.000000    0.409217
  BASE   0.752728  0.752728  0.000000  0.000000    0.000000    0.000000
  Totals 2.974906  2.974906  0.000000  0.000000    0.000000    0.409217
```

ETABS v9.0.0 File:WAFFLE W2THEORY Units:kN-m

C E N T E R S   O F   C U M U L A T I V E   M A S S   &   C E N T E R S   O F   R I G I D I T Y

STORY   DIAPHRAGM   /----------CENTRE OF MASS----------//--CENTRE OF RIGIDITY--/
 LEVEL   NAME   MASS ORDINATE-X ORDINATE-Y   ORDINATE-X   ORDINATE-Y

 STORY1   D1   2.2222   0.450   0.150   0.450   0.150

ETABS v9.0.0 File:WAFFLE W2THEORY Units:kN-m

M O D A L   P E R I O D S   A N D   F R E Q U E N C I E S

MODE NUMBER	PERIOD (TIME)	FREQUENCY (CYCLES/TIME)	CIRCULAR FREQ (RADIANS/TIME)
Mode 1	0.01249	80.04847	502.95938
Mode 2	0.00917	109.05392	685.20598
Mode 3	0.00882	113.41486	712.60661

ETABS v9.0.0 File:WAFFLE W2THEORY Units:kN-m

M O D A L   P A R T I C I P A T I N G   M A S S   R A T I O S

MODE   X-TRANS Y-TRANS Z-TRANS RX-ROTN   RY-ROTN   RZ-ROTN
 NUMBER   %MASS <SUM>   %MASS <SUM>   %MASS <SUM>   %MASS
<SUM>   %MASS <SUM>   %MASS <SUM>

 Mode 1   0.00 < 0>   100.00 <100>   0.00 < 0>   100.00 <100>   0.00 < 0>   0.00 < 0>
 Mode 2   100.00 <100>   0.00 <100>   0.00 < 0>   0.00 <100>   100.00 <100>   0.00 < 0>
 Mode 3   0.00 <100>   0.00 <100>   0.00 < 0>   0.00 <100>   0.00 <100>   100.00 <100>

ETABS v9.0.0 File:WAFFLE W2THEORY Units:kN-m

M O D A L   L O A D   P A R T I C I P A T I O N   R A T I O S
 (STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

TYPE	NAME	STATIC	DYNAMIC
Load	DEAD	0.0000	0.0000
Load	LIVE	0.0000	0.0000
Accel	UX	100.0000	100.0000
Accel	UY	100.0000	100.0000
Accel	UZ	0.0000	0.0000
Accel	RX	100.0000	100.0000
Accel	RY	100.0000	100.0000
Accel	RZ	100.0000	100.0000

```
ETABS v9.0.0   File:WAFFLE W2THEORY   Units:kN-m

TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

LOAD  FX           FY           FZ           MX           MY           MZ

DEAD  0.00         0.00         3.11         0.467        -1.401       0.000
LIVE  0.00         0.00         0.23         0.035        -0.105       0.000
```

Appendix 29: Result of waffle W3 Theoretical (ETABS)

```
ETABS v9.0.0  File:WAFFLE W3 THEORY  Units:kN-m

 PROJECT INFORMATION

  Company Name                      = eng

ETABS v9.0.0  File:WAFFLE W3 THEORY  Units:kN-m
S T O R Y   D A T A

 STORY          SIMILAR TO        HEIGHT    ELEVATION

 STORY1     None                 0.750       0.750
 BASE       None                             0.000

ETABS v9.0.0  File:WAFFLE W3 THEORY  Units:kN-m
S T A T I C   L O A D   C A S E S

 STATIC        CASE         AUTO LAT        SELF WT
 CASE          TYPE         LOAD          MULTIPLIER

 DEAD          DEAD         N/A              0.1000
 LIVE          REDUCE LIVE  N/A              0.0000

ETABS v9.0.0  File:WAFFLE W3 THEORY  Units:kN-m
M A S S   S O U R C E   D A T A

 MASS          LATERAL      LUMP MASS
 FROM          MASS ONLY    AT STORIES

 Masses        Yes          Yes

ETABS v9.0.0  File:WAFFLE W3 THEORY  Units:kN-m
D I A P H R A G M   M A S S   D A T A

 STORY  DIAPHRAGM  MASS-X  MASS-Y   MMI        X-M         Y-M

 STORY1   D1       2.5350  2.5350  0.6810     0.543       0.215

ETABS v9.0.0  File:WAFFLE W3 THEORY  Units:kN-m

A S S E M B L E D   P O I N T   M A S S E S

 STORY   UX         UY         UZ         RX         RY      RZ

 STORY1 2.535048   2.535048   0.000000   0.000000   0.000   0.680962
 BASE   0.752728   0.752728   0.000000   0.000000   0.0000  0.000000
 Totals 3.287776   3.287776   0.000000   0.000000   0.0000  0.680962
```

ETABS v9.0.0 File:WAFFLE W3 THEORY Units:kN-m

C E N T E R S   O F   C U M U L A T I V E   M A S S   &   C E N T E R S   O F   R I G I D I T Y

STORY	DIAPHRAGM		/----------CENTRE OF MASS----------/		//--CENTRE OF RIGIDITY--/	
LEVEL	NAME	MASS	ORDINATE-X	ORDINATE-Y	ORDINATE-X	ORDINATE-Y
STORY1	D1	2.5350	0.543	0.215	0.542	0.215

ETABS v9.0.0 File:WAFFLE W3 THEORY Units:kN-m

M O D A L   P E R I O D S   A N D   F R E Q U E N C I E S

MODE NUMBER	PERIOD (TIME)	FREQUENCY (CYCLES/TIME)	CIRCULAR FREQ (RADIANS/TIME)
Mode 1	0.01145	87.32430	548.67473
Mode 2	0.00993	100.66678	632.50804
Mode 3	0.00892	112.09162	704.29241

ETABS v9.0.0 File:WAFFLE W3 THEORY Units:kN-m

M O D A L   P A R T I C I P A T I N G   M A S S   R A T I O S

MODE NUMBER	X-TRANS %MASS	<SUM>	Y-TRANS %MASS	<SUM>	Z-TRANS %MASS	<SUM>	RX-ROTN %MASS	<SUM>	RY-ROTN %MASS	<SUM>	RZ-ROTN %MASS	<SUM>
Mode 1	0.00	< 0>	100.00	<100>	0.00	< 0>	100.00	<100>	0.00	< 0>	0.00	< 0>
Mode 2	100.00	<100>	0.00	<100>	0.00	< 0>	0.00	<100>	100.00	<100>	0.00	< 0>
Mode 3	0.00	<100>	0.00	<100>	0.00	< 0>	0.00	<100>	0.00	<100>	100.00	<100>

ETABS v9.0.0 File:WAFFLE W3 THEORY Units:kN-m

M O D A L   L O A D   P A R T I C I P A T I O N   R A T I O S
(STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

TYPE	NAME	STATIC	DYNAMIC
Load	DEAD	0.0000	0.0000
Load	LIVE	0.0000	0.0000
Accel	UX	100.0000	100.0000
Accel	UY	100.0000	100.0000
Accel	UZ	0.0000	0.0000
Accel	RX	100.0000	100.0000
Accel	RY	100.0000	100.0000
Accel	RZ	100.0000	100.0000

```
ETABS v9.0.0  File:WAFFLE W3 THEORY  Units:kN-m


TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

LOAD   FX           FY           FZ           MX           MY           MZ

DEAD 0.00         0.00         6.01         1.292        -3.260        0.000
LIVE 0.00         0.00         1.49         0.321        -0.810        0.000
```

Appendix 30: Result of waffle W4 Theoretical (ETABS)

