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Robustness of Simultaneous Estimation Methods to Varying Degrees of Correlation Between Pairs of Random Deviates

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Abstract

This study examined how six estimation methods of a simultaneous equation model cope with varying degrees of correlation between pairs of random deviates using the Variance and Total Absolute Bias (TAB). A two-equation simultaneous system was considered with assumed covariance matrix. The model was structured to have a mutual correlation between pairs of random deviates which is a violation of the assumption of mutual independence between pairs of such random deviates. The correlation between the pairs of normal deviates were generated using three scenarios of $r = 0.0$, 0.3 and 0.5. The performances of various estimators considered were examined at various sample sizes, correlation levels and 50 replications. The sample size, $N = 20,25,30$ each replicated 50 times was considered. OLS is performed best when the variance is used to study the finite sample properties of the estimators in that it produces the least variances in all the cases considered and at all sample sizes. All the estimators revealed an asymptotic pattern under CASE I.

Keywords: Monte Carlo, Random Deviates, Mutual Correlation, Total Absolute Bias, Root Mean Square Error.

Introduction

In 1940, sixty nine years had passed since the term “Monte Carlo methods” was coined by Physicists working on nuclear weapons projects in the Los Alamos National Laboratory. Monte Carlo methods have in the last two decades found extensive use in many fields such as operational research, nuclear physics and econometrics to mention but few, where there are a variety and complexity of problems beyond the available resources of the theoretician Adepoju ([1], [3]). For the last two decades several investigations have concerned themselves with the Monte Carlo Methods, notable among them are; [17], [14], [10], [6], [7], [8], [4], [15], [9] and [16].

Monte Carlo simulation is a method of analysis based on artificially recreating a chance process (usually with a computer), running it many times, and directly observing the results.

In Monte Carlo studies, the econometrician generates data sets and stochastic terms which are free of the problems of multi collinearity, non spherical disturbances, measurement error and even specification error. In the context of simultaneous equation system, the design of Monte Carlo experiments requires the generation of orthogonal normal deviates or mutually independent sequences distributed as $N(0,1)$. These normal deviates are then transformed to ensure that the disturbance terms are distributed as $N(0,\Sigma)$ which are not serially correlated, where Σ is the assumed variance-covariance matrix of the disturbances, however, in real life situation, the errors are not completely free of correlation ([2], [5] and [11]). This study therefore, examined the performance of the estimators of two-equation simultaneous model to varying degrees of correlation between pairs of normal deviates.

The rest of this paper is divided into four sections. Section 2 discusses the general framework, section 3 focuses on the generation of sample data, and section 4 presents and discusses the simulation results while the conclusion is presented in section 5.

The General Framework of the study

Simultaneous equation models (SEM), as the name makes clear, the heart of this class of models lies in a data generation process that depends on more than one equation interacting together to produce the observed data.

Unlike the single-equation model in which a dependent (y) variable is a function of independent (x) variables, other y variables are among the independent variables in each SEM equation. The y variables in the system are jointly (or simultaneously) determined by the equations in the system.

We assume the following two structural equations:

$$Y_{t1} = \beta_{21}Y_{t2} + \gamma_{11}X_{t1} + \gamma_{21}X_{t2} + U_{t1}$$
$$Y_{t2} = \beta_{12}Y_{t1} + \gamma_{12}X_{t1} + \gamma_{32}X_{t3} + U_{t2}$$

These equations can be rewritten as follows;

$$-Y_{t1} = \beta_{21}Y_{t2} + \gamma_{11}X_{t1} + \gamma_{21}X_{t2} + U_{t1}$$
$$\beta_{12}Y_{t1} = Y_{t2} + \gamma_{12}X_{t1} + \gamma_{32}X_{t3} + U_{t2}$$

The two equations above are exactly identified.

