Science Focus Vol.12 (2) 2007 pp 122- 129
Faculty of Pure and Applied Sciences, LAUTECH, Printed in Nigeria

COMPARATIVE PERFORMANCE OF THE LIMITED INFORMATION TECHNIQUES IN A TWO-EQUATION STRUCTURAL MODEL

A.A. ADEPOJU
Department of Statistics,
University of Ibadan, Ibadan, Nigeria.
pojuday@yahoo.com

Abstract

The samples with which we deal in practice are rather small, seldom exceeding 80 observations and frequently much smaller. Thus, it is of great interest to inquire into the properties of estimators for the typical sample sizes encountered in practice. The performances of three simultaneous estimation methods using a model consisting of a mixture of an identified and over identified equations with correlated error terms are compared. The results of the Monte Carlo study revealed that the Two Stage Least Squares (2SLS) and the Limited Information Maximum Likelihood (LIML) estimates are similar and in most cases identical in respect of the just-identified equation. The Total Absolute Biases (TAB) of 2SLS and LIML revealed asymptotic behavior under (upper triangular matrix), P_1, while those of Ordinary Least Squares (OLS) exhibited no such behavior. For both upper and lower triangular matrices (P_1 and P_2), 2SLS estimates showed asymptotic behavior in the middle interval. The OLS is the only stable estimator with a stable behavior of Root Mean Square Error (RMSE) as its estimates increase (decrease) consistently for equation 1 (equation 2) for P_1 (for P_2).

Keywords: Monte Carlo, Identification, Mutual Correlation, Estimator, Random Deviates.

1.0 Introduction

Experience has shown that while large sample properties of estimators can be established, small sample properties typically cannot. The results of the analytical method in the finite-sample area are invariably very complex and exceedingly difficult to interpret which pose a major problem.

It is frequently the case that comparative conclusions can be drawn from the results only by taking steps which effectively destroy their generality, for example, by substituting unknown parameters with specific values. This is one of the reasons why the experimental approach has proved to be a more fruitful method of obtaining finite-sample results, though the analytical approach has recently progressed at a remarkable rate. The alternative experimental approach to the analytical approach is used in this work and is presented in the following section. It is also of interest to determine whether OLS in small samples is sufficiently inferior to simultaneous equation techniques as to warrant its complete exclusion from serious consideration as an estimation procedure for structural parameters.

A detailed survey and review of finite-sample analytical work is given by Basmann [2]. Mariano and McDonald [6] considered the 2SLS and LIML estimators in the just identified case, while Holly and Phillips [4] used an asymptotic expansion to approximate the distribution of the 2SLS estimator. Recently, the "typical" specification of serial independence of the errors in simultaneous equations models has been extended to include the possibility of auto correlated errors. The presence of autocorrelation in the structural disturbances leaves unchanged the ranking of the system estimators established under textbook assumptions, and appears to have little effect on their bias Cragg [3]. Most studies on simultaneous equation models are based on exactly identified equations; however, in real life situations most simultaneous equation models are a mixture of exactly identified and over identified equations Adejumobi [1]. Olubusoye [7] examined a problem using a just identified two-equation model.

2.0 Simulation Procedure

In econometrics, while asymptotic properties of estimators obtained by using various econometric techniques are deduced from postulates, small sample properties of such estimators have always been studied from simulated data using an analytical approach known as Monte Carlo studies. Monte Carlo methods constitute a fascinating, exacting and often indispensable craft with a range of applications that is already very wide yet far from fully explored. The Monte Carlo method provides heuristic solutions to a variety of mathematical problems by performing statistical sampling experiments on a computer. Monte Carlo approach is a useful tool in dealing with the problems of multicollinearity, non spherical disturbances and measurement errors which are problems associated with the data we deal with in practice. Furthermore, it is impossible to obtain real world samples in which the exogenous variables are held constant, hence we are forced to use artificial models and generate through them, artificial data.