```
ETABS v9.0.0  File:WAFFLE W4THEORY  Units:kN-m
 PROJECT INFORMATION

  Company Name                    = eng

 ETABS v9.0.0  File:WAFFLE W4THEORY  Units:kN-m

 S T O R Y   D A T A

 STORY        SIMILAR TO       HEIGHT    ELEVATION

 STORY1       None                0.750       0.750
 BASE         None                            0.000

 ETABS v9.0.0  File:WAFFLE W4THEORY  Units:kN-m

 S T A T I C   L O A D   C A S E S

 STATIC       CASE         AUTO LAT        SELF WT
 CASE         TYPE         LOAD          MULTIPLIER

 DEAD         DEAD         N/A              0.1000
 LIVE         REDUCE LIVE  N/A              0.0000

 ETABS v9.0.0  File:WAFFLE W4THEORY  Units:kN-m

 M A S S   S O U R C E   D A T A

 MASS         LATERAL      LUMP MASS
 FROM         MASS ONLY    AT STORIES

 Masses       Yes          Yes

 ETABS v9.0.0  File:WAFFLE W4THEORY  Units:kN-m

 D I A P H R A G M   M A S S   D A T A

 STORY  DIAPHRAGM  MASS-X    MASS-Y     MMI          X-M          Y-M

 STORY1  D1         1.6142    1.6142    0.1072       0.204        0.182

 ETABS v9.0.0  File:WAFFLE W4THEORY  Units:kN-m

 A S S E M B L E D   P O I N T   M A S S E S

 STORY     UX       UY        UZ          RX           RY           RZ

 STORY1 1.614198 1.614198  0.000000    0.000000     0.000000     0.107212
 BASE   0.752728 0.752728  0.000000    0.000000     0.000000     0.000000
 Totals 2.366926 2.366926  0.000000    0.000000     0.000000     0.107212
```

```
ETABS v9.0.0  File:WAFFLE W4THEORY  Units:kN-m

 C E N T E R S   O F   C U M U L A T I V E   M A S S   &   C E N T E R
S   O F   R I G I D I T Y

 STORY     DIAPHRAGM    /----------CENTRE OF MASS----------//--CENTRE OF
RIGIDITY--/
 LEVEL    NAME   MASS  ORDINATE-X  ORDINATE-Y  ORDINATE-X  ORDINATE-Y

 STORY1  D1    1.6142   0.204       0.182        0.203       0.182


 ETABS v9.0.0  File:WAFFLE W4THEORY  Units:kN-m

 M O D A L   P E R I O D S   A N D   F R E Q U E N C I E S

 MODE      PERIOD               FREQUENCY            CIRCULAR FREQ
 NUMBER    (TIME)              (CYCLES/TIME)         (RADIANS/TIME)

 Mode 1   0.00974            102.65408              644.99462
 Mode 2   0.00934            107.10545              672.96340
 Mode 3   0.00686            145.86851              916.51886


 ETABS v9.0.0  File:WAFFLE W4THEORY  Units:kN-m

 M O D A L   P A R T I C I P A T I N G   M A S S   R A T I O S

 MODE   X-TRANS Y-TRANS Z-TRANS  RX-ROTN       RY-ROTN        RZ-ROTN
 NUMBER         %MASS <SUM>     %MASS <SUM>    %MASS <SUM>     %MASS
<SUM>    %MASS <SUM>     %MASS <SUM>

 Mode 1         0.00 <  0>   100.00 <100>     0.00 <  0>   100.00
<100>     0.00 <  0>    0.00 <  0>
 Mode 2       100.00 <100>     0.00 <100>     0.00 <  0>     0.00
<100>   100.00 <100>     0.00 <  0>
 Mode 3         0.00 <100>     0.00 <100>     0.00 <  0>     0.00
<100>     0.00 <100>   100.00 <100>


 ETABS v9.0.0  File:WAFFLE W4THEORY  Units:kN-m

 M O D A L   L O A D   P A R T I C I P A T I O N   R A T I O S
 (STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

 TYPE         NAME             STATIC      DYNAMIC

 Load         DEAD            0.0000      0.0000
 Load         LIVE            0.0000      0.0000
 Accel        UX            100.0000    100.0000
 Accel        UY            100.0000    100.0000
 Accel        UZ              0.0000      0.0000
 Accel        RX            100.0000    100.0000
 Accel        RY            100.0000    100.0000
```

```
Accel        RZ                    100.0000     100.0000


ETABS v9.0.0   File:WAFFLE W4THEORY   Units:kN-m

TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

LOAD   FX      FY        FZ         MX         MY         MZ

DEAD   0.00    0.00      2.32       0.422      -0.473     0.000
LIVE   0.00    0.00      0.03       0.005      -0.005     0.000
```

Appendix 31: Result of waffle W5 Theoretical (ETABS)

```
ETABS v9.0.0  File:WAFFLE W5THEORY  Units:kN-m

 PROJECT INFORMATION

  Company Name                    = eng

 ETABS v9.0.0  File:WAFFLE W5THEORY  Units:kN-m

 S T O R Y   D A T A

 STORY        SIMILAR TO       HEIGHT   ELEVATION

 STORY1       None                0.750      0.750
 BASE         None                           0.000

 ETABS v9.0.0  File:WAFFLE W5THEORY  Units:kN-m

 S T A T I C   L O A D   C A S E S

 STATIC       CASE         AUTO LAT        SELF WT
 CASE         TYPE         LOAD          MULTIPLIER

 DEAD         DEAD         N/A              0.1000
 LIVE         REDUCE LIVE N/A              0.0000

 ETABS v9.0.0  File:WAFFLE W5THEORY  Units:kN-m

 M A S S   S O U R C E   D A T A

 MASS         LATERAL      LUMP MASS
 FROM         MASS ONLY    AT STORIES

 Masses       Yes          Yes

 ETABS v9.0.0  File:WAFFLE W5THEORY  Units:kN-m

 D I A P H R A G M   M A S S   D A T A

 STORY  DIAPHRAGM  MASS-X  MASS-Y    MMI        X-M         Y-M

 STORY1  D1        2.0627  2.0627  0.7605     0.656       0.150

 ETABS v9.0.0  File:WAFFLE W5THEORY  Units:kN-m

 A S S E M B L E D   P O I N T   M A S S E S

 STORY   UX         UY          UZ         RX          RY          RZ

 STORY1 2.062712  2.062712  0.000000   0.000000    0.000000    0.760534
 BASE   0.752728  0.752728  0.000000   0.000000    0.000000    0.000000
 Totals 2.815440  2.815440  0.000000   0.000000    0.000000    0.760534
```

```
   ETABS v9.0.0  File:WAFFLE W5THEORY  Units:kN-m

 C E N T E R S   O F   C U M U L A T I V E   M A S S   &   C E N T E R
S   O F   R I G I D I T Y

  STORY        DIAPHRAGM   /----------CENTRE OF MASS----------//--CENTRE
OF RIGIDITY--/
  LEVEL        NAME MASS  ORDINATE-X  ORDINATE-Y  ORDINATE-X  ORDINATE-Y

  STORY1       D1   2.0627   0.656       0.150        0.656        0.150


   ETABS v9.0.0  File:WAFFLE W5THEORY  Units:kN-m

 M O D A L   P E R I O D S   A N D   F R E Q U E N C I E S

  MODE      PERIOD              FREQUENCY            CIRCULAR FREQ
  NUMBER  (TIME)              (CYCLES/TIME)        (RADIANS/TIME)

  Mode 1  0.01247              80.18465             503.81500
  Mode 2  0.00954             104.83219             658.68006
  Mode 3  0.00954             104.87250             658.93333