The reduced form model is derived as;

$$\beta Y = \Gamma X + U$$
$$\Rightarrow Y = \beta^{-1}\Gamma X + \beta^{-1}U \text{ i.e } \pi X + V$$

Where, $\pi = \beta^{-1}\Gamma$

and by extension we obtained the following endogenous equations:

$$Y_{t1} = \frac{1}{1-\beta_{21}\beta_{12}}\left(\gamma_{11}X_{t1} + \gamma_{21}X_{t2} + \beta_{21}\gamma_{12}X_{t1} + \beta_{21}\gamma_{32}X_{t3} + U_{t1} + \beta_{21}U_{t2}\right)$$

$$Y_{t2} = \frac{1}{1-\beta_{21}\beta_{12}}\left(\gamma_{12}X_{t1} + \beta_{12}\gamma_{11}X_{t1} + \beta_{12}\gamma_{21}X_{t2} + \gamma_{32}X_{t3} + \beta_{12}U_{t1} + U_{t2}\right)$$

$$Y_{t1} = \left(\frac{\gamma_{11} + \beta_{21}\gamma_{12}}{1-\beta_{21}\beta_{12}}\right)X_{t1} + \left(\frac{\gamma_{21}}{1-\beta_{21}\beta_{12}}\right)X_{t2} + \left(\frac{\beta_{21}\gamma_{32}}{1-\beta_{21}\beta_{12}}\right)X_{t3} + \left(\frac{U_{t1} + \beta_{21}U_{t2}}{1-\beta_{21}\beta_{12}}\right)$$

$$Y_{t2} = \left(\frac{\beta_{12}\gamma_{11} + \gamma_{12}}{1-\beta_{21}\beta_{12}}\right)X_{t1} + \left(\frac{\beta_{12}\gamma_{21}}{1-\beta_{21}\beta_{12}}\right)X_{t2} + \left(\frac{\gamma_{32}}{1-\beta_{21}\beta_{12}}\right)X_{t3} + \left(\frac{\beta_{12}U_{t1} + U_{t2}}{1-\beta_{21}\beta_{12}}\right)$$

Generation of Monte Carlo Data

We will use Monte Carlo simulation to understand the properties of different statistics computed from sample data. In other words, we will test-drive estimators, figuring out how different recipes perform under different circumstances. Our procedure is quite simple: In each case we will set up an artificial environment in which the values of important parameters and the nature of the chance process are specified; then the computer will run the chance process over and over; finally the computer will display the results of the experiment.

The main task is the generation of stochastic dependent (endogenous) variables $Y_{it}\left(i = 1,2; t = 1,\ldots,T\right)$, which are subsequently used in estimating the parameters of the model.

To achieve this, the following have to be assumed

(i) Values of the predetermined variables X_{1t}, X_{2t}, and $X_{3t}\left(t = 1,\ldots,T\right)$

(ii) Values of the parameters, β_{12}, β_{21}, γ_{11}, γ_{12}, γ_{32}.

(iii) Values of the elements Ω

The simulation of the error term $U_{it}\left(i = 1,2,\ldots,T\right)$ is the most complex step in generating stochastic dependent variables. To set up our Monte Carlo experiment, we proceed as follows.

(i) The sample size N is specified as N= 20, 25, 30

(ii) Numerical values are assigned arbitrarily to each of the structural parameters as follows;

$\beta_{12} = 1.5$, $\beta_{21} = 1.8$, $\gamma_{11} = 1.5$, $\gamma_{11} = 1.5$ $\gamma_{12} = 0.5$, $\gamma_{32} = 2.0$ for all cases

The covariance matrix of the disturbances is specified arbitrarily as follows

$$\Omega = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \begin{pmatrix} 5.0 & 2.5 \\ 2.5 & 3.0 \end{pmatrix}$$

The standard random number generator with values obtained from uniform distribution with mean 0 and standard deviation 1 by [12] is used to generate values of the exogenous variables, $X_{it}\left(i = 1,2,3; t = 1,\ldots,T\right)$.

Generation of Random Disturbance Term, U

A 3-stage process is employed here to generate random disturbance terms. In the first stage, independent series of normal deviates of required length (N=20, 25, 30) are generated. At the second stage, these series were then standardized to have a normal distribution with mean zero and variance 1. Lastly, the random disturbance terms were generated assuming three degrees of correlation between the pairs of random deviates.