Consider a two-equation structural model;

$$Y_{1t} = \beta_{12}Y_{2t} + \gamma_{11}X_{1t} + U_{1t}$$
$$Y_{2t} = \beta_{21}Y_{1t} + \gamma_{22}X_{2t} + \gamma_{23}X_{3t} + U_{2t} \qquad (2.0)$$

where the Y's, X's are the endogenous and the predetermined variables respectively, U's are the random disturbance terms and the β's and γ's are the parameters.

The model (2.0) can be written as

$$Y_{1t} - \beta_{12}Y_{2t} = \gamma_{11}X_{1t} + 0X_{2t} + 0X_{3t} + U_{1t}$$
$$-\beta_{21}Y_{1t} + Y_{2t} = 0X_{1t} + \gamma_{22}X_{2t} + \gamma_{23}X_{3t} + U_{2t}$$

Hence,

$$\begin{bmatrix} 1 & -\beta_{12} \\ -\beta_{21} & 1 \end{bmatrix}\begin{bmatrix} Y_{1t} \\ Y_{2t} \end{bmatrix} = \begin{bmatrix} \gamma_{11} & 0 & 0 \\ 0 & \gamma_{22} & \gamma_{23} \end{bmatrix}\begin{bmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \end{bmatrix} + \begin{bmatrix} U_{1t} \\ U_{2t} \end{bmatrix}$$

Therefore, the reduced form model is

$$By = \Gamma X + U$$
$$y = B^{-1}\Gamma X + B^{-1}U$$
$$= \Pi X + V,$$

where

$$\Pi = -B^{-1}\Gamma$$
$$= \frac{1}{1-\beta_{12}\beta_{21}}\begin{bmatrix} \gamma_{11} & \beta_{21}\gamma_{22} & \beta_{21}\gamma_{23} \\ \beta_{12}\gamma_{11} & \gamma_{22} & \gamma_{23} \end{bmatrix}$$

$$B^{-1}\Gamma X = \frac{1}{1-\beta_{12}\beta_{21}}\begin{bmatrix} \gamma_{11} & \beta_{21}\gamma_{22} & \beta_{21}\gamma_{23} \\ \beta_{12}\gamma_{11} & \gamma_{22} & \gamma_{23} \end{bmatrix}\begin{bmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \end{bmatrix}$$

$$= \frac{1}{1-\beta_{12}\beta_{21}}\begin{bmatrix} \gamma_{11}X_{1t} + \beta_{21}\gamma_{22}X_{2t} + \beta_{21}\gamma_{23}X_{3t} \\ \beta_{12}\gamma_{11}X_{1t} + \gamma_{22}X_{2t} + \gamma_{23}X_{3t} \end{bmatrix}$$

But,

$$V = B^{-1}U$$

$$= \frac{1}{1-\beta_{12}\beta_{21}}\begin{bmatrix} 1 & \beta_{21} \\ \beta_{12} & 1 \end{bmatrix}\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

Hence,

$$y_{1t} = \frac{\gamma_{11}}{1-\beta_{12}\beta_{21}}X_{1t} + \frac{\beta_{21}\gamma_{22}}{1-\beta_{12}\beta_{21}}X_{2t} + \frac{\beta_{21}\gamma_{23}}{1-\beta_{12}\beta_{21}}X_{3t} + \frac{1}{1-\beta_{12}\beta_{21}}u_{1t} + \frac{1}{1-\beta_{12}\beta_{21}}u_{2t} \qquad \ldots (2.1)$$

$$y_{2t} = \frac{\beta_{12}\gamma_{11}}{1-\beta_{12}\beta_{21}}X_{1t} + \frac{\gamma_{22}}{1-\beta_{12}\beta_{21}}X_{2t} + \frac{\gamma_{23}}{1-\beta_{12}\beta_{21}}X_{3t} + \frac{\beta_{12}}{1-\beta_{12}\beta_{21}}u_{1t} + \frac{1}{1-\beta_{12}\beta_{21}}u_{2t} \qquad \ldots (2.2)$$

The reduced form of equations (2.1) and (2.2) are

$$y_{1t} = \Pi_{11}X_{1t} + \Pi_{12}X_{2t} + \Pi_{13}X_{3t} + V_{1t} \qquad (2.3)$$
$$y_{2t} = \Pi_{21}X_{1t} + \Pi_{22}X_{2t} + \Pi_{23}X_{3t} + V_{2t} \qquad (2.4)$$

We wish to study the comparative performance of OLS limited information techniques (2SLS and LIML) in the context of the above two-equation model.