   ETABS v9.0.0  File:WAFFLE W5THEORY  Units:kN-m

 M O D A L   P A R T I C I P A T I N G   M A S S   R A T I O S

  MODE    X-TRANS   Y-TRANS  Z-TRANS  RX-ROTN       RY-ROTN        RZ-ROTN
  NUMBER         %MASS <SUM>     %MASS <SUM>     %MASS <SUM>     %MASS
<SUM>     %MASS <SUM>     %MASS <SUM>

 Mode 1          0.00 <  0>   100.00 <100>      0.00 <  0>   100.00
<100>      0.00 <  0>     0.00 <  0>
 Mode 2        100.00 <100>     0.00 <100>      0.00 <  0>     0.00
<100>   100.00 <100>     0.00 <  0>
 Mode 3          0.00 <100>     0.00 <100>      0.00 <  0>     0.00
<100>      0.00 <100>   100.00 <100>


   ETABS v9.0.0  File:WAFFLE W5THEORY  Units:kN-m

 M O D A L   L O A D   P A R T I C I P A T I O N   R A T I O S
  (STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

  TYPE        NAME            STATIC     DYNAMIC

  Load        DEAD            0.0000      0.0000
  Load        LIVE            0.0000      0.0000
  Accel       UX            100.0000    100.0000
  Accel       UY            100.0000    100.0000
  Accel       UZ              0.0000      0.0000
  Accel       RX            100.0000    100.0000
  Accel       RY            100.0000    100.0000
  Accel       RZ            100.0000    100.0000
```

```
ETABS v9.0.0  File:WAFFLE W5THEORY  Units:kN-m


TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

LOAD  FX          FY          FZ          MX          MY          MZ

DEAD  0.00        0.00        3.06        0.460       -2.010      0.000
LIVE  0.00        0.00        0.72        0.108       -0.473      0.000
```

Appendix 32: Result of waffle W6 Theoretical (ETABS)

ETABS v9.0.0 File:WAFFLE W6THEORY Units:kN-m

 PROJECT INFORMATION

  Company Name   = eng

 ETABS v9.0.0 File:WAFFLE W6THEORY Units:kN-m

 S T O R Y   D A T A

 STORY   SIMILAR TO   HEIGHT   ELEVATION

 STORY1   None   0.750   0.750
 BASE   None   0.000

 ETABS v9.0.0 File:WAFFLE W6THEORY Units:kN-m

 S T A T I C   L O A D   C A S E S

 STATIC   CASE   AUTO LAT   SELF WT
 CASE   TYPE   LOAD   MULTIPLIER

 DEAD   DEAD   N/A   0.1000
 LIVE   REDUCE LIVE N/A   0.0000

 ETABS v9.0.0 File:WAFFLE W6THEORY Units:kN-m

 M A S S   S O U R C E   D A T A

 MASS   LATERAL   LUMP MASS
 FROM   MASS ONLY   AT STORIES

 Masses   Yes   Yes

 ETABS v9.0.0 File:WAFFLE W6THEORY Units:kN-m

 D I A P H R A G M   M A S S   D A T A

 STORY DIAPHRAGM MASS-X MASS-Y   MMI   X-M   Y-M

 STORY1 D1   1.6898 1.6898   0.3085   0.430   0.180

 ETABS v9.0.0 File:WAFFLE W6THEORY Units:kN-m

 A S S E M B L E D   P O I N T   M A S S E S

 STORY   UX   UY   UZ   RX   RY   RZ

 STORY1 1.689796 1.689796   0.000000   0.000000   0.000000   0.308515
 BASE   0.752728 0.752728   0.000000   0.000000   0.000000   0.000000
 Totals 2.442524 2.442524   0.000000   0.000000   0.000000   0.308515

ETABS v9.0.0 File:WAFFLE W6THEORY Units:kN-m

C E N T E R S   O F   C U M U L A T I V E   M A S S   &   C E N T E R S   O F   R I G I D I T Y

STORY   DIAPHRAGM /----------CENTRE OF MASS----------//--CENTRE OF RIGIDITY--/
LEVEL   NAME MASS ORDINATE-X ORDINATE-Y ORDINATE-X ORDINATE-Y

STORY1 D1   1.6898 0.430   0.180   0.430   0.180

ETABS v9.0.0 File:WAFFLE W6THEORY Units:kN-m

M O D A L   P E R I O D S   A N D   F R E Q U E N C I E S

MODE NUMBER	PERIOD (TIME)	FREQUENCY (CYCLES/TIME)	CIRCULAR FREQ (RADIANS/TIME)
Mode 1	0.01037	96.47473	606.16863
Mode 2	0.00834	119.94595	753.64261
Mode 3	0.00780	128.27638	805.98424

ETABS v9.0.0 File:WAFFLE W6THEORY Units:kN-m

M O D A L   P A R T I C I P A T I N G   M A S S   R A T I O S

MODE   X-TRANS   Y-TRANS   Z-TRANS   RX-ROTN   RY-ROTN   RZ-ROTN
NUMBER   %MASS <SUM>   %MASS <SUM>   %MASS <SUM>   %MASS
<SUM>   %MASS <SUM>   %MASS <SUM>

 Mode 1   0.00 < 0>   100.00 <100>   0.00 < 0>   100.00
<100>   0.00 < 0>   0.00 < 0>
 Mode 2   100.00 <100>   0.00 <100>   0.00 < 0>   0.00
<100>   100.00 <100>   0.00 < 0>
 Mode 3   0.00 <100>   0.00 <100>   0.00 < 0>   0.00
<100>   0.00 <100>   100.00 <100>

ETABS v9.0.0 File:WAFFLE W6THEORY Units:kN-m

M O D A L   L O A D   P A R T I C I P A T I O N   R A T I O S
 (STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

TYPE	NAME	STATIC	DYNAMIC
Load	DEAD	0.0000	0.0000
Load	LIVE	0.0000	0.0000
Accel	UX	100.0000	100.0000
Accel	UY	100.0000	100.0000
Accel	UZ	0.0000	0.0000
Accel	RX	100.0000	100.0000
Accel	RY	100.0000	100.0000
Accel	RZ	100.0000	100.0000

```
ETABS v9.0.0  File:WAFFLE W6THEORY  Units:kN-m


TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

LOAD    FX      FY          FZ          MX          MY          MZ

DEAD   0.00    0.00        2.70        0.486       -1.160      0.000
LIVE   0.00    0.00        0.24        0.044       -0.105      0.000
```

Appendix 33: Result of waffle W1 Experimental (ETABS)

```
ETABS v9.0.0  File:WAFFLE W1EXPERIMENT  Units:kN-m


 PROJECT INFORMATION

  Company Name                      = eng

 ETABS v9.0.0  File:WAFFLE W1EXPERIMENT  Units:kN-m

 S T O R Y   D A T A

 STORY        SIMILAR TO        HEIGHT    ELEVATION

 STORY1       None                0.750       0.750
 BASE         None                            0.000