Case I: no correlation between the random deviates ($r_{\varepsilon_1,\varepsilon_2} = 0$),

Case II: 0.3 correlation level between the random deviates ($r_{\varepsilon_1,\varepsilon_2} = 0.3$),

Case III: 0.5 correlation level between the random deviates ($r_{\varepsilon_1,\varepsilon_2} = 0.5$).

The samples sizes considered for each scenario are N=20, 25 and 30. The pairs of random normal deviates based on these sample sizes were generated, each replicated 50 times. The deviates were then standardized and appropriately transformed to have a specific variance-covariance matrix Σ assumed in the model. Numerical values were generated for exogenous variables of the model as described above.

Those selected $(\varepsilon_{1t}\varepsilon_{2t})$ are then transformed to be distributed as $N(0,\Sigma)$ where Σ is $Cov(U_t U_t') = \Omega \otimes I_T$ and elements of Ω are decomposed by a non- singular matrix ρ such that

$$\rho\rho' = \Omega$$

Recall, $V = \beta^{-1}U$

$$\begin{pmatrix} V_{t1} \\ V_{t2} \end{pmatrix} = \begin{pmatrix} \beta^* & \beta^*\beta_{21} \\ \beta^*\beta_{12} & \beta^* \end{pmatrix} \begin{pmatrix} U_{t1} \\ U_{t2} \end{pmatrix}$$

According [13], M independent terms of standard normal deviates of length N can be transformed into M series of random normal variables with mean 0 and predetermined covariance matrix. In this model, $M = 2$ i.e. U_{1t}, U_{2t} if the covariance matrix is

$$\Omega = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}$$

Where $\operatorname{var}(U_1) = \sigma_{11}$, $\operatorname{var}(U_2) = \sigma_{22}$ and $\operatorname{cov}(U_1 U_2) = \sigma_{12}$ considering both upper and lower triangular matrices. Let upper triangular matrix be given by

$$P_1 = \begin{pmatrix} \eta_{11} & \eta_{12} \\ 0 & \eta_{22} \end{pmatrix}$$

and lower triangular matrix as

$$P_2 = \begin{pmatrix} \eta_{11} & 0 \\ \eta_{21} & \eta_{22} \end{pmatrix}$$

Then

$$\Omega = P_1 P_1' = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}$$

The pair of standard deviates can be transformed into a pair of random normal variables with mean Z^n variance σ_{11}, σ_{22} and covariance σ_{12} by using

$$\begin{bmatrix} U_{1t} \\ U_{2t} \end{bmatrix} = U_t = \eta_1 \varepsilon_t = \begin{pmatrix} \eta_{11} & \eta_{12} \\ 0 & \eta_{22} \end{pmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

to obtain a pair of random disturbances for the upper triangular matrix:

$$U_{1t} = \eta_{11}\varepsilon_{1t} + \eta_{12}\varepsilon_{2t} = 1.707825128\varepsilon_{1t} + 1.4043$$

$$U_{2t} = \eta_{22}\varepsilon_{2t} = 1.732050808\varepsilon_{2t}$$

where $t = 1,2,...,T$

Similarly, an alternative solution can be obtained for the lower triangular matrix:

$$U'_{1t} = \eta'_{11}\varepsilon_{1t} = 2.236067978\varepsilon_{1t}$$

$$U'_{2t} = \eta'_{12}\varepsilon_{1t} + \eta'_{22}\varepsilon_{2t} = 1.118033989\varepsilon_{1t} + 1.322875656\varepsilon_{2t}$$

Generation of Endogenous Variables

With the numerical values already assigned to the structural parameters, we have all the values required for the generation of the endogenous variables. Considering the upper and lower triangular matrix U_{t1}, U_{t2} defined as;

$$\begin{bmatrix} U_{1t} \\ U_{2t} \end{bmatrix} = \begin{pmatrix} 1.707825128 & 1.443375673 \\ 0 & 1.732050808 \end{pmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

And lower triangular Matrix U'_{1t}, U'_{2t}

$$\begin{bmatrix} U'_{1t} \\ U'_{2t} \end{bmatrix} = \begin{pmatrix} 1.707825128 & 0 \\ 1.443375673 & 1.732050808 \end{pmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