As mentioned earlier, this work uses Monte Carlo approach. The sequence of vectors $\{(x_{1t}, x_{2t}, x_{3t}) : t = 1,2,....T\}$ was generated. The random series were obtained through a standard "random

number generator" computer program using values from Uniform (0, 1) distribution by Kmenta[3] (1971). Since the error terms are to be $N(0,\Sigma)$ and intertemporally independent, we obtained from a standard computer program, two mutually independent $N(0,1)$ sequences $\{(\varepsilon_{1t}, \varepsilon_{2t}) : t = 1,2,...,T\}$. These standardized normal deviates are then screened pair - wise into three categories of;

(i) Relatively highly negatively correlated $(r_{\varepsilon_{1t},\varepsilon_{2t}} \le -0.05)$

(ii) Feebly negatively or positively correlated $(-0.05 < r_{\varepsilon_{1t},\varepsilon_{2t}} < +0.05)$

(iii) Relatively highly positively correlated $(r_{\varepsilon_{1t},\varepsilon_{2t}} \ge +0.05)$

The error terms are then transformed to be distributed as $N(0,\Sigma)$, where Σ is non-singular, it can be shown that $\Sigma = \Omega \otimes I_T$ and can be decomposed by a non-singular P_1 into P_1 and P_1' such that;

$$\Omega = P_1 P_1'$$

Since M independent series of standard normal deviates of length T can be transformed into M series of random normal variables with zero means and predetermined covariance matrix, and here M=2, if we let the predetermined covariance matrix be

$$\Omega = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix},$$

where $\text{var}(u_1) = \sigma_{11}$ and $\text{var}(u_2) = \sigma_{22}$, $\text{cov}(u_1, u_3) = \sigma_{12}$

Since

$$\Omega = P_1 P_1', \qquad (2.5)$$

where

$$P_1 = \begin{bmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{bmatrix} \qquad (2.6)$$

Focusing on the upper triangular matrix

$$\Omega = P_1 P_1' = \begin{bmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{bmatrix} \begin{bmatrix} S_{11} & 0 \\ S_{12} & S_{22} \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix}$$

or

$$S_{11}^2 + S_{12}^2 = \sigma_{11} \qquad (1)$$

$$S_{12} S_{22} = \sigma_{12} \qquad (2)$$

$$S_{22} S_{12} = \sigma_{12} \qquad (3)$$

$$S_{22}^2 = \sigma_{22} \qquad (4)$$

From (4) $\quad S_{22} = \sqrt{\sigma_{22}} \qquad (5)$

From (5) and (2) $\quad S_{12} = \sigma_{12}/S_{22} \qquad (6)$

From (1) and (6) $\quad S_{11} = \sqrt{\sigma_{11} - S_{12}^2} \qquad (7)$

Hence the pair of random disturbances is obtained as follows

$$\left.\begin{aligned} u_{1t} &= s_{11}\varepsilon_{1t} + s_{12}\varepsilon_{2t} \\ u_{2t} &= s_{22}\varepsilon_{2t} \end{aligned}\right\} \quad t = 1,2,...,T \qquad (2.7)$$