 ETABS v9.0.0  File:WAFFLE W1EXPERIMENT  Units:kN-m

 S T A T I C   L O A D   C A S E S

 STATIC       CASE         AUTO LAT        SELF WT
 CASE         TYPE         LOAD           MULTIPLIER

 DEAD         DEAD         N/A              0.1000
 LIVE         REDUCE LIVE  N/A              0.0000

 ETABS v9.0.0  File:WAFFLE W1EXPERIMENT  Units:kN-m

 M A S S   S O U R C E   D A T A

 MASS         LATERAL      LUMP MASS
 FROM         MASS ONLY    AT STORIES

 Masses       Yes          Yes

 ETABS v9.0.0  File:WAFFLE W1EXPERIMENT  Units:kN-m

 D I A P H R A G M   M A S S   D A T A

 STORY  DIAPHRAGM  MASS-X  MASS-Y     MMI       X-M        Y-M

 STORY1  D1        3.0170  3.0170    1.1743    0.677      0.215

 ETABS v9.0.0  File:WAFFLE W1EXPERIMENT  Units:kN-m
```

```
A S S E M B L E D    P O I N T    M A S S E S

STORY     UX        UY        UZ        RX        RY        RZ

STORY1 3.017041 3.017041   0.000000   0.000000   0.000000 1.174264
BASE   0.752728 0.752728   0.000000   0.000000   0.000000 0.000000
Totals 3.769769 3.769769   0.000000   0.000000   0.000000 1.174264


ETABS v9.0.0  File:WAFFLE W1EXPERIMENT  Units:kN-m



C E N T E R S   O F   C U M U L A T I V E   M A S S   &   C E N T E R
S   O F   R I G I D I T Y

 STORY        DIAPHRAGM   /----------CENTRE OF MASS----------//--CENTRE
OF RIGIDITY--/
 LEVEL   NAME   MASS   ORDINATE-X  ORDINATE-Y  ORDINATE-X  ORDINATE-Y

 STORY1  D1    3.0170    0.677       0.215       0.676       0.215


ETABS v9.0.0  File:WAFFLE W1EXPERIMENT  Units:kN-m



M O D A L   P E R I O D S   A N D   F R E Q U E N C I E S

MODE   PERIOD              FREQUENCY           CIRCULAR FREQ
NUMBER (TIME)              (CYCLES/TIME)       (RADIANS/TIME)

 Mode 1 0.01249            80.04585            502.94288
 Mode 2 0.01096            91.26274            573.42070
 Mode 3 0.00990            101.02017           634.72843


ETABS v9.0.0  File:WAFFLE W1EXPERIMENT  Units:kN-m



M O D A L   P A R T I C I P A T I N G   M A S S   R A T I O S

MODE   X-TRANS  Y-TRANS  Z-TRANS  RX-ROTN    RY-ROTN        RZ-ROTN
NUMBER %MASS <SUM>    %MASS <SUM>    %MASS <SUM>    %MASS <SUM>
%MASS <SUM>    %MASS <SUM>

 Mode 1          0.00 <  0>   100.00 <100>     0.00 <  0>   100.00
<100>     0.00 <  0>     0.00 <  0>
 Mode 2        100.00 <100>     0.00 <100>     0.00 <  0>     0.00
<100>   100.00 <100>     0.00 <  0>
 Mode 3          0.00 <100>     0.00 <100>     0.00 <  0>     0.00
<100>     0.00 <100>   100.00 <100>
```

```
ETABS v9.0.0  File:WAFFLE W1EXPERIMENT  Units:kN-m


M O D A L   L O A D   P A R T I C I P A T I O N   R A T I O S
(STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

TYPE          NAME              STATIC    DYNAMIC

Load          DEAD              0.0000     0.0000
Load          LIVE              0.0000     0.0000
Accel         UX              100.0000   100.0000
Accel         UY              100.0000   100.0000
Accel         UZ                0.0000     0.0000
Accel         RX              100.0000   100.0000
Accel         RY              100.0000   100.0000
Accel         RZ              100.0000   100.0000


ETABS v9.0.0  File:WAFFLE W1EXPERIMENT  Units:kN-m


TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

LOAD  FX          FY          FZ          MX          MY          MZ

DEAD  0.00        0.00       15.24       3.277     -10.310       0.000
LIVE  0.00        0.00        6.75       1.452      -4.569       0.000
```

Appendix 34: Result of waffle W2 Experimental (ETABS)

ETABS v9.0.0 File:WAFFLE W2EXPERIMENT Units:kN-m

 PROJECT INFORMATION

  Company Name   = eng

 ETABS v9.0.0 File:WAFFLE W2EXPERIMENT Units:kN-m

 S T O R Y   D A T A

 STORY   SIMILAR TO   HEIGHT   ELEVATION

 STORY1   None   0.750   0.750
 BASE   None   0.000

 ETABS v9.0.0 File:WAFFLE W2EXPERIMENT Units:kN-m

 S T A T I C   L O A D   C A S E S

 STATIC   CASE   AUTO LAT   SELF WT
 CASE   TYPE   LOAD   MULTIPLIER

 DEAD   DEAD   N/A   0.1000
 LIVE   REDUCE LIVE N/A   0.0000

 ETABS v9.0.0 File:WAFFLE W2EXPERIMENT Units:kN-m

 M A S S   S O U R C E   D A T A

 MASS   LATERAL   LUMP MASS
 FROM   MASS ONLY   AT STORIES

 Masses   Yes   Yes

 ETABS v9.0.0 File:WAFFLE W2EXPERIMENT Units:kN-m

 D I A P H R A G M   M A S S   D A T A

 STORY   DIAPHRAGM   MASS-X   MASS-Y   MMI   X-M   Y-M

 STORY1   D1   2.1975   2.1975   0.4053   0.454   0.150

ETABS v9.0.0 File:WAFFLE W2EXPERIMENT Units:kN-m

A S S E M B L E D   P O I N T   M A S S E S

STORY	UX	UY	UZ	RX	RY	RZ
STORY1	2.197480	2.197480	0.000000	0.000000	0.000000	0.405288
BASE	0.752728	0.752728	0.000000	0.000000	0.000000	0.000000
Totals	2.950208	2.950208	0.000000	0.000000	0.000000	0.405288

ETABS v9.0.0 File:WAFFLE W2EXPERIMENT Units:kN-m

C E N T E R S   O F   C U M U L A T I V E   M A S S   &   C E N T E R S   O F   R I G I D I T Y

STORY	DIAPHRAGM	/----------CENTRE OF MASS----------/		//--CENTRE OF RIGIDITY--/	
LEVEL	NAME	MASS	ORDINATE-X	ORDINATE-Y	ORDINATE-X	ORDINATE-Y
STORY1	D1	2.1975	0.454	0.150	0.474	0.150

ETABS v9.0.0 File:WAFFLE W2EXPERIMENT Units:kN-m

M O D A L   P E R I O D S   A N D   F R E Q U E N C I E S

MODE NUMBER	PERIOD (TIME)	FREQUENCY (CYCLES/TIME)	CIRCULAR FREQ (RADIANS/TIME)
Mode 1	0.01283	77.94835	489.76390
Mode 2	0.00922	108.49457	681.69150
Mode 3	0.00892	112.12289	704.48889

ETABS v9.0.0 File:WAFFLE W2EXPERIMENT Units:kN-m

M O D A L   P A R T I C I P A T I N G   M A S S   R A T I O S

MODE NUMBER	X-TRANS %MASS <SUM>	Y-TRANS %MASS <SUM>	Z-TRANS %MASS <SUM>	RX-ROTN %MASS <SUM>	RY-ROTN %MASS <SUM>	RZ-ROTN %MASS <SUM>
Mode 1	0.00 < 0>	99.82 <100>	0.00 < 0>	99.82 <100>	0.00 < 0>	0.16 < 0>
Mode 2	100.00 <100>	0.00 <100>	0.00 < 0>	0.00 <100>	100.00 <100>	0.00 < 0>
Mode 3	0.00 <100>	0.18 <100>	0.00 < 0>	0.18 <100>	0.00 <100>	99.84 <100>

ETABS v9.0.0 File:WAFFLE W2EXPERIMENT Units:kN-m

M O D A L   L O A D   P A R T I C I P A T I O N   R A T I O S
(STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

TYPE	NAME	STATIC	DYNAMIC
Load	DEAD	0.0683	0.0000
Load	LIVE	0.0017	0.0000
Accel	UX	100.0000	100.0000
Accel	UY	100.0000	100.0000
Accel	UZ	0.0000	0.0000
Accel	RX	100.0000	100.0000
Accel	RY	100.0000	100.0000
Accel	RZ	104.3987	100.0000

ETABS v9.0.0 File:WAFFLE W2EXPERIMENT Units:kN-m

TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

LOAD	FX	FY	FZ	MX	MY	MZ
DEAD	0.00	0.00	3.11	0.467	-1.401	0.000
LIVE	0.00	0.00	0.76	0.114	-0.399	0.000

Appendix 35: Result of waffle W3 Experimental (ETABS)