Solving Y_{t1} and Y_{t2} using upper triangular matrix we have;

$$Y_{1t} = -1.411764706X_{t1} - 0.588235294X_{t2} - 2.117647059X_{t3} \; -0.588235294U_{t1} - 0.88235294U_{t2}$$

$$Y_{2t} = -1.411764706X_{t1} - 0.588235294X_{t2} - 2.117647059X_{t3} \; -0.88235294U_{t1} - 0.588235294U_{t2}$$

Solving Y_{t1} and Y_{t2} using lower triangular matrix we have

$$Y_{1t} = -1.411764706X_{t1} - 0.588235294X_{t2} - 2.117647059X_{t3} \; -0.588235294U'_{t1} - 0.88235294U'_{t2}$$

$$Y_{2t} = -1.411764706X_{t1} - 0.588235294X_{t2} - 2.117647059X_{t3} \; -0.88235294U'_{t1} - 0.588235294U'_{t2}$$

Simulation Results

Careful study of Table 1 reveals that OLS houses the smallest TAB in majority of the cases for both equations and across triangular matrices P_1 and P_2 followed by 23LIML estimators.

In most cases, the total absolute bias increases with increase correlation between the random deviates. The total biases of all the estimators for both P_1 and P_2 arising from equation one exceed those of equation two. All the estimators exhibit a consistent pattern for CASE I, when there is no correlation between the random deviates, of decreasing TAB as N increases for both equations and triangular matrices except 23LIML estimators that do not show any consistent pattern.

The TAB decreases as the sample size changes for P_1 and P_2 in case I. The same pattern is noticed in the second and third case.

In Table 2 for upper triangular matrix P_1, and for N = 20, all the estimators except produce variances that increase as the correlation changes across the three cases under consideration except FIML that produced variances that decrease as we move from Case I through II to III. Similar observation is noticed for equation 2 where only OLS gives decreasing values.

For N = 30, all the estimators yield variances that increase with changing correlation levels from 0.0 to 0.3 to 0.5.

However, for N = 25, there is no systematic pattern in the behaviors of the estimators.

Investigating the asymptotic behavior of the estimators under each scenario reveals that for Case I of zero correlation almost all the estimators reveal an asymptotic pattern. However, for the 2^{nd} and 3^{rd} Cases when r = 0.3 and r = 0.5, no such asymptotic behaviors are revealed except for FIML in equation two that revealed an asymptotic behavior.

Also in the same Table 2, for the lower triangular matrix P_2, the behaviors of the estimators are similar to those observed for upper triangular matrix at N = 20. For N = 25 and N = 30, no consistent pattern are noticed. The variances produced by the estimators are minimum at N = 25 in equation two for Cases II and III.

OLS is best when the variance is used to judge the performance of the estimators since it produces the least variances in all the cases considered and at all sample sizes.

Table 1: Perfomances Of Estimators Using Total Absolute Bias

		N = 20			N = 25			N = 30		
P_1	Estimator	I	II	III	I	II	III	I	II	III
EQ1	OLS	3.3760	3.3730	3.4199	3.3741	3.4107	3.3946	3.3724	3.4211	3.4170
	ILS	4.5181	4.4124	4.4797	4.3669	4.6172	4.4894	3.8225	6.1121	4.5708
	23LIML	3.7126	3.3132	3.6537	3.5115	4.4509	4.4509	6.2624	8.0454	3.1665
	FIML	6.8290	5.4856	4.6810	5.6198	5.4444	5.4444	5.0629	6.1430	5.9434
EQ2	OLS	2.9543	2.9455	8.8777	2.9464	2.9169	2.8607	2.9265	2.8906	2.8846
	ILS	4.2106	4.1274	4.1454	4.0548	4.3185	4.1797	4.0531	4.3784	4.2672
	23LIML	3.2050	2.9999	2.9432	2.9529	3.1396	3.1396	2.5441	5.3884	2.7923
	FIML	5.3568	4.5267	4.3447	4.3838	4.4850	4.4850	4.1927	5.6080	5.7476
		N = 20			N = 25			N = 30		
P_2	Estimator	I	II	III	I	II	III	I	II	III
EQ1	OLS	3.4084	3.3298	3.3307	3.3763	3.3512	3.2961	3.3052	3.3722	3.3415
	ILS	4.5154	4.6283	4.4620	4.3562	4.6158	4.5009	4.3334	4.6469	4.6585
	23LIML	2.9670	2.5098	3.5777	3.6065	2.8491	2.8491	8.4369	3.7109	3.6084
	FIML	6.9300	5.9220	5.0014	5.5848	5.2457	5.2457	5.2728	5.3022	5.6120
EQ2	OLS	2.9519	2.9716	2.9809	2.7405	2.9981	2.9653	2.6312	2.9488	2.9771
	ILS	4.0017	4.3325	4.1396	4.0448	4.3035	4.1954	4.0894	4.3166	4.3441
	23LIML	2.5548	2.5059	15.4501	2.5211	2.8224	2.8224	1.6565	2.5316	3.4331
	FIML	4.6817	5.0722	4.5796	4.2567	4.3101	4.3101	4.0238	4.1883	4.8434