An alternative solution is obtained by using the lower triangular matrix,

i.e. $\quad \Omega = P_2 P_2'$,

where $\quad P_2 = \begin{bmatrix} S_{11} & 0 \\ S_{12} & S_{22} \end{bmatrix}$

and going through the above procedure, we obtained a new pair of random disturbances;

$$\left.\begin{aligned} u_{1t}^{*} &= S_{11}\varepsilon_{1t} \\ u_{2t}^{*} &= S_{12}\varepsilon_{1t} + S_{22}\varepsilon_{2t} \end{aligned}\right\} \; t = 1,2,\ldots,T \qquad (2.8)$$

At this stage, the procedures for deriving all the component parts of equations (2.1) and (2.2) which are used in generating stochastic endogenous variables are in place.
Assigning specific values to the structural parameters and the variance-co variance matrix

$$\beta_{12} = 1.5, \beta_{21} = 1.8, \gamma_{11} = 1.2, \gamma_{22} = 0.5 \, and \, \gamma_{23} = 2.0$$

and

$$\Omega = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 5.0 & 2.5 \\ 2.5 & 3.0 \end{bmatrix}$$

such that,

$$B = \begin{bmatrix} 1 & -\beta_{12} \\ -\beta_{21} & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1.5 \\ -1.5 & 1 \end{bmatrix}$$

and

$$\Gamma = \begin{bmatrix} \gamma_{11} & 0 & 0 \\ 0 & \gamma_{22} & \gamma_{23} \end{bmatrix} = \begin{bmatrix} 1.2 & 0 & 0 \\ 0 & 0.5 & 2.0 \end{bmatrix}$$

and the reduced form of the system is
$Y = B^{-1} \gamma x + B^{-1} U$

Since all the quantities in the right-hand are known, the vectors $\{(y_{1t}, y_{2t}) : t = 1, 2, \ldots, T\}$ can easily be generated. The data at our disposal $\{(y_{1t}, y_{2t}, x_{1t}, x_{2t}, x_{3t}) : t = 1, 2, \ldots, T\}$ have been generated with fixed parameter values and error terms that are 'intertemporally independent' and distributed as $N(0, \Sigma)$, the matrix Σ being known.
The next step in utilizing each data set obtained for each sample size and replication is the computation of the estimates of the structural parameters B and Γ using OLS, 2SLS and LIML.

Then the equation which is used in generating Y's becomes
Y=

$$\begin{pmatrix} Y_{1t} \\ Y_{2t} \end{pmatrix} = \begin{pmatrix} 1 & -1.5 \\ -1.8 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1.2 & 0 & 0 \\ 0 & 0.5 & 2.0 \end{pmatrix} \begin{pmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \end{pmatrix} + \begin{pmatrix} 1 & -1.5 \\ -1.8 & 1 \end{pmatrix}$$

3.0 Computations

Computations were carried out using the Time Series Processor (TSP 5.0), an econometric soft ware package.

In this study, three sample sizes N=15, 25 and 40 were used each replicated 50 times. After estimating the parameters, the robustness of each estimator to the inadvertent correlation of the stochastic terms was examined using; average of estimates, absolute bias of estimates, root mean square error and sum of squared residuals of parameter estimates.

Table 1.0 Summary of Estimators using Average R=50, P_1

Estimator	Level of correlation	EQ1					
		β_{12}(1.5)			γ_{11}(1.2)		
		N=15	N=25	N=40	N=15	N=25	N=40
OLS	r<-0.05	1.067074	1.070771	1.063577	-0.40953	-0.50417	-0.56386
	-0.05<r<0.05	1.033484	1.042028	1.046355	-0.49904	-0.53441	-0.50779
	r>0.05	1.013901	1.02698	1.030384	-0.60413	-0.57562	-0.58862
2SLS	r<-0.05	1.083419	1.296365	1.435163	-0.47362	0.029132	0.230474
	-0.05<r<0.05	1.097185	1.355854	1.43023	-0.49016	0.159154	0.302414
	r>0.05	1.134754	1.229802	1.271803	-0.32936	-0.19936	-0.12357
LIML	r<-0.05	1.263064	1.636993	1.540406	-0.07542	1.132359	0.427006
	-0.05<r<0.05	1.113944	1.012208	1.75393	-0.53225	-0.64484	1.017768
	r>0.05	1.23596	1.623244	1.437296	-0.09066	0.696813	0.200169