```
ETABS v9.0.0  File:WAFFLE W3EXPERIMENT  Units:kN-m


 PROJECT INFORMATION

  Company Name                     = eng

 ETABS v9.0.0  File:WAFFLE W3EXPERIMENT  Units:kN-m

 S T O R Y   D A T A

 STORY         SIMILAR TO        HEIGHT    ELEVATION

 STORY1        None                 0.750       0.750
 BASE          None                             0.000

 ETABS v9.0.0  File:WAFFLE W3EXPERIMENT  Units:kN-m

 S T A T I C   L O A D   C A S E S

 STATIC        CASE          AUTO LAT          SELF WT
 CASE          TYPE          LOAD           MULTIPLIER

 DEAD          DEAD          N/A               0.1000
 LIVE          REDUCE LIVE   N/A               0.0000

 ETABS v9.0.0  File:WAFFLE W3EXPERIMENT  Units:kN-m
 M A S S   S O U R C E   D A T A

 MASS          LATERAL       LUMP MASS
 FROM          MASS ONLY     AT STORIES

 Masses        Yes           Yes

 ETABS v9.0.0  File:WAFFLE W3EXPERIMENT  Units:kN-m

 D I A P H R A G M   M A S S   D A T A

 STORY  DIAPHRAGM  MASS-X   MASS-Y   MMI        X-M         Y-M

 STORY1  D1        2.5350   2.5350   0.6810     0.543       0.215
```

ETABS v9.0.0 File:WAFFLE W3EXPERIMENT Units:kN-m

A S S E M B L E D   P O I N T   M A S S E S

STORY   UX   UY   UZ   RX   RY   RZ

STORY1 2.535048 2.535048 0.000000 0.000000   0.000000   0.680962
BASE   0.752728 0.752728 0.000000 0.000000   0.000000   0.000000
Totals 3.287776 3.287776 0.000000 0.000000   0.000000   0.680962

ETABS v9.0.0 File:WAFFLE W3EXPERIMENT Units:kN-m

C E N T E R S   O F   C U M U L A T I V E   M A S S   &   C E N T E R
S   O F   R I G I D I T Y

STORY   DIAPHRAGM /----------CENTRE OF MASS----------//--CENTRE OF
RIGIDITY--/
LEVEL   NAME   MASS ORDINATE-X ORDINATE-Y ORDINATE-X ORDINATE-Y

STORY1   D1   2.5350   0.543   0.215   0.542   0.215

ETABS v9.0.0 File:WAFFLE W3EXPERIMENT Units:kN-m

M O D A L   P E R I O D S   A N D   F R E Q U E N C I E S

MODE   PERIOD   FREQUENCY   CIRCULAR FREQ
NUMBER   (TIME)   (CYCLES/TIME)   (RADIANS/TIME)

Mode 1 0.01145   87.32430   548.67473
Mode 2 0.00993   100.66678   632.50804
Mode 3 0.00892   112.09162   704.29241

ETABS v9.0.0 File:WAFFLE W3EXPERIMENT Units:kN-m

M O D A L   P A R T I C I P A T I N G   M A S S   R A T I O S

MODE X-TRANS   Y-TRANS Z-TRANS RX-ROTN   RY-ROTN   RZ-ROTN
NUMBER   %MASS <SUM>   %MASS <SUM>   %MASS <SUM>   %MASS
<SUM>   %MASS <SUM>   %MASS <SUM>

 Mode 1   0.00 < 0>   100.00 <100>   0.00 < 0>   100.00 <100>
0.00 < 0>   0.00 < 0>
 Mode 2 100.00 <100>   0.00 <100>   0.00 < 0>   0.00 <100>
100.00 <100>   0.00 < 0>
 Mode 3 0.00 <100>   0.00 <100>   0.00 < 0>   0.00 <100>
0.00 <100>   100.00 <100>

```
ETABS v9.0.0  File:WAFFLE W3EXPERIMENT  Units:kN-m

M O D A L   L O A D   P A R T I C I P A T I O N   R A T I O S
(STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

TYPE          NAME             STATIC      DYNAMIC

Load          DEAD             0.0000       0.0000
Load          LIVE             0.0000       0.0000
Accel         UX             100.0000     100.0000
Accel         UY             100.0000     100.0000
Accel         UZ               0.0000       0.0000
Accel         RX             100.0000     100.0000
Accel         RY             100.0000     100.0000
Accel         RZ             100.0000     100.0000


ETABS v9.0.0  File:WAFFLE W3EXPERIMENT  Units:kN-m

TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

LOAD   FX          FY          FZ          MX          MY          MZ

DEAD   0.00        0.00        11.02       2.369       -5.978      0.000
LIVE   0.00        0.00         4.51       0.969       -2.445      0.000
```

Appendix 36: Result of waffle W4 Experimental (ETABS)

```
ETABS v9.0.0  File:WAFFLE W4EXPERIMENT  Units:kN-m

 PROJECT INFORMATION

  Company Name                   = eng

 ETABS v9.0.0  File:WAFFLE W4EXPERIMENT  Units:kN-m

 S T O R Y   D A T A

 STORY        SIMILAR TO       HEIGHT   ELEVATION

 STORY1       None             0.750       0.750
 BASE         None                         0.000

 ETABS v9.0.0  File:WAFFLE W4EXPERIMENT  Units:kN-m

 S T A T I C   L O A D   C A S E S

 STATIC       CASE        AUTO LAT       SELF WT
 CASE         TYPE        LOAD         MULTIPLIER

 DEAD         DEAD        N/A            0.1000
 LIVE         REDUCE LIVE N/A            0.0000

 ETABS v9.0.0  File:WAFFLE W4EXPERIMENT  Units:kN-m

 M A S S   S O U R C E   D A T A

 MASS         LATERAL     LUMP MASS
 FROM         MASS ONLY   AT STORIES

 Masses       Yes         Yes

 ETABS v9.0.0  File:WAFFLE W4EXPERIMENT  Units:kN-m

 D I A P H R A G M   M A S S   D A T A

 STORY   DIAPHRAGM  MASS-X   MASS-Y   MMI     X-M        Y-M

 STORY1  D1         1.6154   1.6154   0.1075  0.204      0.182
```

```
ETABS v9.0.0  File:WAFFLE W4EXPERIMENT  Units:kN-m

A S S E M B L E D   P O I N T   M A S S E S

STORY   UX        UY        UZ        RX        RY        RZ

STORY1 1.615446 1.615446 0.000000 0.000000 0.000000  0.107542
BASE   0.752728 0.752728 0.000000 0.000000 0.000000  0.000000
Totals 2.368174 2.368174 0.000000 0.000000 0.000000  0.107542

ETABS v9.0.0  File:WAFFLE W4EXPERIMENT  Units:kN-m

C E N T E R S   O F   C U M U L A T I V E   M A S S   &   C E N T E R S
O F   R I G I D I T Y

STORY      DIAPHRAGM  /----------CENTRE OF MASS----------//--CENTRE OF
RIGIDITY--/
LEVEL   NAME  MASS  ORDINATE-X  ORDINATE-Y  ORDINATE-X  ORDINATE-Y