Table 2: Perfomances of Estimators Using Variance

		N = 20			N = 25			N = 30		
P_1	Estimator	I	II	III	I	II	III	I	II	III
EQ1	OLS	0.1054	0.1112	0.1857	0.185	0.2356	0.0928	0.2059	0.2205	0.2224
	ILS	0.2385	7.4470	11.0035	7.0050	8.3790	6.1030	9.8757	10.4835	11.5611
	23LIML	0.2651	0.4508	4.1712	5.6919	4.3823	0.1530	0.9880	1.3548	2.1727
	FIML	5.9628	0.9247	0.5237	0.7121	0.4536	0.5872	5.4091	6.2091	7.5431
EQ2	OLS	0.3060	0.2156	0.1506	0.1194	0.1169	0.0958	0.1608	0.1632	0.2063
	ILS	3.0666	8.0353	13.1387	8.0236	9.6987	6.8755	11.101	12.5165	13.4029
	23LIML	0.1431	0.7453	0.8323	1.5454	4.3649	4.4795	2.0624	3.5480	8.3925
	FIML	0.4368	0.3935	0.6943	0.6839	0.6309	0.7557	0.7997	21.7628	22.7202
		N = 20			N = 25			N = 30		
P_2	Estimator	I	II	III	I	II	III	I	II	III
EQ1	OLS	0.0728	0.1202	0.3881	0.1755	0.2373	0.0929	0.1757	0.0642	0.1336
	ILS	0.2017	9.8222	11.0589	7.0025	8.9624	5.7285	10.0756	10.2478	8.6970
	23LIML	6.8917	25.3490	30.8048	8.6678	6.106	18.7943	13.2586	5.6785	4.3195
	FIML	4.4916	5.7443	2.9991	16.5013	0.4703	0.6759	3.2285	1.1838	1.0049
EQ2	OLS	0.2170	0.2032	0.1310	0.7211	1.1063	0.1054	0.8086	0.1488	0.1939
	ILS	0.3766	10.8285	11.4163	8.0465	10.0365	5.7799	10.9420	10.8432	9.1456
	23LIML	0.9232	16.4878	17.3273	25.2765	2.1032	4.4481	11.3215	7.0566	5.6054
	FIML	0.5846	4.4867	5.4418	24.4981	0.5929	0.4772	0.8457	0.8348	0.4576

Conclusion

The finite sampling properties of estimators used in this work are the Total Absolute Bias (TAB) and the Variance. Case I performed better than the other cases considered in respect of the variances of the estimators. OLS happens to be the best estimator since it produces the variances at all levels of correlation and sample sizes. In about 65% of the time, the TAB produced by the lower triangular matrix were better than those produced by the upper triangular matrix.

As the sample size increases from 20 through 25 to 30, the values of the estimators get closer to the true parameters in about 72% of the cases across the upper and lower triangular matrices.

CASE II, where the correlation between the pairs of random deviates is 0.3 has the least proportion of 'best' estimates and hence few 'best' estimators. The most frequent estimator in this interval is the ILS and 23SLIML.

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