Table 1.0 Summary of Estimators using Average R=50, P_1 (continued)

Estimator	Level of correlation	EQ2							
		β_{21} (1.8)			γ_{22} (0.5)			γ_{23} (2.0)	
		N=15	N=25	N=40	N=15	N=25	N=40	N=15	N=25
OLS	r<-0.05	0.895693	0.901183	0.897693	-0.00314	0.080167	0.136797	0.50835	0.488816
	-0.05<r<0.05	0.93639	0.935993	0.92698	0.032062	-0.01435	-0.00525	0.61272	0.624402
	r>0.05	0.948343	0.961693	0.94834	0.059402	-0.00299	0.261901	0.638427	0.753
2SLS	r<-0.05	0.913757	1.044426	1.050831	0.108649	-0.01293	0.453827	0.438724	1.032477
	-0.05<r<0.05	0.93599	0.86421	0.92209	0.07961	0.265288	0.006369	0.511544	0.23652
	r>0.05	0.9748	0.965944	1.02053	-0.36868	-0.07042	0.328487	1.182266	0.772508
LIML	r<-0.05	0.913757	1.044278	1.050831	0.109169	-0.01293	0.453227	0.438724	1.032477
	-0.05<r<0.05	0.93593	0.86381	0.92209	0.079712	0.266248	0.006367	0.570557	0.236394
	r>0.05	1.013139	0.965944	1.01873	-0.38871	-0.07042	0.328487	1.381739	0.772508

Table 2.0 Summary of Estimators using Average R=50, P_2

Estimator	Level of correlation	EQ1					
		β_{12} (1.5)			γ_{11} (1.2)		
		N=15	N=25	N=40	N=15	N=25	N=40
OLS	r<-0.05	0.992898	1.007177	1.005596	-0.58899	-0.65394	-0.61268
	-0.05<r<0.05	1.03872	1.043855	1.046446	-0.50066	-0.53163	-0.57261
	r>0.05	1.0644	1.071153	1.077282	-0.45167	-0.50358	-0.47483
2SLS	r<-0.05	1.138784	1.247508	1.380428	-0.39606	-0.12576	0.196838
	-0.05<r<0.05	1.091362	1.272043	1.405338	-0.42857	-0.08312	0.210843
	r>0.05	1.248948	1.481571	1.433504	-0.04207	0.460571	0.307667
LIML	r<-0.05	1.240382	1.605545	1.452427	-0.21405	0.783268	0.328469
	-0.05<r<0.05	0.698358	1.41948	1.667311	-1.48012	0.300124	0.789091
	r>0.05	1.314913	1.597906	1.628381	-0.15225	0.74354	0.860413

Table 2.0 Summary of Estimators using Average R=50, P_2 (continued)

Estimator	Level of correlation	EQ2							
		β_{21} (1.8)			γ_{22} (0.5)			γ_{23} (2.0)	
		N=15	N=25	N=40	N=15	N=25	N=40	N=15	N=25
OLS	r<-0.05	0.96792	0.96679	0.919973	0.07633	0.071831	0.137499	0.693704	0.701117
	-0.05<r<0.05	0.936266	0.938178	0.923779	0.062311	0.011683	0.131808	0.606942	0.67296
	r>0.05	0.907646	0.91977	0.913413	-0.02132	-0.04718	0.10778	0.571083	0.663349
2SLS	r<-0.05	1.042884	1.090812	0.950122	0.046108	0.385726	0.074536	0.960521	0.899887
	-0.05<r<0.05	0.995573	0.765274	1.009423	-0.08116	-0.06946	0.078527	1.614764	0.263587
	r>0.05	1.14819	0.870026	0.903442	0.142534	-0.22502	0.164284	1.018524	0.651609
LIML	r<-0.05	1.042999	1.090812	0.94256	0.040113	0.385726	0.074535	0.960521	0.899887
	-0.05<r<0.05	1.179751	0.765274	1.009423	-0.08116	-0.06948	0.078527	1.614764	0.263587
	r>0.05	1.14837	0.870026	0.903442	0.142534	-0.22502	0.164284	1.018164	0.651609