STORY1  D1   1.6154   0.204       0.182        0.204       0.182


ETABS v9.0.0  File:WAFFLE W4EXPERIMENT  Units:kN-m

M O D A L   P E R I O D S   A N D   F R E Q U E N C I E S

MODE      PERIOD            FREQUENCY            CIRCULAR FREQ
NUMBER    (TIME)            (CYCLES/TIME)        (RADIANS/TIME)

Mode 1  0.00973            102.72929             645.46719
Mode 2  0.00934            107.06637             672.71782
Mode 3  0.00686            145.82088             916.21964

ETABS v9.0.0  File:WAFFLE W4EXPERIMENT  Units:kN-m

M O D A L   P A R T I C I P A T I N G   M A S S   R A T I O S

MODE   X-TRANS  Y-TRANS Z-TRANS RX-ROTN   RY-ROTN      RZ-ROTN
NUMBER %MASS <SUM>    %MASS <SUM>    %MASS <SUM>    %MASS <SUM>
%MASS <SUM>    %MASS <SUM>

Mode 1          0.00 < 0>  100.00 <100>    0.00 < 0>  100.00
<100>    0.00 < 0>    0.00 < 0>
Mode 2        100.00 <100>    0.00 <100>    0.00 < 0>    0.00
<100>  100.00 <100>    0.00 < 0>
Mode 3          0.00 <100>    0.00 <100>    0.00 < 0>    0.00
<100>    0.00 <100>  100.00 <100>
```

```
ETABS v9.0.0  File:WAFFLE W4EXPERIMENT  Units:kN-m

M O D A L   L O A D   P A R T I C I P A T I O N   R A T I O S
(STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

TYPE          NAME             STATIC     DYNAMIC

Load          DEAD             0.0000      0.0000
Load          LIVE             0.0000      0.0000
Accel         UX             100.0000    100.0000
Accel         UY             100.0000    100.0000
Accel         UZ               0.0000      0.0000
Accel         RX             100.0000    100.0000
Accel         RY             100.0000    100.0000
Accel         RZ             100.0000    100.0000


ETABS v9.0.0  File:WAFFLE W4EXPERIMENT  Units:kN-m


TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

LOAD   FX         FY         FZ         MX         MY         MZ

DEAD   0.00       0.00       2.45       0.446      -0.498     0.000
LIVE   0.00       0.00       1.58       0.287      -0.321     0.000
```

Appendix 37: Result of waffle W5 Experimental (ETABS)

```
ETABS v9.0.0  File:WAFFLE W5EXPERIMENT  Units:kN-m
 PROJECT INFORMATION

  Company Name                      = eng

 ETABS v9.0.0  File:WAFFLE W5EXPERIMENT  Units:kN-m

 S T O R Y   D A T A

 STORY         SIMILAR TO        HEIGHT   ELEVATION

 STORY1        None                0.750      0.750
 BASE          None                           0.000

 ETABS v9.0.0  File:WAFFLE W5EXPERIMENT  Units:kN-m

 S T A T I C   L O A D   C A S E S

 STATIC        CASE         AUTO LAT       SELF WT
 CASE          TYPE         LOAD          MULTIPLIER

 DEAD          DEAD         N/A              0.1000
 LIVE          REDUCE LIVE N/A               0.0000

 ETABS v9.0.0  File:WAFFLE W5EXPERIMENT  Units:kN-m

 M A S S   S O U R C E   D A T A

 MASS          LATERAL      LUMP MASS
 FROM          MASS ONLY    AT STORIES

 Masses        Yes          Yes

 ETABS v9.0.0  File:WAFFLE W5EXPERIMENT  Units:kN-m

 D I A P H R A G M   M A S S   D A T A

 STORY  DIAPHRAGM MASS-X  MASS-Y      MMI        X-M         Y-M

 STORY1  D1        2.2570  2.2570      0.8492      0.668       0.147


 ETABS v9.0.0  File:WAFFLE W5EXPERIMENT  Units:kN-m

 A S S E M B L E D   P O I N T   M A S S E S
 STORY     UX        UY         UZ          RX          RY          RZ

 STORY1 2.749433 2.749433   0.000000    0.000000    0.000000 0.849182
 BASE   0.752728 0.752728   0.000000    0.000000    0.000000 0.000000
 Totals 3.502160 3.502160   0.000000    0.000000    0.000000 0.849182
```

ETABS v9.0.0 File:WAFFLE W5EXPERIMENT Units:kN-m

C E N T E R S   O F   C U M U L A T I V E   M A S S   &   C E N T E R S
O F   R I G I D I T Y

STORY   DIAPHRAGM   /----------CENTRE OF MASS----------//--CENTRE OF
RIGIDITY--/
LEVEL NAME MASS ORDINATE-X ORDINATE-Y ORDINATE-X ORDINATE-Y

STORY1 D1   2.2570   0.668   0.147   0.655   0.169

ETABS v9.0.0 File:WAFFLE W5EXPERIMENT Units:kN-m

M O D A L   P E R I O D S   A N D   F R E Q U E N C I E S

MODE NUMBER	PERIOD (TIME)	FREQUENCY (CYCLES/TIME)	CIRCULAR FREQ (RADIANS/TIME)
Mode 1	0.01392	71.85817	451.49820
Mode 2	0.01073	93.21213	585.66912
Mode 3	0.01045	95.73303	601.50835
Mode 4	0.00074	1342.60301	8435.82349
Mode 5	0.00043	2303.89344	14475.78944
Mode 6	0.00015	6552.63176	41171.39961
Mode 7	0.00000	11613.02352	72966.77872
Mode 8	0.00000	13258.95988	83308.50193
Mode 9	0.00000	22731.23830	142824.58252

ETABS v9.0.0 File:WAFFLE W5EXPERIMENT Units:kN-m

M O D A L   P A R T I C I P A T I N G   M A S S   R A T I O S

MODE NUMBER	X-TRANS %MASS <SUM>	Y-TRANS %MASS <SUM>	Z-TRANS %MASS <SUM>	RX-ROTN %MASS <SUM>	RY-ROTN %MASS <SUM>	RZ-ROTN %MASS <SUM>
Mode 1	0.00 < 0>	100.00 <100>	0.00 < 0>	100.00 <100>	0.00 < 0>	0.00 < 0>
Mode 2	0.07 < 0>	0.00 <100>	0.00 < 0>	0.00 <100>	0.07 < 0>	99.96 <100>
Mode 3	99.93 <100>	0.00 <100>	0.00 < 0>	0.00 <100>	99.93 <100>	0.04 <100>
Mode 4	0.00 <100>	0.00 <100>	0.00 < 0>	0.00 <100>	0.00 <100>	0.00 <100>
Mode 5	0.00 <100>	0.00 <100>	0.00 < 0>	0.00 <100>	0.00 <100>	0.00 <100>
Mode 6	0.00 <100>	0.00 <100>	0.00 < 0>	0.00 <100>	0.00 <100>	0.00 <100>
Mode 7	0.00 <100>	0.00 <100>	0.00 < 0>	0.00 <100>	0.00 <100>	0.00 <100>
Mode 8	0.00 <100>	0.00 <100>	0.00 < 0>	0.00 <100>	0.00 <100>	0.00 <100>
Mode 9	0.00 <100>	0.00 <100>	0.00 < 0>	0.00 <100>	0.00 <100>	0.00 <100>