Table 3.0 Summary of Total Absolute Bias R=50, P_1

Level of correlation	OLS			2SLS			LIML	
	N=15	N=25	N=40	N=15	N=25	N=40	N=15	N=25
R<-0.05	4.941557	4.961139	5.058924	4.929073	3.610526	3.220135	4.350709	2.440805
-0.05<r<0.05	4.88493	4.946335	4.946682	4.865835	4.118974	3.748477	4.832112	5.266177
r>0.05	4.944047	4.836939	4.936597	4.40622	4.301533	3.962273	3.848532	3.258402

Table 4.0 Summary of Total Absolute Bias R=50, P_2

Level of correlation	OLS			2SLS			LIML		
	N=15	N=25	N=40	N=15	N=25	N=40	N=15	N=25	N=40
R<-0.05	4.858138	4.907023	5.050227	4.207758	3.50183	3.837962	4.230036	2.445852	3.641911
-0.05<r<0.05	4.856425	4.864958	4.991839	3.808026	4.851684	3.506159	5.068027	4.321039	3.000559
r>0.05	4.929852	4.896483	4.914466	3.48387	3.761242	3.839642	3.528268	3.55775	3.348781

Table 5.0 Summary of Estimators using Root Mean Square Error R=50, P_1

Estimator	Level of correlation	EQ1					
		β_{12}(1.5)			γ_{11}(1.2)		
		N=15	N=25	N=40	N=15	N=25	N=40
OLS	r<-0.05	0.436156	0.431532	0.437016	1.62725	1.717513	1.766868
	-0.05<r<0.05	0.467685	0.458839	0.454077	1.712024	1.742782	1.710205
	r>0.05	0.491549	0.47391	0.469965	1.8375	1.78329	1.791164
2SLS	r<-0.05	0.809737	0.342553	0.546158	2.432383	1.397053	1.632861
	-0.05<r<0.05	0.60476	0.584134	0.478114	1.981421	1.89306	1.422934
	r>0.05	0.601253	0.609297	0.471323	1.944759	2.068539	1.641591
LIML	r<-0.05	1.289207	2.257132	1.21125	3.556873	7.0543	3.240905
	-0.05<r<0.05	1.363863	3.018664	1.82203	4.056048	7.426581	4.09952
	r>0.05	1.517892	1.770641	1.08149	3.985818	4.528709	2.666646

Table 5.0 Summary of Estimators using Root Mean Square Error R=50, P_1 (continued)

Estimator	Level of correlation	EQ2								
		β_{21}(1.8)			γ_{22}(0.5)			γ_{23}(2.0)		
		N=15	N=25	N=40	N=15	N=25	N=40	N=15	N=25	N=40
OLS	r<-0.05	0.905521	0.89962	0.902529	0.701269	0.547223	0.433591	1.576979	1.542993	1.611507
	-0.05<r<0.05	0.863759	0.864116	0.873127	0.622535	1.606007	0.560126	1.462256	1.436591	1.428301
	r>0.05	0.857112	0.838529	0.85181	0.584447	0.577407	0.326965	1.484068	1.279606	1.60844
2SLS	r<-0.05	1.132223	0.953103	0.996496	1.260255	1.249345	1.043329	2.726721	2.178451	2.013747
	-0.05<r<0.05	1.011447	1.500281	1.071224	1.316546	1.573114	0.8141	2.612439	3.512402	2.196302
	r>0.05	1.299193	1.048964	0.961267	2.43981	1.127184	1.235409	4.393985	2.256035	1.870449
LIML	r<-0.05	1.132223	0.95321	0.996496	1.260009	1.249345	1.043022	2.726721	2.178451	2.013755
	-0.05<r<0.05	1.01154	1.50064	1.071224	1.316484	1.57258	0.814102	2.578634	3.512839	2.196302
	r>0.05	1.2449	1.048964	0.962872	2.453764	1.127184	1.235409	4.048931	2.256035	1.870449