```
ETABS v9.0.0  File:WAFFLE W5EXPERIMENT  Units:kN-m

M O D A L   L O A D   P A R T I C I P A T I O N   R A T I O S
(STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

TYPE          NAME           STATIC     DYNAMIC

Load          DEAD           0.5812      0.0000
Load          LIVE           0.0041      0.0000
Accel         UX           100.0000    100.0000
Accel         UY           100.0000    100.0000
Accel         UZ             0.0000      0.0000
Accel         RX           100.0000    100.0000
Accel         RY           100.0000    100.0000
Accel         RZ            99.7702    100.0000


ETABS v9.0.0  File:WAFFLE W5EXPERIMENT  Units:kN-m


TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

LOAD  FX         FY         FZ         MX         MY         MZ

DEAD  0.00       0.00       3.59       0.593      -2.353     0.000
LIVE  0.00       0.00       1.72       0.258      -1.129     0.000
```

Appendix 38: Result of waffle W6 Experimental (ETABS)

```
ETABS v9.0.0  File:WAFFLE W6EXPERIMENT  Units:kN-m

 PROJECT INFORMATION

  Company Name                       = eng

 ETABS v9.0.0  File:WAFFLE W6EXPERIMENT  Units:kN-m

 S T O R Y   D A T A

 STORY          SIMILAR TO       HEIGHT   ELEVATION

 STORY1     None                 0.750       0.750
 BASE       None                             0.000

 ETABS v9.0.0  File:WAFFLE W6EXPERIMENT  Units:kN-m

 S T A T I C   L O A D   C A S E S

 STATIC       CASE         AUTO LAT        SELF WT
 CASE         TYPE         LOAD          MULTIPLIER

 DEAD         DEAD         N/A              0.1000
 LIVE         REDUCE LIVE  N/A              0.0000

 ETABS v9.0.0  File:WAFFLE W6EXPERIMENT  Units:kN-m

 M A S S   S O U R C E   D A T A

 MASS         LATERAL      LUMP MASS
 FROM         MASS ONLY    AT STORIES

 Masses       Yes          Yes

 ETABS v9.0.0  File:WAFFLE W6EXPERIMENT  Units:kN-m

 D I A P H R A G M   M A S S   D A T A

 STORY  DIAPHRAGM  MASS-X   MASS-Y      MMI        X-M         Y-M

 STORY1  D1         1.6898   1.6898    0.3085      0.430       0.180



 ETABS v9.0.0  File:WAFFLE W6EXPERIMENT  Units:kN-m

 A S S E M B L E D   P O I N T   M A S S E S

 STORY   UX         UY         UZ         RX          RY          RZ

 STORY1 1.689796   1.689796   0.000000   0.000000    0.000000    0.308515
 BASE   0.752728   0.752728   0.000000   0.000000    0.000000    0.000000
```

```
   Totals 2.442524  2.442524  0.000000  0.000000   0.000000   0.308515



   ETABS v9.0.0  File:WAFFLE W6EXPERIMENT  Units:kN-m


  C E N T E R S   O F   C U M U L A T I V E   M A S S   &   C E N T E R S
O F   R I G I D I T Y

  STORY    DIAPHRAGM  /----------CENTRE OF MASS----------//--CENTRE OF
RIGIDITY--/
  LEVEL   NAME   MASS ORDINATE-X  ORDINATE-Y  ORDINATE-X  ORDINATE-Y

  STORY1  D1    1.6898  0.430      0.180        0.430        0.180



   ETABS v9.0.0  File:WAFFLE W6EXPERIMENT  Units:kN-m

  M O D A L   P E R I O D S   A N D   F R E Q U E N C I E S


  MODE    PERIOD              FREQUENCY            CIRCULAR FREQ
  NUMBER  (TIME)            (CYCLES/TIME)         (RADIANS/TIME)

  Mode 1  0.01037             96.47473              606.16863
  Mode 2  0.00834            119.94595              753.64261
  Mode 3  0.00780            128.27638              805.98424



   ETABS v9.0.0  File:WAFFLE W6EXPERIMENT  Units:kN-m

  M O D A L   P A R T I C I P A T I N G   M A S S   R A T I O S


  MODE   X-TRANS  Y-TRANS  Z-TRANS  RX-ROTN     RY-ROTN      RZ-ROTN
  NUMBER        %MASS <SUM>    %MASS <SUM>    %MASS <SUM>     %MASS
<SUM>    %MASS <SUM>    %MASS <SUM>

 Mode 1  0.00 <  0>   100.00 <100>    0.00 <  0>   100.00 <100>
0.00 <  0>     0.00 <  0>
 Mode 2 100.00 <100>     0.00 <100>    0.00 <  0>     0.00 <100>
100.00 <100>     0.00 <  0>
 Mode 3 0.00 <100>     0.00 <100>     0.00 <  0>     0.00 <100>
0.00 <100>   100.00 <100>

  ETABS v9.0.0  File:WAFFLE W6EXPERIMENT  Units:kN-m

  M O D A L   L O A D   P A R T I C I P A T I O N   R A T I O S
  (STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

  TYPE        NAME            STATIC     DYNAMIC

  Load        DEAD            0.0000      0.0000
  Load        LIVE            0.0000      0.0000
  Accel       UX            100.0000    100.0000
  Accel       UY            100.0000    100.0000
  Accel       UZ              0.0000      0.0000
  Accel       RX            100.0000    100.0000
  Accel       RY            100.0000    100.0000
```