Table 6.0 Summary of Estimators using Root Mean Square Error R=50, P_2

Estimator	Level of correlation	EQ1					
		β_{12}(1.5)			γ_{11}(1.2)		
		N=15	N=25	N=40	N=15	N=25	N=40
OLS	r<-0.05	0.510122	0.494626	0.501262	1.81861	1.862027	1.81824
	-0.05<r<0.05	0.46242	0.456944	0.454163	1.714671	1.738476	1.776118
	r>0.05	0.440366	0.430319	0.423331	1.667882	1.710873	1.678407
2SLS	r<-0.05	0.605268	0.660274	0.741436	2.121374	2.186771	2.136287
	-0.05<r<0.05	1.218544	0.61113	0.473233	3.623711	2.063201	1.468571
	r>0.05	0.57849	0.518955	0.375886	1.814418	1.600259	1.265556
LIML	r<-0.05	1.692143	1.13838	1.242624	5.138109	3.392945	3.304945
	-0.05<r<0.05	3.508064	1.625238	1.057219	8.986473	4.460552	2.399121
	r>0.05	3.046523	0936647	0.778139	7.653294	2.389923	2.186194

Table 6.0 Summary of Estimators using Root Mean Square Error R=50, P_2 (continued)

Estimator	Level of correlation	EQ2								
		β_{21} (1.8)			γ_{22} (0.5)			γ_{21} (2.0)		
		N=15	N=25	N=40	N=15	N=25	N=40	N=15	N=25	N=40
OLS	r<-0.05	0.832829	0.833769	0.899442	0.637005	0.57612	0.459278	1.373695	1.354464	[illegible]
	-0.05<r<0.05	0.86398	0.861902	0.876361	0.57803	0.571873	0.418968	1.442819	1.360709	[illegible]
	r>0.05	0.894405	0.880426	0.886701	0.642272	0.606852	0.435069	1.514568	1.362601	[illegible]
2SLS	r<-0.05	1.026163	1.406742	1.085789	1.148885	1.988452	1.088622	2.539541	2.840852	[illegible]
	-0.05<r<0.05	1.260182	1.744164	1.009576	2.401548	2.944427	1.167258	5.618237	2.997798	[illegible]
	r>0.05	1.702187	1.163565	1.058411	1.779497	1.352837	0.673501	3.294118	2.153446	[illegible]
LIML	r<-0.05	1.026113	1.406742	1.094484	1.55008	1.988452	1.088622	2.539541	2.840852	[illegible]
	-0.05<r<0.05	1.607053	1.744164	1.009576	2.401548	2.944436	1.167258	5.618237	2.997809	[illegible]
	r>0.05	1.70207	1.163565	1.058411	1.779497	1.352837	0.673501	3.294261	2.153446	[illegible]

Table 7.0 Summary of Sum of Squared Residuals for Three Correlation Levels R=50, P_1

Estimator	Level of correlation	EQ1			EQ2		
		N=15	N=25	N=40	N=15	N=25	N=40
OLS	r<-0.05	8.519252	14.19056	22.9875	5.652294	9.582751	16.77818
	-0.05<r<0.05	7.872591	13.48913	22.01066	5.293851	9.039257	15.69381
	r>0.05	7.976691	11.86255	19.50978	5.506655	7.679018	14.58404
2SLS	r<-0.05	52.23134	31.7618	129.0425	53.20173	69.03816	160.5889
	-0.05<r<0.05	28.26443	89.056	134.4355	34.25902	287.9793	135.8645
	r>0.05	37.30773	81.22743	96.78534	98.89281	95.32503	122.4366
LIML	r<-0.05	158.917	864.4592	464.3107	53.20173	69.03816	160.5889
	-0.05<r<0.05	191.4413	1699.326	1124.543	34.25903	287.9793	135.8645
	r>0.05	300.5057	763.9346	470.4372	98.89281	95.32503	122.4366

Table8.0 Summary of Sum of Squared Residuals for Three Correlation Levels R=50, P_2

Estimator	Level of correlation	EQ1			EQ2		
		N=15	N=25	N=40	N=15	N=25	N=40
OLS	r<-0.05	8.138315	14.0019	21.92372	5.932354	9.971365	16.92984
	-0.05<r<0.05	7.629978	13.73168	21.76534	5.294499	9.180186	16.11784
	r>0.05	8.45091	11.76171	19.15493	5.629083	7.617836	13.30717
2SLS	r<-0.05	31.65482	83.26091	203.0776	53.74694	257.6117	150.7368
	-0.05<r<0.05	146.3053	79.31666	124.7349	259.9118	388.9549	148.6921
	r>0.05	46.27148	100.0351	111.6931	331.2791	120.0217	130.6516
LIML	r<-0.05	263.5214	272.0442	18856.09	53.74694	257.6117	150.7368
	-0.05<r<0.05	1282.805	508.3275	461.4283	259.9118	388.9549	148.6921
	r>0.05	1139.261	227.3939	270.9071	331.2791	120.0217	130.6516

4.0 Results

Tables 1-8 summarize the relative performance of three estimators using average, total absolute bias (TAB), root mean square error (RMSE), and sum of squared residuals (RSS) of parameter estimates. Tables 1-2 reveal that the parameters of the over-identified equation are under-estimated in 93 percent of the cases while those of the just- identified equation are under-estimated in 98 percent of the cases at the same level for triangular matrices, upper (P_1) and lower (P_2). For tables 3-4, the estimates of 2SLS decreased asymptotically for P_1. Tables 5-6 show that, OLS is the only estimator with stable behavior of RMSE as correlation changes through the three critical levels i.e. RMSE increases (decreases) consistently for equation 1 (equation 2) for P_1 (for P_2).

5.0 Conclusions

Findings on 'best' estimates show that the three estimators vary in their relative performance in estimating the parameters of the model depending on the identifiability status of the equation, the correlation status of the error term and the assumed triangular matrix (P_1 or P_2). There is evidence to suggest that there is a greater scope for estimating the just identified equation at the three ranges of the correlation coefficient. Also the OLS appears to be best for estimating the over-identified equation with negatively correlated errors using P_1 and with positively correlated errors using P_2.

References:

[1] **Adejumobi, A. A. (2006).** "Robustness of Simultaneous Estimation Techniques To over-identification and Correlated Random Deviates". PhD Thesis, Unpublished, University of Ibadan.

[2] **Basmann, R. L. (1974).** "Exact Finite Sample Distributions for Some Econometric Estimators and Test Statistics: A Survey and Appraisal," In Frontiers of Quantitative Economics, vol. II Edited by M. D. Intriligator and D. A. Kendrick Amsterdam: North - Holland 209 – 285

[3] **Cragg, J. G. (1967).** "On the Relative Small Sample Properties of Several structural Equation Estimators," Economical, vol. 35, pp 89-110.

[4] **Holly, A. and Phillips, P. (1979).** "A Saddle point Approximation to the distribution of the k-class Estimator of a Coefficient in a Simultaneous System," Econometrica, 47, 1527-1547.

[5] **Kmenta, J. (1971).** "Elements of Econometrics," New York: Macmillan.

[6] **Mariano, R. and Mc Donald, J. (1979).** "A Note on the Distribution Functions of LIML and 2SLS Structural Coefficients in the Exactly Identified Case," Journal of the American Statistical Association, 74, 847 - 848.

[7] **Olubusoye, O. E. (2001).** "Relative Sensitivity of Estimators of Simultaneous equations to violation of Mutual Independence of Random Deviates". PhD Thesis, Unpublished, University of Ibadan