```
Accel        RZ              100.0000    100.0000
```

TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

LOAD	FX	FY	FZ	MX	MY	MZ
DEAD	0.00	0.00	2.70	0.486	-1.160	0.000
LIVE	0.00	0.00	1.11	0.200	-0.477	0.000

Appendix 39: Test results for panel W1

Load (kN)	Deflections (mm)	Crack width
2	0.10	-
4	0.15	-
6	0.23	-
8	0.31	-
10	0.40	-
12	0.49	-
14	0.80	-
16	0.99	-
18	1.19	0.60

Appendix 40 : Test results for panel W2

Load (kN)	Deflections (mm)	Crack width (mm)
1	0.34	-
2	0.71	-
3	0.89	-
4	1.34	-
5	1.51	-
6	1.70	-
7	1.90	-
8	2.70	-
9	3.64	0.35

Appendix 41 : Test results for panel W3

Load (kN)	Deflection (mm)	Crack width (mm)
2	0	-
4	1.11	-
6	1.79	-
8	2.45	-
10	2.99	-
12	3.90	0.34

Appendix 42: Test results for panel W4

Load (kN)	Deflection (mm)	Crack width (mm)
1	1.45	-
2	3.31	-
3	4.69	-
4	5.90	-
5	6.16	-
6	6.60	0.25

Appendix 43: Test results for panel W5

Load (mm)	Deflection (mm)	Crack width (mm)
1	1.44	-
2	3.77	-
3	4.69	-
4	6.05	-
5	7.34	-
6	7.35	-
7	7.72	-
8	8.17	0.35

Appendix 44: Test results for panel W6

Load (kN)	Deflection (mm)	Crack width (mm)
2	0.43	-
4	0.78	-
6	1.22	-
8	1.69	-
10	2.40	-
12	3.29	0.62

Appendix 45: Test results for panel S1

Load (kN)	Deflection (mm)	Crack width (mm)
2	0.73	-
4	1.47	-
6	2.32	-
8	2.86	-
10	3.49	-
12	3.56	1.20

Appendix 46: Test results of panel S2

Load (kN)	Deflection (mm)	Crack width (mm)
1	1.10	-
2	1.58	-
3	2.38	-
4	4.01	-
5	5.49	-
6	7.38	-
7	9.28	0.75

Appendix 47: Test results for panel S3

Load (kN)	Deflection (mm)	Crack width (mm)
2	0.71	-
4	1.89	-
6	3.33	-
8	4.29	-
10	5.08	-
12	6.34	-
14	7.44	0.90

Appendix 48: Test results for panel S4

Load (kN)	Deflection (mm)	Crack width (mm)
1	1.19	-
2	1.99	-
3	2.62	-
4	3.57	-
5	4.82	-
6	6.44	0.70

Appendix 49: Test results for panel S5

Load (kN)	Deflection (mm)	Crack width (mm)
1	2.14	-
2	6.18	-
3	7.52	-
4	9.30	-
5	11.75	-
6	12.18	1.10

Appendix 50: Test results for panel S6

Load (kN)	Deflection (mm)	Crack width (mm)
2	0.14	-
4	0.91	-
6	1.79	-
8	2.82	-
10	3.89	1.30

Appendix 51: Analysis of variance table for W1 and S1

Source of variation	Degree of freedom	Sum of square	Mean square	Variance ratio F
Specimens	1	12.83	12.83	21.93**
Error	13	7.61	0.585	
Total	*14*	*20.44*		

F** >$P_{0.05}$ significant

$P_{0.05}$ = 4.67   (F tabulated)

Appendix 52: Analysis of variance table for W2 and S2

Source of variation	Degree of freedom	Sum of square	Mean square	Variance ratio F
Specimens	1	34.83	34.83	7.01**
Error	14	69.60	4.97	
Total	*15*	*104.43*		

F**>$P_{0.05}$ significant

$P_{0.05}$ = 4.60 (F tabulated)

Appendix 53: Analysis of variance table for W3 and S3

Source of variation	Degree of freedom	Sum of square	Mean square	Variance ratio F
Specimens	1	14.46	14.46	3.637*
Error	11	43.74	3.98	
Total	*12*	*58.20*		

F* < $P_{0.05}$ no significance

$P_{0.05}$= 4.96 (F tabulated)

Appendix 54: Analysis of variance table for W4 and S4

Source of variation	Degree of freedom	Sum of square	Mean square	Variance ratio F
Specimens	1	4.30	4.30	0.591*
Error	14	101.96	7.28	
Total	*15*	*106.26*		

F* < $P_{0.05}$ no significance

$P_{0.05}$= 4.60 (F tabulated)

Appendix 55: Analysis of variance table for W5 and S5

Source of variation	Degree of freedom	Sum of square	Mean square	Variance ratio F
Specimens	1	18.94	18.94	2.076*
Error	12	109.42	9.12	
Total	*13*	*128.36*		

F* < $P_{0.05}$ no significance

$P_{0.05}$= 4.75 (F tabulated)

Appendix 56: Analysis of variance table for W6 and S6

Source of variation	Degree of freedom	Sum of square	Mean square	Variance ratio F
Specimens	1	0.00	0.00	0.000*
Error	10	61.532	6.153	
Total	11	61.532		

F*< $P_{0.05}$ no significance

$P_{0.05}$= 4.96 (F tabulated)

Appendix 57: Correlation analysis for W1 and W3

		W1	W3
W1	Pearson Correlation	1	.984**
	Sig. (2-tailed)	.	.000
	N	9	6
W3	Pearson Correlation	.984**	1
	Sig. (2-tailed)	.000	.
	N	6	6

** Correlation is significant at 0.01 levels

Appendix 58: Correlation analysis for W2 and W4

		W2	W4
W2	Pearson Correlation	1	.988**
	Sig. (2-tailed)	.	.000
	N	9	8
W4	Pearson Correlation	.988**	1
	Sig. (2-tailed)	.000	.
	N	8	8

** Correlation is significant at 0.01 levels

Appendix 59: Correlation analysis for W5 and W6

		W5	W6
W1	Pearson Correlation	1	.919**
	Sig. (2-tailed)	.	.003
	N	8	7
W3	Pearson Correlation	.919**	1
	Sig. (2-tailed)	.003	.
	N	7	7

** Correlation is significant at 0.01 levels

Appendix 60: Correlation analysis for S1 and S3

		S1	S3
W1	Pearson Correlation	1	.983**
	Sig. (2-tailed)	.	.000
	N	6	6
W3	Pearson Correlation	.983**	1
	Sig. (2-tailed)	.000	.
	N	6	6

** Correlation is significant at 0.01 levels

Appendix 61: Correlation analysis for S2 and S4

		S2	S4
W1	Pearson Correlation	1	.997**
	Sig. (2-tailed)	.	.000
	N	7	7
W3	Pearson Correlation	.997**	1
	Sig. (2-tailed)	.000	.000
	N	7	8

** Correlation is significant at 0.01 levels

Appendix 62: Correlation analysis for S5 and S6

		S5	S6
W1	Pearson Correlation	1	.974**
	Sig. (2-tailed)	.	.005
	N	6	5
W3	Pearson Correlation	.974**	1
	Sig. (2-tailed)	.005	.
	N	5	5

** Correlation is significant at 0.01 levels

Appendix 63: Estimation of flexural crack width at service for Slab S1

Modulus of elasticity of steel $E_s = 200$ kN/mm².

Calculate neutral axis position and steel stress.

Instantaneous modulus of elasticity $= 20$ kN/mm²

$\quad E_c = 20/2 = 10$ kN/mm²

The neutral axis position is giving by

$$\frac{1}{2}bx^2 + \frac{E_s}{E_c}A_s x - \frac{E_s}{E_s}A_s d = 0$$

In this case A_s = area of distribution bars in the slab = 377mm²

Effective depth, d $= 40 - (5+3) = 32$ mm

Thus $\qquad \frac{1}{2} \times 1000 x^2 + \frac{200}{10} \times 377 \times x - \frac{200}{10} \times 377 \times 32 = 0$

$$x = -\frac{15 \pm \sqrt{15^2 + 4 \times 483}}{2} = 15.71$$

The neutral axis   X = 15.71 mm

The stress in the reinforcement at the crack section,

$$f_s = \frac{M}{(d - x/3)A_s} = \frac{3.684 \times 10^6}{(32 - 4.8)377} = 359 \text{ N/mm}^2$$

$$\varepsilon_1 = \frac{y}{(d-x)} \cdot \frac{f_s}{E_s} = \frac{y}{17.7} \cdot \frac{359}{200 \times 10^3} = y101 \times 10^{-6}$$

y is the distance from the neutral axis to the bottom of the slab = 25.7mm

$\varepsilon_1 = 25.7 \times 101 \times 10^{-6} = 2.58 \times 10^{-3}$

ε_m = the average concrete strain,

$$\varepsilon_m = \varepsilon_1 - \frac{b_t(h-x)(a'-x)}{3E_s A_s(d-x)} = (3.31 - \left(\frac{1000(40-25.7)(40-25.7)}{3 \times 200 \times 377 \times 17.7}\right)) \times 10^{-3} = 2.529 \times 10^{-3}$$

$$w_{max} = \frac{3a_{cr}\varepsilon_m}{1 + 2\left(\frac{a_{cr} - c_{min}}{h - x}\right)} = \frac{3 \times 4.07 \times 2.529 \times 10^{-3}}{1 + 2\left(\frac{4.07 - 5}{14.3}\right)} = 0.035mm$$

The estimated maximum crack width at service = 0.035 mm.

Appendix 64: Analysis of variance table of crack width for Waffle and Solid slabs

Source of variation	Degree of freedom	Sum of square	Mean square	Variance ratio F
Specimens	1	0.770	0.770	15.46*
Error	10	0.498	0.049	
Total	*11*	*1.268*		

$F^* > P_{0.05}$ there is significance difference

$P_{0.05}$= 4.96 (F tabulated)